# DCMOTODR 

# SPEED CONTROLS 

SHIISMS ORAIS • STOUHOA
ELECTRO. CRAFT CORP.

AN ENGINEERING HANDBOOK BY
ELECTRO-CRAFT CORP

## DC MOTORS

## SPEED CONTROLS

## SERVO SYSTEMS

## AN ENGINEERING HANDBOOK

by


ELECTRO-CRAFT CORPORATION
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## Foreword to the Fifth Edition

Two years have passed since the publication of the expanded fourth edition of this handbook, and the number of copies of all previous editions in the hands of engineers all over the world is now 80,000 .

Electro-Craft continues to grow. We now operate four manufacturing plants in Hopkins and Winnebago, Minnesota, Amery, Wisconsin, and Santa Barbara, California. Today we offer the widest range of DC motors, tachometers, speed and position control systems, and optical encoders. We are the only company in the business that offers in-house capabilities in motors, electronics and encoders, enabling ElectroCraft to offer "system solutions" to a wide variety of customer problems. Our current annualized rate of revenues is approximately $\$ 30$ million.

Since the last edition, we have significantly expanded our product offerings which is
reflected in the appropriate product section of this book. The Brushless Direct Current Motors (now marketed under the registered trademark, BRUDIC ${ }^{\circledR}$ ) are now in volume production.

A major change in this edition is in Chapter 4 which is rewritten and expanded with emphasis on switching amplifiers. The discussion focusses on Pulse Width Modulated (PWM) amplifiers. This chapter now presents a simplified method of analyzing digital systems and includes theory and operation of control amplifiers and specifically microprocessor based systems. In keeping with the rapidly expanding uses for digital control methods from the computer industry to all types of industrial applications, this major revision hopefully will be a benefit to a large number of readers.

## ELECTRO-CRAFT CORPORATION July 1980

## Foreword to the Fourth Edition

Two and a half years have passed since the publication of the expanded third edition of this handbook. Now almost sixty thousand copies are in the hands of engineers all over the world.

Many changes have taken place at ElectroCraft also. Since the third edition was published, the sales of Electro-Craft have doubled. Renco Corporation, a manufacturer of optical encoders in Santa Barbara, California, was acquired. A fourth manufacturing plant was opened in Amery, Wisconsin. Many new products and successful new applications were completed.

Electro-Craft today offers the widest variety of DC Motors and Tachometers,

Speed Control Systems, Servo Amplifiers, Digital Positioning Systems, and Optical Encoders.

The fourth edition has been expanded to a total of 550 pages. An entire chapter has been added describing the technology, application, and implementation of optical position encoders which are increasingly being used as digital position transducers in instrumentation and control systems. The product data has been completely updated and expanded to include our new range of Digital Positioning Systems (DPS).

## ELECTRO-CRAFT CORPORATION

July 1978

## Foreword to the Third Edition

Since its first publication back in 1972 Electro-Craft's engineering handbook, "DC MOTORS - SPEED CONTROLS - SERVO SYSTEMS", has become a standard reference in the field of motor control. In all 35,000 copies have been sold averaging about 1,000 per month. Many of these books are used by professional engineers to provide day to day assistance in their system design work, and many are playing a useful role in colleges and universities helping to train the engineers of the future.

As a result of the continuous feedback received from users of this book, we have prepared a new, third edition, which has been expanded by about 100 pages to a total of little over 500. Apart from the correction of errors this new edition includes the following changes:

Chapter 5 - The application section has been expanded to include. more applications and notes on the application of the
new P6000 series servomotor control systems.

Chapter 6 - An entirely new chapter discussing the new and exciting developments made by Electro-Craft in the field of DC Brushless Motors.

Chapter 7 - Data sheets on Electro-Craft products have been revised, updated, and expanded. The new models now available in the MOTOMATIC range of Speed Controls, Servomotor Controls, and Digital Positioning Systems are described in full.

Once again Electro-Craft Corporation cordially invites your comments and criticisms so that the next edition of this book will remain one of the foremost reference works in its field.

## ELECTRO-CRAFT CORPORATION August 1975

## Foreword to the Second Edition

Since October 1972, when the first edition of this book was published, we have distributed 15000 copies to interested engineers and designers. The overwhelming majority of the comments which we received from our readers have been favorable and complimentary to our effort for creating this book. We have also received some constructive criticism and requests for expanding the treatment and discussion on certain subjects. In preparing this expanded second edition, we have taken into account all these requests which we could accommodate. All the changes and additions enlarge this book by about 30 percent from the first edition to approximately 400 pages.

Listed below are the nature of the various changes and additions by chapter:

Chapter 1 - Recognizing the need for inclusion of the metric system, we revised the Terminology and Symbols sections and have added the Systems of Units section.

Chapter 2 - The treatment of Motor Equations, Transfer Functions, Power Dissipation and Thermal Characteristics has been re written and expanded.

Chapter 3 - Minor revisions.
Chapter 4 - Servo Components treatment has been revised and expanded. The section on Servo Amplifiers has been rewritten, and
a new section Phase-Locked Servo Systems has been added.

Chapter 5 - The Applications section has been expanded, sections of Application Classification and Specification, and Motor Selection Criteria have been added.

Chapter 6 - Data sheets on motors and amplifiers have been revised, updated and expanded. Electro-Craft's new line of servo amplifiers is also presented in this chapter.

Two new Appendices have been added; a description of the SI System of Units (Metric System) and a Units' Conversion Factors and Tables.

It is the continuing interest of the authors and editors of this engineering handbook to make it a most useful, relevant and informative source of knowledge regarding all aspects of DC motors, speed controls and servo systems. To this end we again wish to solicit the constructive comments of our readers. These comments, together with the results of our continually expanding search for valuable, pertinent technical information, will be incorporated into future editions of this book.

## ELECTRO-CRAFT CORPORATION

September 1973

## Foreword to the First Edition

## THE ELECTRO-CRAFT STORY

Electro-Craft is a growing, publicly held corporation started in 1960. For close to ten of these twelve years, the company has specialized in the development and manufacture of fractional horsepower permanent magnet DC motors, generators and control amplifiers.

In the early years, business centered around patented, integrally constructed motor-generators and matching transistorized "Class $A^{\prime \prime}$ control amplifiers, marketed under the MOTOMATIC ${ }^{\circledR}$ trademark. These smaller size, transistorized motor speed control systems found their first applications mostly in office equipment, medical electronics, and instrumentation products.

Over the years, the product line has expanded into larger sizes, higher performance servomotors, low ripple tachometer-generators, and increasingly sophisticated servo control amplifiers to meet the growing demands of our customers.

During the past decade, Electro-Craft has sold over half a million precision DC motors and controls to more than 1000 customers for approximately 300 different applications.

There are many motor manufacturers who do not make matching controllers, and vice versa; Electro-Craft is one of the very few
companies which make both. It is this special "system-oriented" approach, coupled with constant technological innovation, that is perhaps most responsible for the company's success in the field.

## WHY THIS BOOK

Around the turn of this century, AC motors won out over DC machines as the generally accepted sources of electric motive power. The deciding factor then was the state of technology and materials, rather than the inherent characteristics of each device.

Automation and the need for controlled speed and torque, rather than just raw motive power, have brought about a resurgent interest in the use of DC motors. High energy permanent magnets, epoxy resins, improved brushes and other new materials and technologies have also made possible DC machines which are more reliable, smaller and less expensive to produce.

The combination of the above factors produced a great deal of change during the last decade in the variety and performance of DC motors. The greatest innovation took place in the fractional horsepower sizes. The concurrent rapid advances in semiconductor technology, especially in low cost, high power SCR's and transistors, facilitated the emergence of a varietv of matching control
amplifiers, from economy models to the very sophisticated ones.

Due to this fast growth of technology, much of what is available today is not described in the standard textbooks. The purpose of this book is to offer a basic background in DC motors, speed controls and servo systems, and to describe new developments in the field. The presentation is aimed at the designer and user in industry, with emphasis on many practical problems from testing to applications engineering.

The selection of the power drive system for an application is greatly facilitated by proper communication: a specific understanding of the problem between the user and the manufacturer. Another aim of this book is to make this task simpler by attempting to share the accumulated technical knowledge of our company, which has specialized in this field for the past decade.

## HOW TO USE THE BOOK

This book was prepared as a compendium of articles on many related subjects. The editors made no attempt to homogenize the styles of the different authors. An effort was made, however, to make the chapters self-contained and to organize the subject matter in a meaningful way. The
index and the definitions in Chapter 1 should help the reader to decide how much background he needs to cover.

Some of the information presented is standard reference material, and some probably will not be found elsewhere. Some chapters are quite theoretical, while others focus on the more practical "how to" aspects.

This is a handbook which will probably be consulted as a reference as problems arise, rather than being read cover to cover as a student's textbook.

## ABOUT THE AUTHORS

Approximately twenty people contributed to the preparation of this book. They comprise the technical staff of Electro-Craft, with backgrounds in design engineering, research and development, testing, teaching, systems development, sales, application engineering, and - most of all - practical problem solving. The authors collectively have over 200 man-years of experience in DC motor and control technology.

In addition to their regular daily work, the authors gave their time generously over the past year to make this book possible.

## ELECTRO-CRAFT CORPORATION

October 1972

## Chapter 1

## Terminology, Symbols, Systems of Units

### 1.1. TERMINOLOGY

Acceleration, Maximum from Stall - the angular acceleration of an unloaded motor, initially at rest, when the peak armature current $\mathbf{I}_{\mathbf{p k}}$ is applied.

Ambient Temperature - the temperature of the cooling medium immediately surrounding the motor or another device.

Analog-to-Digital Converter - a device for converting a voltage level into a digital word.

Armature Reaction - the production of a magnetic field shifted by 90 electrical degrees with respect to the direction of stator magnetic field. This magnetic field is produced by armature current.

Bandwidth - the frequency range in which the magnitude of system gain expressed in dB is within the $\mathbf{3} \mathbf{d B}$ band.
$B C D$ - a decimal notation in which the individual decimal digits are each represented by a group of binary bits. Usually associated with codes of four or more bits used to define the Arabic numbers 0 through 9.

BIT - abbreviation of Binary Digit, a unit of information equal to 1 binary decision or having only a value 0 or 1 .

Block Diagram - a simplified representation of a system, with each component represented by a block, and each block positioned in the order of signal flow through the system.

Bode Plot - a plot of the magnitude of system gain in dB and the phase of system gain in degrees versus the sinusoidal input signal frequency in logarithmic scale.

Boolean Equation - mathematical expression of logic relationships, employing Boolean algebra.

Break Frequency - the frequency(-ies) at which the asymptotes of the gain curve in the Bode plot intersect.

Buffer - a circuit or component which isolates one electrical circuit from another.

Byte - a. single group of bits processed together, operated on as a unit.

Characteristic Equation - the characteristic equation of a servo system is $\mathbf{1 + G H}=\mathbf{0}$
where $\mathbf{G}$ is the transfer function of the forward signal path and $\mathbf{H}$ is the transfer function of the feedback signal path.

Circulating Current - the current in armature conductors which are short-circuited during commutation.

Coding - the term used for naming quantized parts. In digital systems, all information is represented in schemes of binary or Boolean variables which take on either one of only two values. In general, we can say that with " n " Boolean variables, we can code $2^{\mathrm{n}}$ quanta.

Cogging - the non-uniform rotation of a motor armature caused by the tendency of the armature to prefer certain discrete angular positions.

Coupling Ratio - a general term to define the relative motion between motor armature and the driven load.

Critical Damping - a critically damped system's response to a step disturbance is the return to its equilibrium state without overshooting the equilibrium state in minimum possible time.

Crossover Frequency - the frequency at which the magnitude of the product of the forward path gain and the feedback path gain is unity.

Damping (in servo theory) - a term to describe the amplitude decay of an oscillatory signal.

Damping Ratio - a measure of system damping expressed as the ratio between the actual damping and the critical damping.

Dead Band - a range of input signals for which there is no system response.

Decibel (dB)-a measure of system gain (A).

$$
A_{d B}=20 \log _{10} A
$$

Dielectric Test - a high voltage test of the motor insulation ability to withstand an AC voltage. Test criterion limits the leakage current to a specified maximum at test voltage of specified magnitude and frequency, applied between the motor case and winding.

Digital-to-Analog Converter - A device for converting a digital word into an analog voltage.

DIP - abbreviation for dual-in-line package.
Duty Cycle - the ratio of operating time to total cycle time.

## Duty Cycle $=\frac{\text { Operating Time }}{\text { Operating Time + Off Time }}$

Dynamic Braking - a method for braking a DC servomotor by controlling armature current during deceleration.

ECL - Emitter-Coupled Logic
Efficiency - the ratio of output power to input power.

$$
\eta=\frac{\mathbf{P}_{\mathbf{o}}}{\mathbf{P}_{\mathbf{i}}}
$$

Electrical Time Constant - the electrical time constant of a DC servomotor is the ratio of armature inductance to armature resistance.

Enable - an input which, when true, allows the circuit to function.

Encoder - a device for translating mechanical motion into a unique electronic signal or combination of signals.

Fall Time - the time for the amplitude of system response to decay to $37 \%$ of its steady-state value after the removal of steady-state forcing signal.

Fan-Out - the number of gates one gate can drive.

Field Weakening - a method of increasing the speed of a wound field motor: reducing stator magnetic field intensity by reducing magnet winding current.

Flutter - flutter is an error of the basic cycle of encoder per one revolution.

Flux Biasing - a method for controlling the torque constant of a servomotor by varying the magnetic field intensity of a separate wound field assembly.

Form Factor - the form factor of a harmonic signal is the ratio of its RMS value to its average value in one half-wave.

Full Load Current - the armature current of a motor operated at its full load torque and speed with rated voltage applied.

Full Load Speed - the speed of a motor operated with rated voltage and full load torque.

Gain - the ratio of system output signal to system input signal.

Gain Margin - the magnitude of the system gain at the frequency for which the phase angle of the product of the forward path and feedback path gains is $-180^{\circ}$.

Gate - a logic circuit having 2 or more inputs, the output state of which depends upon the combination of logic signals at the inputs.

Hunting - the oscillation of system response about theoretical steady-state value due to insufficient damping.

Hysteresis - the difference between the reponse of a system to an increasing and a decreasing input signal.

IC - abbreviation for integrated circuit.

Incremental Motion System - a control system which changes the load position in discrete steps rapidly and repetitively.

Inertial Match - an inertial match between motor and load is obtained by selecting the coupling ratio such that the load moment of inertia referred to the motor shaft is equal to the motor moment of inertia.

Inhibit - an input which, when true, prevents the circuit from functioning.

Inverter - a logic gate in which the output signal is the logic inversion of the input signal.

Lag Network - an electrical network which increases the delay between system input signal and system output signal.

Lead Network - an electrical network which decreases the delay between system input signal and system output signal.

Lead Screw - a device for translating rotary motion into linear motion, consisting of an externally threaded screw and an internally threaded carriage (nut).

Lead Screw, Ball - a lead screw which has its threads formed as a ball bearing race; the carriage contains a circulating supply of balls for increased efficiency.

Least Significant Bit - the bit in a number that is the least important or having the least weight.

## LED - Light Emitting Diode.

Linearity - for a speed control system it is the maximum deviation between actual and set speed expressed as a percentage of set speed.

Loop Gain - the product of the forward path and feedback path gains.

Mechanical Time Constant - the time for an unloaded motor to reach $63.2 \%$ of its final velocity after the application of a DC armature voltage.

Moiré Fringe - the third optical pattern created by two like images being nearly superimposed on one another. The third or fringe pattern is often used in high resolution optical encoders to serve as the mask or window to electrically displace the sensor signals.

Monolithic - existing as one large undifferentiated whole. In electronics, this refers to a circuit built on one semiconductor chip.

Monostrophic Code - a binary code in which only one bit changes between any two adjacent code positions.

Most Significant Bit - the bit in a number that is the most important or that has the most weight.

MOS and CMOS - Metal Oxide Semiconductor and Complementary MOS.

Motor Constant - the ratio of the motor torque to motor input power.

Negative Logic - a form of electronic logic circuits in which the more positive voltage level represents logic 0 and the more negative voltage level represents logic 1.

No Load Speed - motor speed with no external load.

Parallel Data - all the bits are available simultaneously.

Phase and Phase Angle - the separation in electrical degrees between any specified transitions of any two channels of an encoder.

Phase-Locked Servo System - a hybrid control system in which the output of an optical tachometer is compared to a reference square wave signal to generate a system error signal proportional to both shaft velocity and position errors.

Phase Margin - the phase angle of the loop gain minus $180^{\circ}$ at the crossover frequency.

Pole - a term of root locus plotting for the frequency(-ies) at which system gain goes to infinity.

Polystrophic Code - a binary code in which two or more bits are changed simultaneously when going from one adjacent code position to another.

Positive Logic - a form of electronic logic circuit in which the more positive voltage level represents logic 1 and more negative voltage level represents logic 0 .

Propagation Delay - the time required for a circuit output to go to the state demanded by the inputs. Alternatively, the time required for a signal to travel from one point in a circuit (or signal channel) to another point.

Pull-Down Resistor - a resistor connected between the negative supply voltage or ground of an analog or logic circuit and the circuit input or output terminal.


#### Abstract

Pull-Up Resistor - a resistor connected between the positive supply voltage of an analog or logic circuit and the circuit input or output terminal.


Quantization - The process of breaking up a continuous variable into discrete quanta or parts.

Quantizing Error - the inherent fixed error associated with digitizing an analog variable in that a continuous form of data is being replaced by noncontinuous increments.

Resolution - the number of unique electrically identified positions occuring in $360^{\circ}$ of input shaft rotation for rotary encoders, or per unit of linear displacement, i.e., positions per inch, millimeter, for linear encoders.

Ringing - a dampened oscillation in a system as a result of a sudden change in state.

Rise Time - the time required for a signal to rise from $10 \%$ of its final value to $90 \%$ of its final value.

Slew Speed (Encoders) - the maximum velocity at which an encoder will be required to perform.

Speed Regulation - for a speed control system, speed regulation is the variation in actual speed expressed as a percentage of set speed.

Speed Regulation Constant - the slope of the motor speed-torque characteristic.

Symmetry - the relative duty cycles of the " 1 " and " 0 " states of a square wave. Equal duty cycles would be said to have " $50 / 50$ " symmetry.

System Order - the degree of the system characteristic equation.

System Stiffness - a measure of system accuracy when subjected to disturbance signals.

System Type - is given by the number of poles located at the origin of the loop gain characteristic in the complex plane.

Torque Ripple - the cyclical variation of generated torque at a frequency given by the product of motor angular velocity and number of commutator segments.

Transfer Function - the ratio of the Laplace transforms of system output signal and system input signal.

TTL - Transistor-Transistor Logic or $\mathrm{T}^{2} \mathrm{~L}$.
"U" Scan - a parallel reading method for preventing ambiguity in the readout of polystrophic codes at the code position transition.
"V" Scan - a serial reading method for preventing ambiguity in the readout of polystrophic codes at the code position transitions.

Zero - a term of root locus plotting for the frequency(-ies) at which system gain goes to zero.

### 1.2. SYMBOLS FOR QUANTITIES AND THEIR UNITS

| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| a | linear acceleration $a=\frac{d v}{d t}$ | $\mathrm{m} / \mathrm{s}^{2}$ | $\mathrm{in} / \mathrm{s}^{2}$ |
| A | gain | - | - |
| $A_{d B}$ | gain expressed in dB | - | - |
| AC | alternating current |  |  |
| B | magnetic flux density | T | line/in ${ }^{2}$ |
| C | capacitance | F | F |
| CW | clockwise rotation |  |  |
| ccw | counterclockwise rotation |  |  |
| D | viscous damping factor $D=\frac{T_{D}}{\omega}$ | $\mathrm{Nm} / \mathrm{rad} \mathrm{s}^{-1}$ | oz-in/rad s ${ }^{-1}$ |
| dB | decibel | - | - |
| DC | direct current |  |  |
| e | (2.71828) base of natural logarithms | - | - |
| E | internal voltage (emf) | V | V |
| $\mathrm{E}_{\mathrm{g}}$ | internally generated voltage (counter emf) $E_{g}=K_{E} \cdot n$ | V | V |
| f | frequency | Hz | $\mathrm{Hz}, \mathrm{c} / \mathrm{s}, \mathrm{cps}$ |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $f(t)$ | real function of time |  |  |
| f(s) | Laplace transform of $f(t)$ |  |  |
| F | force | $N$ | oz, lb |
| $F_{f}$ | friction force | $N$ | oz, lb |
| g | ( $9.80665 \mathrm{~m} / \mathrm{s}^{2}$ or $386.09 \mathrm{in} / \mathrm{s}^{2}$ ) standard gravitational acceleration | $\mathrm{m} / \mathrm{s}^{2}$ | $\mathrm{in} / \mathrm{s}^{2}$ |
| $\mathrm{G}_{\mathrm{m}}(\mathrm{s})$ | motor transfer function |  |  |
| H | magnetic field intensity (magnetizing force) | A/m | A-turn/in |
| i(t) | current (instantaneous value) | A | A |
| I | current | A | A |
| $I_{a}$ | armature current | A | A |
| $\mathrm{I}_{\mathrm{ad}}$ | maximum armature pulse current to avoid demagnetization | A | A |
| $\mathrm{I}_{\mathrm{ao}}$ | armature current at no load | A | A |
| $I_{p k}$ | peak current | A | A |
| j | $(\sqrt{-1})$ imaginary operator | - | - |
| J | moment of inertia | $\mathrm{kg} \mathrm{m}{ }^{2}$ | oz-in-s ${ }^{2}$ |
| $J_{c}$ | capstan moment of inertia | kg m ${ }^{2}$ | $0 z-\mathrm{in}-\mathrm{s}^{2}$ |
| $J_{L}$ | load moment of inertia | $\mathrm{kg} \mathrm{m}{ }^{2}$ | oz-in-s ${ }^{2}$ |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $J_{m}$ | armature (rotating part of motor) moment of inertia | $\mathrm{kg} \mathrm{m}^{2}$ | oz-in-s ${ }^{2}$ |
| $K_{D}$ | damping constant (not equal to D ) $K_{D}=\frac{1}{R_{m}}$ | Nm/krpm | oz-in/krpm |
| $K_{E}$ | voltage constant | V/krpm, <br> V/rad s-1 | V/krpm, <br> $\mathrm{V} / \mathrm{rad} \mathrm{s}^{-1}$ |
| $K_{M}$ | motor constant | Nm/W | oz-in/W |
| $K_{T}$ | torque constant | Nm/A | oz-in/A |
| 1 | length | m | in |
| L | inductance | H | H |
| $L_{a}$ | armature inductance | H | H |
| m | mass | kg | Oz-s ${ }^{2} / \mathrm{in}$ |
| n | rotational speed, shaft speed | r/min, rpm, krpm | rpm, krpm |
| $\mathrm{n}_{0}$ | shaft speed at no load $n_{o}=\frac{V}{K_{E}}$ | r/min, rpm, krpm | rpm, krpm |
| $N$ | a) gear ratio (coupling ratio) <br> b) number of coil turns | - | - |
| $p$ | a) number of poles <br> b) pole of transfer function | - $s^{-1}$ | - $s^{-1}$ |
| P | a) power <br> b) lead screw pitch | $\begin{aligned} & \mathrm{W} \\ & \mathrm{~mm} \end{aligned}$ | W turns/in |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $\mathrm{P}_{\mathrm{i}}$ | input power | W | W |
| $\mathrm{P}_{\mathrm{L}}$ | power loss $P_{L}=P_{i}-P_{0}$ | W | W |
| $\mathrm{P}_{0}$ | output power | W | W, hp |
| $\mathrm{P}_{\text {th }}$ | thermal flux | W | W |
| PM | permanent magnet |  |  |
| PR | power rate $P R=\frac{T_{o s}{ }^{2}}{J_{a}}$ | $\mathrm{kg} \mathrm{m}{ }^{2} \mathrm{~s}^{-4}$ | oz-in/s ${ }^{2}$ |
| $r$ | radius | m | in |
| R | a) resistance b) motor terminal resistance $R=R_{a}+R_{b}$ | $\Omega$ $\Omega$ | $\Omega$ $\Omega$ |
| $\mathrm{R}_{\mathrm{a}}$ | armature winding resistance | $\Omega$ | $\Omega$ |
| $\mathrm{R}_{\mathrm{b}}$ | brush resistance | $\Omega$ | $\Omega$ |
| $\mathrm{R}_{\mathrm{m}}$ | speed regulation constant $\quad R_{m}=\frac{R}{K_{E} K_{T}}$ | krpm/Nm | krpm/oz-in |
| $\mathrm{R}_{\text {th }}$ | thermal resistance | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ | ${ }^{0} \mathrm{C} / \mathrm{W}$ |
| S | Laplace operator $s=\sigma+\mathrm{j} \omega$ | $\mathrm{s}^{-1}$ | $s^{-1}$ |
| SI | international system of units (metric system) |  |  |
| t | time | S | s |
| $t_{i}$ | time interval | s | s |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $t_{f}$ | fall time | S | S |
| $t_{r}$ | rise time | s | s |
| $t_{s}$ | settling time | S | s |
| $t_{R}$ | $\left(1.69 \tau_{m}\right)$ theoretical reversing time - time interval from full speed in one direction to $63.2 \%$ of full speed in reverse direction, when motor supply voltage is stepwise reversed | s | s |
| T | torque | Nm | oz-in, lb-in |
| $\mathrm{T}_{\mathrm{a}}$ | accelerating torque | Nm | oz-in |
| $\mathrm{T}_{\mathrm{D}}$ | damping torque $T_{D}=D \cdot \omega$ | Nm | oz-in |
| $\mathrm{T}_{\mathrm{f}}$ | internal friction torque | Nm | oz-in |
| $\mathrm{T}_{\mathrm{g}}$ | internally generated torque |  |  |
|  | $T_{g}=T_{o}+T_{f}+T_{D}$ | Nm | oz-in |
| Tgs | internally generated torque at stall $T_{g s}=T_{o s}+T_{f}$ | Nm | oz-in |
| $\mathrm{T}_{\mathrm{L}}$ | load torque | Nm | oz-in |
| $T_{L R}$ | rated load torque | Nm | oz-in |
| $\mathrm{T}_{\mathrm{m}}$ | motor opposing torque |  |  |
|  | $T_{m}=T_{f}+T_{D}$ | Nm | oz-in |
| $\mathrm{T}_{0}$ | motor output torque $T_{o}=T_{g}-T_{f}-T_{D}=T_{L}+T_{a}$ | Nm | oz-in |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $\mathrm{T}_{\text {os }}$ | motor output torque at stall |  |  |
|  | $\mathrm{T}_{\text {os }}=\mathrm{T}_{\mathrm{gs}}-\mathrm{T}_{\mathrm{f}}$ | Nm | oz-in |
| $\mathrm{T}_{\mathrm{pk}}$ | peak torque $T_{p k}=K_{T} \mathbf{l}_{p k}$ | Nm | oz-in |
| v | linear velocity $v=\frac{d x}{d t}$ | $\mathrm{m} / \mathrm{s}$ | in/s |
| $v(t)$ | voltage (instantaneous value) | V | V |
| V | a) voltage | V | V |
|  | b) motor terminal voltage | V | V |
| $\mathrm{V}_{\mathrm{ad}}$ | maximum terminal voltage a motor can withstand at stall to avoid demagnetization $V_{a d}=I_{a d} R$ | V | V |
| $V_{b}$ | voltage drop across brushes | V | V |
| $\mathrm{V}_{\mathrm{pk}}$ | peak voltage | V | V |
| $V_{r}$ | ripple voltage | V | V |
| $\mathrm{V}_{\mathrm{S}}$ | supply voltage | V | V |
| w | weight | kg | oz, lb (mass) |
| W | energy (work) | J | $\mathrm{lb}-\mathrm{ft}$ |
| x | linear displacement | m | in |
| $x, y, z$ | rectangular coordinates | m | in |
| X | reactance | $\Omega$ | $\Omega$ |
| z | zero of transfer function | $\mathrm{s}^{-1}$ | $s^{-1}$ |
| Z | impedance | $\Omega$ | $\Omega$ |


| quantity symbol | definition | unit symbol |  |
| :---: | :---: | :---: | :---: |
|  |  | SI | British |
| $a$ | angular acceleration $a=\frac{\mathrm{d} \omega}{\mathrm{dt}}$. | $\mathrm{rad} / \mathrm{s}^{2}$ | $\mathrm{rad} / \mathrm{s}^{2}$ |
| $a_{m}$ | maximum theoretical acceleration from stall | $\mathrm{rad} / \mathrm{s}^{2}$ | $\mathrm{rad} / \mathrm{s}^{2}$ |
|  | $a_{m}=\frac{K_{T} I_{p k}-T_{f}}{J_{a}}$ |  |  |
| $\epsilon$ | permittivity (dielectrical constant) $\epsilon=\epsilon_{\mathbf{r}} \epsilon_{\mathbf{o}}$ in SI system | F/m |  |
| $\epsilon_{\text {o }}$ | $\left(8.85416 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)$ permittivity of vacuum | $\mathrm{F} / \mathrm{m}$ |  |
| $\epsilon_{\mathrm{r}}$ | relative permittivity | - | - |
| $\zeta$ | damping ratio | - | - |
| $\eta$ | efficiency | -, \% | -, \% |
| $\theta$ | angular displacement | rad, ${ }^{\circ}, \mathrm{r}$ | rad, ${ }^{\circ}$, r |
| $\Theta$ | temperature | ${ }^{\circ} \mathrm{C}, \mathrm{K}$ | ${ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{F}$ |
| $\Theta_{a}$ | armature temperature $\Theta_{a}=\Theta_{A}+\Theta_{r}$ | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ |
| $\Theta_{\text {A }}$ | ambient temperature | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{F}$ |
| $\Theta_{c}$ | case temperature | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ |
| $\Theta_{h}$ | motor housing temperature | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ |
| $\Theta_{r}$ | temperature rise | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ |
| $\mu$ | permeability $\mu=\mu_{\mathbf{r}} \mu_{\mathbf{o}} \text { in SI system }$ | H/m | line/A-turn-in |
| $\mu_{0}$ | $\left(1.25664 \times 10^{-6} \mathrm{H} / \mathrm{m}\right)$ permeability of vacuum | H/m |  |



### 1.3. SYSTEMS OF UNITS

The U.S.A. has been a member nation of the Metric Convention since 1875, but until recently, except for sporadic use of metric units in science and part of industry, the United States was the only industrialized nation which has not established a national policy committing itself to the use of metric system.

Since 1972, however, the situation has dramatically changed. The Senate Commerce Committee passed legislation which will convert the United States to the use of international system (SI) of metric measurement. The bill does not establish a mandatory date for the change, but does set the conversion to the metric system within ten years as a national goal.

Finally, the Metric Conversion Act became a Public Law 94-168 on December 23, 1975.

The editors felt the necessity to update the handbook also in this respect. Therefore, the Appendix A.1. introduces a comprehensive description of SI system and explaines its basic philosophy.

Further, all equations in the book are presented in SI units and, where necessary, in British units at the same time. Finally, the reader can find exact conversion factors between SI and British units in Appendix A. 2 .

The British system of units is used in this book in its technical form, briefly explained in Appendix A.2. Therefore, the units oz and $I b$ throughout the book are units of force unless otherwise specified.

To help the reader keep both SI and British units in order, all important equations in the book are provided with a listing of units to be used. The following example:

$$
P=T \omega \quad[\mathrm{~W} ; \mathrm{Nm}, \mathrm{rad} / \mathrm{s}]
$$

shows that this equation is correct only if the torque is expressed in newtonmetres, angular velocity in radians per second, and power in watts. If other units are used, a conversion factor must be introduced.

## Chapter 2

## DC Motors and Generators

### 2.1. BASIC THEORY

## HISTORICAL BACKGROUND

The direct current (DC) motor is one of the first machines devised to convert electrical power into mechanical power. Its origin can be traced to disc-type machines conceived and tested by Michael Faraday, the experimenter who formulated the fundamental concepts of electromagnetism.

Faraday's primitive design was quickly improved upon; and many DC machines were built in the 1880's when DC was the principal form of electric power generation. With the advent of 60 Hz AC as the electric power standard in the United States and 50 Hz in Europe, and invention of the induction motor with its lower manufacturing costs, the DC machine became less important. In recent years, the use of DC machines has become almost exclusively associated with applications where the unique characteristics of the DC motor (e.g., high starting torque for traction motor application) justify its cost, or where portable equipment must be run from a DC (or battery) power supply.

The ease with which the DC motor lends itself to speed control has long been recognized. Compatibility with the new thyristor (SCR) and transistor amplifiers, plus better performance due to the availability of new
improved materials in magnets, brushes and epoxies, has also revitalized interest in DC machines. The need for new high-performance motors with highly sophisticated capabilities has produced a superabundance of new shapes and sizes quite unlike DC machines of 10 years ago.

## CONCEPT OF TORQUE AND POWER

An understanding of electro-mechanical energy conversion, as exemplified by a motor, is based upon acquaintance with several fundamental concepts from the field of mechanics.

The first concept is that of torque. If a force is applied to a lever which is free to pivot about a fixed point, the lever, unless restrained, will rotate. The motive action for this rotation, termed torque, is defined as the product of force and the perpendicular distance from the pivot to the force vector. This concept is illustrated in Fig. 2.1.1.

Referring to Fig. 2.1.1, it can be seen that by applying this definition we can write an equation for torque ( $T$ ) so that:

$$
\begin{array}{cll}
T=F \cdot r & {[\mathrm{Nm} ; \mathrm{N}, \mathrm{~m}]}  \tag{2.1.1}\\
& \text { or } & {[o z-\mathrm{in} ; \mathrm{oz}, \mathrm{in}]}
\end{array}
$$



Fig. 2.1.1 Applying a force $F$ to a lever with radius $r$ will produce a torque equal to $F \cdot r$ at the pivot point.

If several forces of different magnitudes are applied at different points along the lever, a resultant torque can be calculated. A counterclockwise torque direction will be adopted as the positive one, clockwise will be negative, and resultant torque will then be the sum of all the torques applied to the lever with the appropriate plus or minus signs assigned to each of the $F_{i} \cdot r_{i}$ products. Then if we look at Fig. 2.1.2, the resultant torque can be written as:

$$
\begin{align*}
& T=\Sigma\left(F_{i} \cdot r_{i}\right)  \tag{2.1.2}\\
& T=F_{1} r_{1}-F_{2} r_{2}-F_{3} r_{3} \tag{2.1.3}
\end{align*}
$$

This definition of torque, while elementary, nevertheless correctly describes the turning moment which can be observed at the output shaft of an electric motor at stall. The measurable shaft output torque is the resultant one that occurs after the various negative torques (bearing and brush friction torque, gear friction torque, etc.) are subtracted from the developed electromagnetic torque.


Fig. 2.1.2. The resultant torque of several applied forces at various radii is the sum of all torques applied with appropriate plus and minus signs.

Having established the concept of torque it is then necessary to develop an understanding of power which is defined as the rate at which work is done or energy is expended.

Work (or energy) can be also defined in terms of torque acting through a given displacement. If a torque $\mathbf{T}$ acts through an angle $\theta$, as shown in Fig. 2.1.3, the work performed will be:

$$
\begin{align*}
& \mathrm{W}=\mathrm{T} \cdot \theta {[\mathrm{~J} ; \mathrm{Nm}, \mathrm{rad}] }  \tag{2.1.4}\\
& \text { or } \quad \\
& {[\mathrm{oz}-\mathrm{in} ; \mathrm{oz}-\mathrm{in}, \mathrm{rad}] }
\end{align*}
$$

If the above work $W$ is performed in the time interval $\mathbf{t}$, we can describe the power expended as:

$$
\begin{equation*}
\mathbf{P}=\frac{\mathrm{W}}{\mathbf{t}}=\frac{\mathbf{T} \cdot \theta}{\mathbf{t}} \quad[\mathrm{W} ; \mathrm{Nm}, \mathrm{rad}, \mathrm{~s}] \tag{2.1.ヶ}
\end{equation*}
$$



Fig. 2.1.3. Work, defined as a torque acting through a given angular displacement.

## LAW OF ELECTROMAGNETIC INDUCTION

If a current-carrying conductor is placed in a magnetic field, a force will act upon it. The magnitude of this force is a function of the magnetic flux density, $B$, the current, I , and the orientation of the field and current vectors. Referring to Fig. 2.1.4, the force acting upon the conductor can be written as:


Fig. 2.1.4. A current-carrying conductor in a magnetic field experiences a force acting upon it.

$$
\begin{equation*}
F=B L I \quad[N ; T, m, A] \tag{2.1.6}
\end{equation*}
$$

where $L$ is the length of the conductor.
The direction in which force on a conductor will act is a function of the direction of current and magnetic field vectors.

Vector notation is a convenient means of including the directional information in an expression for force. If the conductor is considered to be of unit length, electro magnetic force can be written as the cross product of two vectors:

$$
\begin{equation*}
F=I \times B \tag{2.1.7}
\end{equation*}
$$

or

$$
\begin{equation*}
|\mathbf{F}|=|\mathbf{I}||\mathbf{B}| \sin \theta \tag{2.1.8}
\end{equation*}
$$

Since two vectors, $\mathbf{B}$ and $\mathbf{I}$, will determine a plane, their vector product will be a vector, perpendicular to the plane, and will be proportional to the sine of the angle between them. This is shown in Fig. 2.1.5. The analogy of the "screw" can also be used to determine the direction. The force vector will point in the direction that a right hand screw advances when its head is rotated from the current vector, I, towards the magnetic flux density vector, B.

The exact opposite effect on the foregoing discussion is also observed: if a conductor was moved through a magnetic field, an electric voltage appeared (or was generated) across the conductor.


Fig. 2.1.5. The direction of the force on a current carrying conductor is perpendicular to the plane determined by the current and field vectors.

The magnitude of this generated voltage, $E$, was found to depend upon magnetic flux density, B, length of conductor, $L$, and speed of the conductor, $\mathbf{v}$, as it moved through the field. An expression for the generated voltage is:

$$
\begin{equation*}
E=B L v \quad[V ; T, m, m / s] \tag{2.1.9}
\end{equation*}
$$

It occured to Faraday that the best way to obtain a useful function from the electromagnetic effect was to make the conductors a part of a rotational body by fixing them to a shaft which could turn continuously through a magnetic field. The problem of introducing current (motor action) or picking off a voltage (generator action) was solved by the use of sliding contacts called brushes. The type of machine built by Faraday (Fig. 2.1.6) would today be called a homo-polar (single pole) machine, and with other design improvements is used to provide large currents at low voltages.


Fig. 2.1.6. An example of the type of machine built by Faraday in the 1880's would be this single pole, or homo-polar machine as it would be called today.

A rudimentary DC motor can be seen to be comprised of three main parts: currentcarrying conductors called an armature; a circuit for magnetic field to provide the means for energy conversion, which can be provided either by permanent magnets or by electromagnets (for the "wound-field" motors); some type of sliding contact arrangement for introducing current to moving conductors (usually carbon brushes and commutator).

## MAGNETIC CIRCUIT PRINCIPLES

Since electric motor design is based upon the placement of conductors in magnetic field, a discussion of magnetic circuit principles will help understand mutor action.

If a conductor were wound into a coil with many turns, the magnetic contribution of each individual turn would add to the magnetic field intensity which exists in the space enclosed by the coil. In this way, extremely strong magnetic fields can be developed. The force which acts to push the magnetic flux through a space is called variously magnetomotance, magnetomotive force, or simply $m m f$. The unit of $m m f$ is A-turn and its dimension is [A]. Thus, coils with 10 turns and 2 A , or 5 turns and 4 A will each develop a magnetomotance of 20 A-turn.

## Magnetic Flux ( $\Phi$ )

The term magnetic flux is used to describe "how much magnetism" there is in the space around a coil or permanent magnet, or in the air gap of a motor. The unit of magnetic flux is the line (or 1 kilo-line $=10^{3}$ lines) in the British system or the weber in the SI system. One weber is equal to $10^{8}$ lines.

## Magnetic Flux Density (B)

Magnetic flux density is a measure of concentration of magnetic flux in an area. It is defined as the ratio of total flux to the given area and is specified in terms of lines per square inch in the British system. The SI unit for magnetic flux density is the tes/a and is equal to one weber per square meter.

## Permeability ( $\mu$ )

Permeability is the degree to which a medium will support a magnetic field, also stated as the ease with which flux will penetrate a medium. The unit of permeability is line per A-turn/in in the British system. In the SI system the permeabiltiy is given as the product

$$
\mu=\mu_{\mathrm{o}} \mu_{\mathrm{r}}
$$

where $\mu_{o}$ is the permeability of vacuum [ $\mathrm{H} / \mathrm{m}$ ] and $\mu_{\mathrm{r}}$ is the relative permeability. The relative permeability is used to describe the ability of different materials to support magnetic fields. Vacuum or air has a relative permeability of one, while ferromagnetic materials such as cold-rolled steel will have a relative permeability of several hundred. This means that for a given value of $m m f$, a magnetic circuit which contains iron may have several hundred times the flux of one constructed with non-ferrous materials. This is the reason most motors are built with steel materials.

## Magnetic Field Intensity (H)

Niagnetic field intensity has the properties of a gradient and gives an indication of how a $m m f$ is used up around a magnetic circuit. The unit of magnetic field intensity is A-turn/in in the British system and $A / m$ in the SI system.

This discussion of electromagnetism is portrayed in Fig. 2.1.7a and 2.1.7b. In Fig. 2.1.7a a coil of N turns is excited by the


Fig. 2.1.7a. With air surrounding N turns of wire, the magnetic flux is $\Phi_{1}$
current I. The resultant flux is $\Phi_{1}$. If the same coil is now placed around an iron core with relative permeability $\mu_{r}$, as shown in Fig. 2.1.7b, the magnetic field conditions will change greatly, with flux increasing to $\Phi_{2}=\mu_{r} \Phi_{1}$.

## WORKING EQUATIONS FOR VOLTAGE AND TORQUE

## Voltage

Using the laws outlined previously useful expressions can be derived to describe such machine characteristics as torque and generated voltage in terms of design parameters (e.g., number of conductors, magnet size, etc.). We will first derive an expression for generated voltage using the equation $\mathbf{E}=\mathrm{Blv}$ (the voltage generated in a single conductor) as a starting point.

Velocity $\mathbf{v}$ of an armature conductor can be rewritten as:


Fig. 2.1.7b. If $N$ turns of wire are placed around an iron core with relative permeability $\mu_{\mathrm{r}}$, the magnetic flux is increased $\mu_{r}$ - times.

$$
\begin{equation*}
\mathrm{v}=\mathrm{r} \omega \quad[\mathrm{~m} / \mathrm{s} ; \mathrm{m}, \mathrm{rad} / \mathrm{s}] \tag{2.1.10}
\end{equation*}
$$

where $\omega$ is angular velocity, and $r$ is radius of rotation.

The magnetic flux density $B$ can be rewritten in terms of pole flux and area:

$$
\begin{equation*}
\mathrm{B}=\frac{\Phi}{\mathrm{A}} \quad\left[\mathrm{~T} ; \mathrm{Wb}, \mathrm{~m}^{2}\right] \tag{2.1.11}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{A}=\frac{2 \pi \mathrm{r} L}{\mathrm{p}} \quad\left[\mathrm{~m}^{2} ; \mathrm{m}, \mathrm{~m},-\right] \tag{2.1.12}
\end{equation*}
$$

where
$r$ is the radius of rotation of the armature
$L$ is the length of the conductor
$p$ is the number of poles.

Then, the generated voltage per conductor will be:

$$
\begin{equation*}
E^{\prime}=\frac{\Phi p \omega}{2 \pi} \quad[\mathrm{~V} ; \mathrm{Wb},-, \mathrm{rad} / \mathrm{s}] \tag{2.1.13}
\end{equation*}
$$

If there are many conductors ( $\mathbf{z}$ ) moving through the field, their cumulative effect will be $z$-times the above voltage.

$$
\begin{equation*}
E=\frac{z \Phi p \omega}{2 \pi}[\mathrm{~V} ;-, \mathrm{Wb},-, \mathrm{rad} / \mathrm{s}] \tag{2.1.14}
\end{equation*}
$$

or

$$
\begin{equation*}
E=\frac{z \Phi p n}{60} \quad[\mathrm{~V} ;-, \mathrm{Wb},-, \mathrm{rpm}] \tag{2.1.15}
\end{equation*}
$$

Finally, if we introduce the factor $\mathrm{n}^{\prime}$, the number of parallel conductor paths in the armature, the average generated voltage will be:

$$
\begin{equation*}
E=\left[\frac{z \Phi p}{60 n^{\prime}}\right] n \quad[V ; W b, r p m] \tag{2.1.16}
\end{equation*}
$$

in the SI system, or

$$
\begin{equation*}
E=\left[\frac{z \Phi p}{60 \cdot 10^{8} n^{\prime}}\right] n \quad[V \text {; lines, } r p m] \tag{2.1.17}
\end{equation*}
$$

in the British system.
The quantity enclosed in brackets in (2.1.16) and (2.1.17) will be a design constant and is commonly called the voltage constant, $\mathrm{K}_{\mathrm{E}}$, of the motor. It is an important motor (or generator) characteristic since it will determine the speed of the motor at a given value of applied voltage. Using the new motor parameter the voltage equation becomes:

$$
\begin{equation*}
E_{g}=K_{E} n \quad[V ; V / k r p m, k r p m] \tag{2.1.18}
\end{equation*}
$$

## Torque

The basic equation for torque is derived from the equation for force ( $F=B L I$ ) in a manner similar to that used in obtaining the voltage equation. After the appropriate substitutions are made, we obtain the following expression for torque:

$$
\begin{equation*}
\mathbf{T}=\left[\frac{\mathrm{z} \Phi \mathrm{p}}{2 \pi \mathrm{n}^{\prime}}\right], \quad[\mathrm{Nm} ; \mathrm{Wb}, \mathrm{~A}] \tag{2.1.19}
\end{equation*}
$$

If it is desired to express the torque and magnetic flux in British units, a conversion factor must be introduced and (2.1.19) then becomes:

$$
\begin{equation*}
\mathrm{T}=\left[2.254 \times 10^{-7} \frac{\mathrm{z} \mathrm{\Phi p}}{\mathrm{n}^{\prime}}\right] \tag{2.1.20}
\end{equation*}
$$

[oz-in; lines, A]

The quantity in brackets in (2.1.19) and (2.1.20) will also be a constant for a given motor design and is called the torque constant, $\mathrm{K}_{\mathrm{T}}$, of the motor. Substituting, the torque equation becomes:

$$
T=K_{T} I \quad[\mathrm{Nm} ; \mathrm{Nm} / \mathrm{A}, \mathrm{~A}]
$$

$$
[o z-i n ; o z-i n / A, A]
$$

If the torque and voltage equation are each solved for a common factor, the pole flux, $\Phi$, and the results equated, we obtain other useful relationships:

$$
\begin{equation*}
K_{T}=K_{E} \quad\left[\mathrm{Nm} / \mathrm{A} ; \mathrm{V} / \mathrm{rad} \mathrm{~s}^{-1}\right] \tag{2.1.22}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{K}_{\mathrm{T}}=9.5493 \times 10^{-3} \mathrm{~K}_{\mathrm{E}} \tag{2.1.23}
\end{equation*}
$$

$$
[\mathrm{Nm} / \mathrm{A} ; \mathrm{V} / \mathrm{krpm}]
$$

$$
\begin{equation*}
K_{T}=1.3524 \mathrm{~K}_{\mathrm{E}} \tag{2.1.24}
\end{equation*}
$$

[oz-in/A;V/krpm]

## COMMUTATION

A practical machine based upon the principles discussed earlier, for maximum performance and minimum cost, must get many conductors into the magnetic field existing around its armature. This allows the cumulative effect of many conductors to add to the output torque and/or voltage characteristic of the machine. A practical machine will therefore have the entire armature surface covered with conductors. Or, as in the case with conventionally constructed motors, the conductors are placed in deep slots of an iron laminated rotor structure. In either case, there are many conductors enveloping the entire periphery of the rotor structure. This multiplicity of conductors leads to another constructional feature of the motor: the commutator.

Recalling the vector diagram of Fig. 2.1.5, it was seen that in order to have the force vector $\mathbf{F}$ remain constant in magnitude and direction, it is necessary to have the vectors B and I remain fixed in space with respect to each other. This condition is not met if direction of current through the conductors were to remain as the rotor turns on its axis of rotation. In Fig. 2.1.8, current flowing into the paper is indicated by a


Fig. 2.1.8. By means of a commutator switching current in the rotor, the vector of resultant electromagnetic field remains fixed in space.
circled x , while current flow out of the paper is indicated by a circled dot. The rotor member shown in Fig. 2.1.8 has six full pitch coils equally spaced around its periphery.

As the rotor turns, the resultant electromagnetic field will rotate with it unless some provision is made to switch the direction of current in individual coils as they pass a fixed point in space. This switching is accomplished by means of a commutator. The most commonly used commutator is a cylinder, consisting of segments of conductive material (usually copper) interspersed with, and insulated from one another, by an insulating material, as shown in Fig. 2.1.9.


Fig. 2.1.9. The end view of a commutator showing the interspersing of conductive and insulating materials.


Fig. 2.1.10. A representation of current flow through a motor armature.

The machine conductors (which constitute the armature) are connected in sequence to segments of the commutator. The connection of the coils to the commutator is shown symbolically in Fig. 2.1.10.

Current flow is in at brush A and out at brush $B$. The small radially oriented arrows show current direction in the individual coil sides. If the rotation is clockwise, it can be seen that $1 / 6$ of a revolution after the instant shown, the current in coils

3-3' and 6-6' will have changed directions. As successive commutator segments pass under the brushes, their current directions will also change. As a result of this switching (commutation), current flow. in the armature occupies a fixed position in space independent of rotation and a steady unidirectional torque will result.

Referring again to Fig. 2.1.10, it can be seen that at an instant of time, each of the brushes may be contacting two adjacent commutator segments. At this condition the coil connected to those two segments will be "shorted" through the brush. The result is a short circuit current through the coil. In some cases this short circuit current can produce undesirable vibration.

By action of the commutator, the wound armature of the DC motor can be regarded, as shown in Fig. 2.1.11, as a wound core with an axis of magnetization fixed in space.

The axis of magnetization is determined by the position of the brushes and for this reason is also called the brush axis. In order for a motor to be bidirectional, i.e., have equal characteristics for both directions of rotation, the brush axis must be at an angle of 90 electrical degrees with the main field, in a position called the neutral position or neutral axis. Most DC motors are built so that the structure holding the brushes can be rotated over a limited range. Then after the motor is assembled, the brush axis can be adjusted to the neutral position by actual observance of machine characteristics. If a machine is not required to function in a bidirectional role, it is possible to move the

brush axis sightly off the neutral position so that commutation is optimum for one direction of rotation.

Fig. 2.1.11. The brush axis, or neutral axis of a motor is determined by the position of the brushes.

### 2.2. MOTOR COMPARISON

There are two large families of DC motors, the integral horsepower types having power ratings of one horsepower or more, and the fractional horsepower motors, with power ratings of less than one horsepower. For the purposes of this manual, we will confine our considerations to the latter classification.

Fractional horsepower DC motors can be subdivided into types based on the type of motor magnetic field employed (permanent magnet, or wound in straight series, split series, shunt, compound), with further delineation based on duty cycle (continuous or intermittent), cooling method or enclosure (fan-cooled, no cooling, closed frame, open frame), and by hot-spot temperature (the hottest part of the motor insulation).

## GENERAL DESCRIPTION

The class of fractional horsepower DC motors which utilize electromagnets to generate the stator magnetic field are called wound-field motors. The electromagnets can be energized individually or in conjunction with the armature. When the electromagnets are energized in conjunction with the armature, the motors are known as self-excited motors. Various configurations of electromagnet windings for the selfexcited motors are possible, such as: series, shunt, or compound.

The class of fractional horsepower DC motors which utilize permanent magnets differs
from wound-field types in that no external power is required in the stator structure. These various motors are described and compared below.

## Wound-Field Motors

Straight-series motors provide very large torque at start-up due to their use of coils in series with the armature to produce the stator magnetic flux. Because the field winding carries the full armature current, it consists of a few turns of heavy gage wire. As motor speed increases, current reduces and so does the stator magnetic flux. This in turn causes another increase in the motor speed. Under no-load conditions this type of motor could theoretically run away because of the steep speed curve, but internal friction and losses in the windings provide sufficient load to hold motor speed within safe operating limits.

The straight-series motor is usually employed where large starting torques are required. A typical straight-series motor is shown in Fig. 2.2.1, along with its speedtorque curve.

Split-series motors are quite similar to straight-series motors, except that they have two field coils, oppositely connected. This feature accommodates applications where rapid polarity switching is desired for rapid changes of direction. A representative application for motors of this type is their use in aircraft actuators. A typical splitseries motor is shown in Fig. 2.2.2.
Shunt motors have armature and field coils connected in parallel. Line current in the


Fig. 2.2.1. Straight-series motor.
armature is a function of the load configuration. The shunt motor has in the past been popular for both fixed speed and


Fig. 2.2.2. Split-series motor.


Fig. 2.2.3. Typical shunt motor.
variable speed applications. The torquespeed characteristics are non-linear at higher current levels, as shown in Fig. 2.2.3.

Compound motors have both series and shunt field windings. When the series winding aids the shunt winding, the motor is termed a "cumulative compound" motor, and when the series winding opposes the shunt winding, the motor is termed a "differential compound" motor. In general, small compound motors have a strong shunt field and a weak series field to help start the motor.

The compound motor exhibits high starting torque and relatively flat speed-torque characteristic at rated load. In reversing applications, the polarity of both fields, or of
the armature, must be switched. Because rather complex circuits are required for control, most compound motors built are large bidirectional types. A typical compound motor is shown in Fig. 2.2.4.


Fig. 2.2.4. Typical compound motor.

## Permanent Magnet Motors

Since the stator magnetic field of Permanent Magnet (PM) motors is generated by permanent magnets, no power is used in the field structure. The stator magnetic flux remains essentially constant at all levels of armature current and, therefore, the speedtorque curve of the PM motor is linear over an extended range (see Fig. 2.2.5). With



Fig. 2.2.5. Typical permanent magnet motor.
modern ceramic magnets, the stalled torque will tend to be higher and the speed-torque curve will tend to be more linear than for a comparable wound-field motor.
A comparison of a permanent magnet motor and a shunt motor is shown in Fig. 2.2.6. Note the non-linear characteristic of the wound-field motor at higher torque levels. The reason for the non-linear torque-speed curve in the case of the shunt motor is that the armature reaction flux (which always is orthogonal to the main stator flux in any DC motor) tends to follow the low-reluctance path through the pole shoe, and at higher current levels causes a net effect of an angular shift in pole location and a lower effective flux level. This is illustrated in Fig. 2.2.7.


Torque


Fig. 2.2.6. Stalled torque tends to be higher, and speed-torque curves more linear for the permanent magnet motors, compared to a shunt motor.

In the case of the ceramic permanent magnet (Fig. 2.2.8.), the armature reaction flux stays orthogonal to the permanent magnet flux, since the permeability of ceramic


Fig. 2.2.7. Demagnetization effect in wound-field motors is caused by a component of the armature reaction field.


Fig. 2.2.8. Due to the coercive strength of the permanent magnet motor, it is virtually insensitive to demagnetization effects.
magnet material is very low (almost equal to that of air). In addition, the high coercive force of the magnet material resists any change in flux whenever the armature reaction field enters. The result is a linear torque-speed characteristic.

The PM motor offers several advantages in addition to those discussed above. Perhaps the most obvious advantage is that electrical power need not be supplied to generate the stator magnetic flux. Since the conversion of electrical power to mechanical power takes place in the armature winding, the power supplied to the field winding results mostly in an $\mathrm{RI}^{2}$ loss (heat loss) in the winding itself. The PM motor thus simplifies power supply requirements, while at the same time it requires less cooling.

Another benefit of the PM motor is a reduced frame size for a given output power. This characteristic is also portrayed in Figs. 2.2.7 and 2.2.8. Because of the high coercive strength of permanent magnets,
their radial dimension is typically onefourth that of the wound-field motor for a given air gap.
The significant advantages of PM motors over wound-field types are summarized as follows:

1. Linear torque-speed characteristic.
2. High stall (accelerating) torque.
3. No need for electric power to generate the magnetic flux.
4. A smaller frame and lighter motor for a given output power.

### 2.3. MOTOR EQUATIONS AND TRANSFER FUNCTION

This section presents the equations of a DC motor and the derivation of its transfer function. It is assumed that the reader is familiar with the Laplace transformation and with the concept of the transfer function. Readers who wish to review this material are directed to the first section of Chapter 4. For simplicity in notations, the Laplace transform of a time variable is denoted by the same letter, followed by the argument $s$. For example, the Laplace transform of $x(t)$ is $x(s)$.

The description of motor equations starts with the relationships between electrical variables. These are known as the electrical equations.

### 2.3.1. ELECTRICAL EQUATIONS

The first characteristic to be considered is the motor impedance. The best way to determine this is by direct measurements. Locking the motor shaft at fixed position, and applying sinusoidal voltage at varying frequencies, one can measure the resulting current and evaluate the impedance as the ratio of the voltage to the current.

When this is done, it appears that the motor impedance at stall is equal to a resistance, $\mathbf{R}$, in series with a parallel combination of an inductance, $L_{a}$, and another resistance, $\mathbf{R}_{\mathbf{L}}$. When the motor rotates, the armature coils move in the stator magnetic field. The induced emf appears across the armature terminals as internally generated voltage
(counter emf), $\mathbf{E}_{\mathbf{g}}$. Therefore, the equivalent electric circuit of the motor is the impedance at stall, connected in series to a voltage source, $\mathrm{E}_{\mathrm{g}}$. This equivalent circuit is shown in Fig. 2.3.1. The physical explanation for


Fig. 2.3.1. Motor equivalent circuit.
this model is that $R_{L}$ represents the losses in the magnetic circuit. This model was found to be accurate for the motor. However, the resistance $\mathbf{R}_{\mathbf{L}}$ is usually larger than $\mathbf{R}$ (typically, about 5-10 times), and hence, the effect of $R_{L}$ on the motor operation is insignificant. Therefore, it is possible to ignore this resistance in most practical applications, and approximate the motor equivalent circuit by $R, L_{a}$, and $E_{g}$. This assumption will be made throughout the book. However, in moving coil motors, the resistance $\mathbf{R}_{\mathbf{L}}$ affects the measurement of the inductance $L_{a}$, and must be considered. Let the motor voltage and current be $V$ and $I_{a}$, respectively. The relation between these variables is given by

$$
\begin{equation*}
V=L_{a} \frac{d I_{a}}{d t}+R I_{a}+E_{g} \tag{2.3.1}
\end{equation*}
$$

where $\mathbf{E}_{\mathbf{g}}$, the internally generated voltage, is proportional to the motor velocity, $\omega$.

$$
\begin{equation*}
E_{g}=K_{E} \omega \tag{2.3.2}
\end{equation*}
$$

The above equations can be combined into

$$
\begin{equation*}
V=L_{a} \frac{d I_{a}}{d t}+R I_{a}+K_{E} \omega \tag{2.3.3}
\end{equation*}
$$

Equation (2.3.3) is known as the electrical equation of the motor.

### 2.3.2. DYNAMIC EQUATIONS

Since the magnetic field in the motor is constant, the current produces a proportional torque

$$
\begin{equation*}
T_{g}=K_{T} I_{a} \tag{2.3.4}
\end{equation*}
$$

Let us denote the moment of inertia of the motor by $\mathbf{J}_{\mathbf{m}}$, and let $\mathbf{T}_{\mathrm{f}}$ represent the constant friction torque in the motor. Also, denote all the viscous friction torques and other torques which are proportional to the velocity by $\mathrm{D} \omega$. Then, the opposing torque in the motor, $\mathbf{T}_{\mathrm{m}}$, is given by

$$
\begin{equation*}
T_{m}=T_{f}+D \omega \tag{2.3.5}
\end{equation*}
$$

Now assume that the motor is coupled to a load. Denote the load moment of inertia by $J_{L}$ and the load opposing torque by $T_{L}$. The relation between the torques and velocity is the following:

$$
\begin{equation*}
T_{g}=\left(J_{m}+J_{L}\right) \frac{d \omega}{d t}+D \omega+T_{f}+T_{L} \tag{2.3.6}
\end{equation*}
$$

Equation (2.3.6) is the dynamic equation of the motor, and along with (2.3.3) and (2.3.4), it describes the relations between the electrical and mechanical variables. Note that equation (2.3.6) is based on a tacit
assumption that motor velocity is the same as that of the load. While this assumption holds in most cases, for high-performance servo systems one has to investigate the case where the deflections of the motor shaft and other elastic parts lead to torsional resonance. The following discussion of the transfer function is based on the equal velocity assumption, and section 2.3.4. describes the torsional resonance.

### 2.3.3. MOTOR TRANSFER FUNCTION

Equation (2.3.3), (2.3.4), and (2.3.6) provide a model for the motor which describes the relation between its variables. However, when the motor is used as a component in a system, it is desired to describe it by the appropriate transfer function between the motor voltage and its velocity. For this purpose we can assume that $\mathrm{T}_{\mathrm{L}}=\mathbf{0}$ and $\mathrm{T}_{\mathrm{f}}=\mathbf{0}$, since neither affects the transfer function. If we now apply the Laplace transformation to the motor equations, we get:

$$
\begin{gather*}
V(s)=\left(s L_{a}+R\right) I_{a}(s)+K_{E} \omega(s)  \tag{2.3.7}\\
T_{g}(s)=K_{T} I_{a}(s)  \tag{2.3.8}\\
T_{g}(s)=\left(J_{m}+J_{L}\right) s \omega(s)+D \omega(s) \tag{2.3.9}
\end{gather*}
$$

Let us define the total moment of inertia, J, by

$$
\begin{equation*}
J=J_{m}+J_{L} \tag{2.3.10}
\end{equation*}
$$

Now combine (2.3.8) with (2.3.9) to obtain an expression for the current:

$$
\begin{equation*}
I_{a}(s)=\frac{1}{K_{T}}(s J+D) \omega(s) \tag{2.3.11}
\end{equation*}
$$

Next, combine (2.3.11) with (2.3.7) to form

$$
\begin{align*}
V(s)= & \frac{1}{K_{T}}\left(s L_{a}+R\right)(s J+D) \omega(s)+ \\
& +K_{E} \omega(s) \tag{2.3.12}
\end{align*}
$$

and the corresponding transfer function is

$$
\begin{align*}
G_{m}(s) & =\frac{\omega(s)}{V(s)} \\
& =\frac{K_{T}}{\left(s L_{a}+R\right)(s J+D)+K_{E} K_{T}} \tag{2.3.13}
\end{align*}
$$

The transfer function has two poles, which, in all practical cases, are negative and real. Therefore, it may be written as

$$
\begin{align*}
G_{m}(s) & =\frac{1 / K_{E}}{\left(s \tau_{1}+1\right)\left(s \tau_{2}+1\right)} \\
& =\frac{K_{T}}{J L_{a}} \frac{1}{\left(s-p_{1}\right)\left(s-p_{2}\right)} \tag{2.3.14}
\end{align*}
$$

$\tau_{1}$ and $\tau_{2}$ are the time constants, whereas $\mathrm{p}_{1}$ and $\mathrm{p}_{2}$ are the poles of the transfer function. The poles are the roots of the characteristic equation

$$
\begin{equation*}
s^{2} L_{a} J+s\left(L_{a} D+R J\right)+R D+K_{E} K_{T}=0 \tag{2.3.15}
\end{equation*}
$$

The time constants are related to the poles by

$$
\begin{equation*}
\tau_{1}=-\frac{1}{p_{1}} \quad \tau_{2}=-\frac{1}{p_{2}} \tag{2.3.16}
\end{equation*}
$$

Clearly, the time constants $\tau_{1}$ and $\tau_{2}$ are functions of all the motor parameters.

In the case where some of the parameters are negligible, the transfer function may be simplified considerably. For example, the following discussion considers a common case where the damping factor, D , is negligible and the inductance, $L_{a}$, is small.

## Low Inductance and No Damping

If we assume that the motor damping factor is zero, $D=0$, the transfer function (2.3.13) becomes

$$
\begin{equation*}
G_{m}(s)=\frac{\omega(s)}{V(s)}=\frac{K_{T}}{s^{2} L_{a} J+s R J+K_{E} K_{T}} \tag{2.3.17}
\end{equation*}
$$

The poles in this case are the roots of

$$
\begin{equation*}
s^{2} L_{a} J+s R J+K_{E} K_{T}=0 \tag{2.3.18}
\end{equation*}
$$

These are

$$
\begin{equation*}
p_{1,2}=\frac{-R J \pm \sqrt{(R J)^{2}-4 L_{a} J K_{E} K_{T}}}{2 L_{a} J} \tag{2.3.19}
\end{equation*}
$$

In all practical motors the inductance, $L_{a}$, is small so that

$$
\begin{equation*}
R^{2} J^{2}-4 L_{a} J K_{E} K_{T}>0 \tag{2.3.20}
\end{equation*}
$$

and therefore the two poles are negative real. They are defined as

$$
\begin{equation*}
p_{1}=\frac{-R J+\sqrt{(R J)^{2}-4 L_{a} J K_{E} K_{T}}}{2 L_{a} J} \tag{2.3.21}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{2}=\frac{-R J-\sqrt{(R J)^{2}-4 L_{a} J K_{E} K_{T}}}{2 L_{a} J} \tag{2.3.22}
\end{equation*}
$$

When the inductance $\mathbf{L}_{\mathbf{a}}$ is much smaller than the term $\frac{\mathbf{R}^{2} \mathrm{~J}}{\mathrm{~K}_{\mathrm{E}} \mathrm{K}_{\mathrm{T}}}$, one can use the approximation

$$
\begin{equation*}
\sqrt{1-x}=1-\frac{x}{2} \tag{2.3.23}
\end{equation*}
$$

which is valid for small values of $\mathbf{x}$.
The approximation (2.3.23) gives

$$
\begin{align*}
& \sqrt{(R J)^{2}-4 L_{a} J K_{E} K_{T}} \\
& =R J \sqrt{1-\frac{4 L_{a} K_{E} K_{T}}{R^{2} J}} \\
& =R J\left(1-\frac{2 L_{a} K_{E} K_{T}}{R^{2} J}\right) \tag{2.3.24}
\end{align*}
$$

Now, when (2.3.24) is substituted in (2.3. 21 ) and (2.3.22), it is found that

$$
\begin{align*}
p_{1} & =\frac{-R J+R J\left(1-\frac{2 L_{a} K_{E} K_{T}}{R^{2} J}\right)}{2 L_{a} J} \\
& =\frac{-K_{E} K_{T}}{R J} \tag{2.3.25}
\end{align*}
$$

and

$$
\begin{align*}
p_{2} & =\frac{-R J-R J\left(1-\frac{2 L_{a} K_{E} K_{T}}{R^{2} J}\right)}{2 L_{a} J} \approx \frac{-2 R J}{2 L_{a} J} \\
& =\frac{-R}{L_{a}} \tag{2.3.26}
\end{align*}
$$

In view of (2.3.25) and (2.3.26), the transfer function can be written as

$$
\begin{equation*}
G_{m}(s)=\frac{K_{T} / L_{a} J}{\left(s+\frac{K_{E} K_{T}}{R J}\right)\left(s+\frac{R}{L_{a}}\right)} \tag{2.3.27}
\end{equation*}
$$

or

$$
\begin{equation*}
G_{m}(s)=\frac{1 / K_{E}}{\left(s \tau_{m}+1\right)\left(s \tau_{e}+1\right)} \tag{2.3.28}
\end{equation*}
$$

where

$$
\begin{aligned}
& \tau_{m}=\frac{\mathrm{RJ}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}} \quad \text { [Metric Units] (2.3.29) } \\
& \tau_{\mathrm{m}}=\frac{10_{4} \mathrm{RJ}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}} \quad \text { [English Units] }
\end{aligned}
$$

is the mechanical time constant, and

$$
\begin{equation*}
\tau_{\mathrm{e}}=\frac{\mathrm{L}_{\mathrm{a}}}{\mathrm{R}} \tag{2.3.30}
\end{equation*}
$$

is the electrical time constant.

Note, however, that the poles of the transfer function will be negative reciprocals of $\tau_{\mathrm{m}}$ and $\tau_{\mathrm{e}}$ only if the two time constants are sufficiently different, or if

$$
\begin{equation*}
\tau_{\mathrm{m}}>10 \tau_{\mathrm{e}} \tag{2.3.31}
\end{equation*}
$$

However, if the two time constants are close (as for the Electro-Craft ET-4000 motor), the poles will be functions of the motor parameters, and equation (2.3.18) is to be used to determine them.

Furthermore, if the motor inductance is large enough so that

$$
\begin{equation*}
4 L_{a} J K_{E} K_{T}>R^{2} J^{2} \tag{2.3.32}
\end{equation*}
$$

the two poles of the transfer function become complex. However, this is not the case for the majority of DC motors.

### 2.3.4. TORSIONAL RESONANCE

In the previous discussion of the motor equation it was assumed that the velocity of all the parts in the motor-load system is identical; i.e., the motor and load may be approximated by a single body. This assumption is not accurate for high-performance servo systems, since the mechanical parts of the system are elastic, and they deflect under torque. Consequently, the instantaneous velocities of various parts are different, and at some frequencies will be in opposite directions. This condition allows the system to store a large amount of mechanical energy, which results in notice-
able angular vibrations. This phenomenon is called torsional resonance.

Since many motors are coupled to both an inertial load and a tachometer, it is reasonable to approximate this mechanical system by three solid bodies, coupled by inertialess shafts. The approximate system is shown in Fig. 2.3.2. Let us denote the


Fig. 2.3.2. Three-body model for load-motortachometer system.
moments of inertia and angular positions of the load, motor, and tachometer by $J_{1}, J_{m}, J_{2}$, and $\theta_{1}, \theta_{m}, \theta_{2}$, respectively. Also, denote the stiffness and damping factors of the equivalent shafts by $\mathrm{K}_{1}, \mathrm{D}_{1}$ and $K_{2}, D_{2}$. Note that this model is an overall approximation and, hence, $\mathrm{K}_{1}$ and $D_{1}$ may be influenced by characteristics of the motor armature, shaft, coupling, or the load.

In order to derive the dynamic equations for this system, let $\mathbf{T}_{1}$ and $\mathbf{T}_{2}$ be the torques delivered from the motor to $J_{1}$ and $\mathrm{J}_{2}$, respectively. The dynamic equations of the three bodies are:

$$
\begin{align*}
& T_{g}=J_{m} \ddot{\theta}_{m}+D \dot{\theta}_{m}+T_{1}+T_{2}  \tag{2.3.33}\\
& T_{1}=J_{1} \ddot{\theta}_{1}  \tag{2.3.34}\\
& T_{2}=J_{2} \ddot{\theta}_{2} \tag{2.3.35}
\end{align*}
$$

The deflections of the shafts are described by the following equations:

$$
\begin{aligned}
& T_{1}=K_{1}\left(\theta_{m}-\theta_{1}\right)+D_{1}\left(\dot{\theta}_{m}-\dot{\theta}_{1}\right)(2.3 .36) \\
& T_{2}=K_{2}\left(\theta_{m}-\theta_{2}\right)+D_{2}\left(\dot{\theta}_{m}-\dot{\theta}_{2}\right)(2.3 .37)
\end{aligned}
$$

It may be useful to consider the electrical analog of this mechanical system. The analogy between the variables is the following:

A torque $\mathbf{T}_{\mathbf{g}}$ is equivalent to a proportional current.

A velocity $\dot{\theta}$ is equivalent to a proportional voltage.

A moment of inertia J is equivalent to a proportional capacitance.

A damping factor $D$ is equivalent to resistance of 1/D.

A stiffness factor $K$ is equivalent to inductance of $1 / K$.

$$
\begin{equation*}
V=L_{a} \frac{d I_{a}}{d t}+R I_{a}+K_{E} \dot{\theta}_{m} \tag{2.3.41}
\end{equation*}
$$

Now consider the electrical equations,

$$
\begin{equation*}
J_{1} \ddot{\theta}_{1}+D_{1} \dot{\theta}_{1}+K_{1} \theta_{1}=D_{1} \dot{\theta}_{m}+K_{1} \theta_{m} \tag{2.3.38}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{J}_{2} \ddot{\theta}_{2}+\mathrm{D}_{2} \dot{\theta}_{2}+\mathrm{K}_{2} \theta_{2}=\mathrm{D}_{2} \dot{\theta}_{\mathrm{m}}+\mathrm{K}_{2} \theta_{\mathrm{m}} \tag{2.3.39}
\end{equation*}
$$

$$
\begin{equation*}
T_{g}=K_{T} I_{a} \tag{2.3.42}
\end{equation*}
$$



Fig. 2.3.3. Electrical analogy of three-body mechanical system.

The above equations describe the behavior of the system. In order to simplify the analysis, substitute (2.3.42) in (2.3.38) and apply the Laplace transformation to all the equations.

$$
\begin{align*}
& \left(s L_{a}+R\right) I_{a}(s)+K_{E} s \theta_{m}(s)=V(s) \\
& -K_{T} I_{a}(s)+\left(s^{2} J_{m}+s D\right) \theta_{m}(s)+ \\
& \quad+s^{2} J_{1} \theta_{1}(s)+s^{2} J_{2} \theta_{2}(s)=0 \\
& \left(s D_{1}+K_{1}\right) \theta_{m}(s)- \\
& \quad-\left(s^{2} J_{1}+s D_{1}+K_{1}\right) \theta_{1}(s)=0 \\
& \left(s D_{2}+K_{2}\right) \theta_{m}(s)- \\
& \quad-\left(s^{2} J_{2}+s D_{2}+K_{2}\right) \theta_{2}(s)=0 \tag{2.3.43}
\end{align*}
$$

The system equations (2.3.43) may be written in matrix form for simplification.
(See Below)
The matrix equation (2.3.44) is a complete model and includes (2.3.13) as a special case. Note also that if the mechanical system is simple and can be approximated by two bodies $J_{m}$ and $J_{1}$, we can obtain the
corresponding model by crossing the fourth row and column in the matrix of (2.3.44).

Equation (2.3.44) enables us to derive the transfer function between the input voltage, V , and the velocities of the three bodies. The transfer function of the motor velocity is given by (2.3.45), whereas (2.3.46) is the transfer function for either the tachometer or the load velocity (when subscripts 1 and 2 are interchanged).

$$
\begin{align*}
G_{m}(s)= & \frac{s \theta_{m}(s)}{V(s)} \\
= & \frac{1}{\Delta}\left[K_{T}\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\right. \\
& \left.\cdot\left(s^{2} J_{2}+s D_{2}+K_{2}\right)\right] \tag{2.3.45}
\end{align*}
$$

$$
\begin{align*}
G_{2}(s) & =\frac{s \theta_{2}(s)}{V(s)} \\
& =\frac{1}{\Delta}\left[K_{T}\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\left(s D_{2}+K_{2}\right)\right] \tag{2.3.46}
\end{align*}
$$

where $\Delta$ is the determinant of the matrix in (2.3.44), divided by s.
$\left[\begin{array}{lllll}s L_{a}+R & s K_{E} & 0 & 0 \\ -K_{T} & s^{2} J_{m}+s D & s^{2} J_{1} & s^{2} J_{2} \\ 0 & s D_{1}+K_{1} & -\left(s^{2} J_{1}+s D_{1}+K_{1}\right) & 0 \\ 0 & s_{2}+K_{2} & 0 & -\left(s^{2} J_{2}+s D_{2}+K_{2}\right)\end{array}\right]\left[\begin{array}{l}I_{a}(s) \\ \theta_{m}(s) \\ \theta_{1}(s) \\ \theta_{2}(s)\end{array}\right]=\left[\begin{array}{l}V(s) \\ 0 \\ 0 \\ 0\end{array}\right]$
$\Delta=\left(s L_{a}+R\right)\left[\left(s J_{m}+D\right)\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\left(s^{2} J_{2}+s D_{2}+K_{2}\right)+\right.$

$$
\begin{align*}
& +s J_{2}\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\left(s D_{2}+K_{2}\right)+ \\
& \left.+s J_{1}\left(s D_{1}+K_{1}\right)\left(s^{2} J_{2}+s D_{2}+K_{2}\right)\right]+ \\
& +K_{E} K_{T}\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\left(s^{2} J_{2}+s D_{2}+K_{2}\right) \tag{2.3.47}
\end{align*}
$$

Using the notations,

$$
R_{i}(s)=s^{2} J_{i}+s D_{i}+K_{i} \quad i=1,2
$$

$$
\begin{equation*}
Q_{i}(s)=s D_{i}+K_{i} \quad i=1,2 \tag{2.3.48}
\end{equation*}
$$

The transfer functions may be written as

$$
\begin{align*}
G_{m}(s) & =\frac{s \theta_{m}(s)}{V(s)} \\
& =\frac{1}{\Delta}\left[K_{T} R_{1}(s) R_{2}(s)\right]  \tag{2.3.49}\\
G_{2}(s) & =\frac{s \theta_{2}(s)}{V(s)} \\
& =\frac{1}{\Delta}\left[K_{T} R_{1}(s) Q_{2}(s)\right] \tag{2.3.50}
\end{align*}
$$

where

$$
\begin{align*}
\Delta & =\left(s L_{a}+R\right)\left[\left(s J_{m}+D\right) R_{1} R_{2}+s J_{2} R_{1} Q_{2}+s J_{1} Q_{1} R_{2}\right]+ \\
& +K_{E} K_{T} R_{1} R_{2} \tag{2.3.51}
\end{align*}
$$

Note that the two transfer functions, (2.3. 49) and (2.3.50), have the same poles, but the zeros may be different. Let us consider the zeros and the poles of the tachometer velocity transfer function.

The zeros are the roots of the equation

$$
\begin{equation*}
\left(s^{2} J_{1}+s D_{1}+K_{1}\right)\left(s D_{2}+K_{2}\right)=0 \tag{2.3.52}
\end{equation*}
$$

This equation has one real root and two complex ones.

$$
\begin{equation*}
z_{1}=-\frac{K_{2}}{D_{2}} \tag{2.3.53}
\end{equation*}
$$

and

$$
\begin{equation*}
z_{2,3}=\frac{-D_{1} \pm j \sqrt{4 K_{1} J_{1}-D_{1}^{2}}}{2 J_{1}} \tag{2.3.54}
\end{equation*}
$$

The poles of the transfer function are the roots of the characteristic equation,

$$
\begin{equation*}
\Delta=0 \tag{2.3.55}
\end{equation*}
$$

Since $\Delta$ is a polynomial of sixth order, there will be six roots. It would be impossible to determine the roots exactly, but we can find reasonable approximations.
Note that at low frequencies we may perform the following approximations:

$$
R_{i}(s)=K_{i}
$$

$$
\begin{align*}
& Q_{i}(s) \approx K_{i}  \tag{2.3.56}\\
& s L_{a}+R \approx R
\end{align*}
$$

Under these conditions equation (2.3.55) becomes

$$
\begin{align*}
& s R K_{1} K_{2}\left(J_{m}+J_{1}+J_{2}\right)+ \\
& +R D K_{1} K_{2}+K_{E} K_{T} K_{1} K_{2}=0 \tag{2.3.57}
\end{align*}
$$

Define J, the total moment of inertia, as

$$
\begin{equation*}
J=J_{m}+J_{1}+J_{2} \tag{2.3.58}
\end{equation*}
$$

Then (2.3.57) becomes

$$
\begin{equation*}
\left(s R J+R D+K_{E} K_{T}\right) K_{1} K_{2}=0 \tag{2.3.59}
\end{equation*}
$$

and the low frequency pole is

$$
\begin{equation*}
p_{1}=-\frac{K_{E} K_{T}+R D}{R J} \tag{2.3.60}
\end{equation*}
$$

If the motor damping factor, $D$, is small and can be ignored, the pole $p_{1}$ becomes

$$
\begin{equation*}
p_{1}=-\frac{K_{E} K_{T}}{R J} \tag{2.3.61}
\end{equation*}
$$

which corresponds to the mechanical time constant of the motor.

In order to determine the other poles of the motor transfer function, we assume that the other poles correspond to frequencies much higher than $p_{1}$. This assumption is realistic, especially for moving coil motors, where the other poles are about 50 times larger than $p_{1}$. Thus, assume that for these poles,

$$
\begin{equation*}
|s| \gg \frac{J}{J_{m}}\left|p_{1}\right|=\frac{K_{E} K_{T}+R D}{R J_{m}} \tag{2.3.62}
\end{equation*}
$$

It follows that

$$
\begin{equation*}
|s| R J_{m} \gg K_{E} K_{T} \tag{2.3.63}
\end{equation*}
$$

and if we multiply both sides by $R_{1}(s) R_{2}(s)$, it becomes
$\left|s R \quad J_{m} R_{1}(s) R_{2}(s)\right| \gg$

$$
\begin{equation*}
\Rightarrow\left|K_{E} K_{T} R_{1}(s) R_{2}(s)\right| \tag{2.3.64}
\end{equation*}
$$

Note, however, that the two terms in (2.3.64) are parts of $\Delta$, as given by equation (2.3.51). This suggests that the term on the right-hand side is small and, hence, it may be ignored.

Similarly, we may conclude from (2.3.62) that

$$
\begin{equation*}
|s| \gg \frac{R D}{R J_{m}} \tag{2.3.65}
\end{equation*}
$$

and therefore

$$
\begin{equation*}
\left|J_{m} s\right| \gg D \tag{2.3.66}
\end{equation*}
$$

Thus, we may approximate the term $\left(\mathbf{s} \mathbf{J}_{\mathrm{m}}+\right.$ D) by $\mathrm{sJ} \mathrm{m}_{\mathrm{m}}$.

In view of the above simplifications, equation (2.3.55) becomes

$$
\begin{align*}
& \Delta=s\left(s L_{a}+R\right) x \\
& x\left[J_{m} R_{1} R_{2}+J_{2} R_{1} Q_{2}+J_{1} Q_{1} R_{2}\right]=0 \tag{2.3.67}
\end{align*}
$$

The root at zero represents the low frequency pole, $p_{1}$. The second pole is the root of

$$
\begin{equation*}
\left(s L_{a}+R\right)=0 \tag{2.3.68}
\end{equation*}
$$

or

$$
\begin{equation*}
p_{2}=-\frac{R}{L_{a}} \tag{2.3.69}
\end{equation*}
$$

which corresponds to the electrical time constant of the motor. The remaining four poles are the roots of the equation

$$
\begin{equation*}
J_{m} R_{1} R_{2}+J_{2} R_{1} Q_{2}+J_{1} Q_{1} R_{2}=0 \tag{2.3.70}
\end{equation*}
$$

In order to approximate them, assume that the tachometer moment of inertia, $\mathbf{J}_{2}$, is much smaller than that of the motor or the load. This assumption is reasonable in most cases. Consequently, we choose to ignore the term $\mathrm{J}_{\mathbf{2}} \mathbf{R}_{\mathbf{1}} \mathbf{Q}_{\mathbf{2}}$ in equation (2.3. 70). The remaining terms in the equation are

$$
\begin{equation*}
\left(J_{m} R_{1}+J_{1} Q_{1}\right) R_{2}=0 \tag{2.3.71}
\end{equation*}
$$

or, explicitly,

$$
\begin{aligned}
& {\left[J_{m}\left(s^{2} J_{1}+s D_{1}+K_{1}\right)+\right.} \\
& \left.+J_{1}\left(s D_{1}+K_{1}\right)\right]\left[s^{2} J_{2}+s D_{2}+K_{2}\right]=0
\end{aligned}
$$

Note that one pair of poles is the solution of

$$
\begin{equation*}
\mathrm{s}^{2} \mathrm{~J}_{2}+\mathrm{sD} \mathrm{D}_{2}+\mathrm{K}_{2}=0 \tag{2.3.72}
\end{equation*}
$$

and it is determined mostly by the tachometer parameters. Therefore, this pair is called the tachometer resonance poles:

$$
\begin{equation*}
p_{3,4}=\frac{-D_{2} \pm j \sqrt{4 J_{2} K_{2}-D_{2}^{2}}}{2 J_{2}} \tag{2.3.73}
\end{equation*}
$$

The frequency of the tachometer poles is

$$
\begin{equation*}
\omega_{T} \approx \sqrt{\frac{\mathrm{~K}_{2}}{\mathrm{~J}_{2}}} \quad[\mathrm{rad} / \mathrm{s}] \tag{2.3.74}
\end{equation*}
$$

Similarly, the other pair of poles is the solution of

$$
\begin{equation*}
s^{2} J_{m} J_{1}+s\left(J_{m}+J_{1}\right) D_{1}+\left(J_{m}+J_{1}\right) K_{1}=0 \tag{2.3.75}
\end{equation*}
$$

or, if we define the equivalent moment of inertia, $\mathrm{J}_{\mathrm{e}}$, as

$$
\begin{equation*}
J_{e}=\frac{J_{m} J_{1}}{J_{m}+J_{1}} \tag{2.3.76}
\end{equation*}
$$

equation (2.3.75) becomes

$$
\begin{equation*}
s^{2} J_{e}+s D_{1}+K_{1}=0 \tag{2.3.77}
\end{equation*}
$$

The poles are

$$
\begin{equation*}
p_{5,6}=\frac{-D_{1} \pm j \sqrt{4 K_{1} J_{e}-D_{1}^{2}}}{2 J_{e}} \tag{2.3.78}
\end{equation*}
$$

and the frequency of the poles is approximately

$$
\begin{equation*}
\omega_{\mathrm{L}}=\sqrt{\frac{K_{1}}{J_{e}}} \quad[\mathrm{rad} / \mathrm{s}] \tag{2.3.79}
\end{equation*}
$$

Clearly, these poles are formed by the interaction between the motor and the load, and they are called the load resonance poles. In summary, the transfer function between the motor voltage and the tachometer velocity has three zeros and six poles.

$$
\begin{align*}
G_{2}(s) & =\frac{s \theta_{2}(s)}{V(s)} \\
& =\frac{K_{T} D_{2}}{J_{m} J_{2} L_{a}} \frac{\left(s-z_{1}\right)\left(s-z_{2}\right)\left(s-z_{3}\right)}{\left(s-p_{1}\right)\left(s-p_{2}\right)\left(s-p_{3}\right)\left(s-p_{4}\right)\left(s-p_{5}\right)\left(s-p_{6}\right)} \tag{2.3.80}
\end{align*}
$$

The first zero, $\mathbf{z}_{\mathbf{1}}$, is negative real and the other two are complex. They are given by equations (2.3.53) and (2.3.54). On the other hand, the pole $p_{1}$ is at low frequency and is caused by the mechanical load. The second pole, $\mathrm{p}_{2}$, corresponds to the electrical time constant of the motor. The remaining poles are two complex pairs, where the first one is determined mostly by the tachometer and the other pair is caused by the motor and the load.

Note that the zero $\mathbf{z}_{1}$ and the pole $\mathrm{p}_{2}$ are negative real, and correspond to high frequencies; therefore, their contribution to the motor transfer function is insignificant and they may be ignored. Thus, the transfer function may be reduced to the form

$$
\begin{align*}
G_{2}(s) & =\frac{s \theta_{2}(s)}{V(s)} \\
& =\frac{K_{T} K_{2}}{R J_{m} J_{2}} \frac{\left(s-z_{2}\right)\left(s-z_{3}\right)}{\left(s-p_{1}\right)\left(s-p_{3}\right)\left(s-p_{4}\right)\left(s-p_{5}\right)\left(s-p_{6}\right)} \tag{2.3.81}
\end{align*}
$$

The transfer function was examined for several motors, under various load conditions, and was found to be in excellent agreement with the theoretical analysis given above. For example, when the ElectroCraft M-1030 motor with the M-110 tachometer was loaded by an inertial load, $\mathrm{J}_{\mathrm{L}}=0.0006 \mathrm{oz}-\mathrm{in}^{\mathrm{s}} \mathrm{s}^{2}$ (approximately 42.4 $\mathrm{gcm}^{2}$ ), the following poles and zeros were measured:

$$
\begin{aligned}
& p_{1}=-20 \mathrm{~Hz} \\
& p_{3,4}=-200 \pm \mathrm{j} 4900 \mathrm{~Hz}
\end{aligned}
$$

$$
\begin{aligned}
& p_{5,6}=-177 \pm j 4000 \mathrm{~Hz} \\
& z_{2,3} \cong \pm j 2900 \mathrm{~Hz}
\end{aligned}
$$

The pole-zero distribution of the transfer function is shown in Fig. 2.3.4, and the motor-tachometer frequency response is shown in Fig. 2.3.5.


Fig. 2.3.4. Poles and zeros of motor-tachometer transfer function.

The analysis of the resonance phenomenon was based on the assumption model of the three bodies with inertialess shafts, where the stiffness and damping were constant for each shaft. However, as the motor temperature rises due to power dissipation, the mechanical properties of the armature and coupling change and, consequently, the locations of the poles and zeros change


Fig. 2.3.5. Frequency response of motor-tachometer with inertial load.
along with it. In general, the damping and the shaft compliance tend to increase with rising temperature, resulting in increase in the real part of poles and zeros, and decrease in their imaginary part.

### 2.3.5. SPEED-TORQUE CURVE

When a constant voltage, V , is applied to the motor terminals, the motor shaft will accelerate according to (2.3.3.) and 2.3.6.) and attain a final steady state velocity.

Under steady state conditions the current is constant and the motor equations become

$$
\begin{align*}
& V=R I_{a}+K_{E} \omega  \tag{2.3.82}\\
& T_{g}=K_{T} I_{a} \tag{2.3.83}
\end{align*}
$$

When the two equations are combined they become

$$
\begin{equation*}
V=\frac{T_{g} R}{K_{T}}+K_{E} \omega \tag{2.3.84}
\end{equation*}
$$

Equation (2.3.84) shows the relation between the velocity and the generated torque at steady state conditions. Fig. 2.3.6 shows that the equation forms a straight line in the speed-torque plane.


Fig. 2.3.6. DC motor speed-torque curve.

If the torque is zero, the no-load velocity $\omega_{\mathrm{NL}^{\prime}}$ is given by

$$
\begin{equation*}
\omega_{\mathrm{NL}}=\frac{\mathrm{V}}{\mathrm{~K}_{\mathrm{E}}} \tag{2.3.85}
\end{equation*}
$$

On the other hand, the generated torque at stall, $\mathrm{T}_{\mathrm{gs}}$, equals

$$
\begin{equation*}
T_{g s}=\frac{V K_{T}}{R} \tag{2.3.86}
\end{equation*}
$$

Accordingly, the relation between the speed and the torque can be expressed by the equation

$$
\begin{equation*}
\omega=\omega_{N L}-R_{m} T_{g} \tag{2.3.87}
\end{equation*}
$$

where

$$
\begin{equation*}
R_{m}=\frac{R}{K_{T} K_{E}} \tag{2.3.88}
\end{equation*}
$$

$R_{m}$, the slope constant of the speed-torque curve, is called the speed regulation constant. For a given motor, we can plot a family of speed-torque lines that correspond to various terminal voltages. For example, if the motor parameters are:

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{T}}=13.5 \quad \mathrm{oz}-\mathrm{in} / \mathrm{A} \\
& \mathrm{~K}_{\mathrm{E}}=10 \mathrm{~V} / \mathrm{krpm} \\
& \mathrm{R}=1 \Omega
\end{aligned}
$$

and the applied voltages are 10, 20, and 30 V, the no-load velocity will be 1000,2000 , and 3000 rpm, respectively. The complete family of speed-torque lines is shown in Fig. 2.3.7.

This family of speed-torque characteristics used to be important to users of DC motors in the past, so that they could graphically


Fig. 2.3.7. Speed-torque curves for terminal voltages of 10,20 , and 30 V .
trace motor speed regulation in response to load and voltage changes. However, since the permanent magnet DC motors have linear characteristics, the designer can calculate all needed information from the knowledge of the motor most basic parameters, namely the $K_{T}, \mathbf{R}, \mathbf{J}$ and $\mathbf{R}_{\mathrm{th}}$. The value of $\mathbf{R}_{m}$ can be calculated from $\mathbf{R}$ and $\mathrm{K}_{\mathrm{T}}$, and static and dynamic characteristics can be derived, as we have just seen in this chapter.

### 2.4. POWER DISSIPATION IN DC MOTORS

### 2.4.1.ORIGINS OF POWER DISSIPATION

The DC motor is used to produce mechanical power from electric power, and as such it may be viewed as an energy converter. It converts electric power into mechanical power during acceleration and steady run, and back to electric form during deceleration. However, the motor is not an ideal converter, due to the armature resistance and other losses. Therefore, it has heat losses as by-products of the energy conversion. The power flow through the motor and the distribution of losses are illustrated in Fig. 2.4.1. The figure shows that the motor losses can be grouped into two divisions. The load sensitive losses are dependent upon the generated torque, and the speed sensitive losses are determined by rotational speed. Since those losses are important in establishing the limits of mo-
tor application, let us consider them in detail.

## Winding Losses $\mathbf{I}_{\mathrm{a}}^{2} \mathrm{R}$

These losses are caused by the resistance of the motor and equal $I_{a}^{2}$ R. Clearly, they depend on the generated torque, as the armature current, $\mathbf{I}_{\mathbf{a}}$, is proportional to it.

## Brush Contact Losses

As the carbon brush slides over the conductive surface of the commutator, a film is formed which is necessary in providing lubrication for proper brush function. It has some of the properties of a dielectric material. Thus, it is not uncommon for this apparent contact resistance to be of sizable value with respect to the actual winding resistance. Current flow through the brush and the film creates heat with resulting loss of power.


Fig. 2.4.1. Power losses in a permanent magnet motor.

The problem of brush and contact losses is complex, owing to three factors: (1) the carbon has a negative temperature coefficient of resistivity which causes the resistance of the brush itself to decrease with temperature; (2) the contact film between brush and commutator breaks down as current density increases, thereby also decreasing the apparent resistance of the brushcommutator interface; and (3) at increasing rotational speeds a cushion of air is swept along by the commutator, producing an aerodynamic lift under the brush. This brush lift effect tends to increase the apparent contact resistance. To summarize the characteristics of the losses it can be said that:
(1) They are unpredictable, depending upon factors of current density, rotational speed, and even atmospheric conditions (humidity promotes the film-forming process).
(2) Increasing load (current) tends to decrease the percentage of machine loss at the brush-commutator interface.
(3) Increasing speed tends to increase the proportional loss at brush-commutator interface.

## Eddy Current Losses

Eddy currents are phenomena associated with a change of magnetic field in a medium that can also support a flow of electric current. In the case of the PM motor, the medium that experiences the change of magnetic field and allows current to flow
is the iron of the armature. Fig. 2.4.2. shows how all parts of the armature iron undergo a change of magnetization as it rotates in the magnetic field.

The circulating currents (eddy currents) which are produced in the iron are proportional to speed, and can have significant heating effect on the motor, particularly when the motor operates at high speed. Eddy currents, along with hysteresis losses, usually determine the maximum speed that may be obtained from an iron core armature.


Fig. 2.4.2. All parts of the armature undergo a change of magnetization as it rotates in the magnetic field, thus causing the magnetic domain boundaries to shift.

Eddy current effect is not present in all PM motors. There is a type of low inertia motor, designed for maximum acceleration capability, which does not have moving iron in its magnetic field. These "nonferrous armature" motors have neither iron eddy currents nor hysteresis effects, and consequently require lower power inputs to obtain high rotational speeds. Typical of this type are Electro-Craft's MCM ${ }^{\circledR *}$ Moving Coil Motors.

[^0]

Fig. 2.4.3. Short circuit current in a commutated coil.

Eddy current effects can be minimized in iron core armatures by using laminated sheet steel for the core. The use of thinner laminations results in reduced eddy currents. Silicon steel with high electrical resistance will further reduce eddy current heating.

## Hysteresis Effect

Fig. 2.4.2. shows how the iron core of the armature experiences changing magnetization as it rotates in the stator field. As the armature rotates, magnetic domain boundaries shift. The resistance to this magnetic domain shifting also causes heat generation that increases with motor speed. Hysteresis loss is a characteristic of the type of steel used. It is usually combined with eddy current effect and called "iron loss."

## Windage and Friction

This category of motor heat generation includes all those factors which produce a
mechanical drag on the machine, such as bearing friction, friction of the brushes on the commutator, and air resistance. The last item is usually insignificant in smaller sized servomotors. The magnitude of windage and the friction losses will depend upon the mechanical features of design; i.e., coefficient of friction between brush and commutator and size of bearing and lubricant.

## Short Circuit Currents

This term does not imply a fault in a machine, but rather describes a normal current which exists briefly in the armature coil that is commutated.

As shown in Fig. 2.4.3, at the instant the brush is riding on two commutator segments, it actually provides a short circuit connection between the ends of the coil that is connected to those two segments.

It can be seen that the start of coil \#1 is connecced to segment \#1, while the end of coil \#1 is connected to segment \#2. It can easily be surmised that any voltage which exists across the coil terminations will cause a current to flow through the coil. This current can not be measured directly, but in cases where it becomes excessive, it will cause arcing at the trailing edge of the motor brush.

The effect of short circuit current is to contribute a component of loss which increases with motor speed. It appears as a viscous drag on the armature. It will also impose a maximum speed limitation on motors which are not otherwise limited by their iron losses.

Short circuit current effects can be minimized by proper motor design so that the brushes only commutate coils which are in regions of low magnetic flux. This tends to minimize the voltage that will exist across the ends of the coil.

### 2.4.2. POWER DISSIPATION

The speed sensitive losses, when combined together, act as a velocity dependent opposing torque, $\mathbf{T}_{\mathrm{m}}$, which contains a constant term, $\mathrm{T}_{\mathrm{f}}$, and a velocity dependent term, D $\omega$.

$$
\begin{equation*}
T_{m}=T_{f}+D \omega \tag{2.4.1}
\end{equation*}
$$

The terms $\mathbf{T}_{f}$ and $\mathbf{D}$ may be interpreted as constant friction and viscous damping, but they represent all the velocity sensitive losses.

Now suppose that the motor is coupled to a load with a moment of inertia, $J_{L}$, and a load torque, $T_{L}$. The electric and dynamic equations will be (2.3.3) and (2.3.6). Since we are interested in power dissipation, the inductance, $L_{a}$, has very small effect and may be ignored. The simplified equations for the motor would be:

$$
\begin{align*}
& V=R I_{a}+K_{E} \omega  \tag{2.4.2}\\
& I_{a}=\frac{1}{K_{T}}\left[\left(J_{m}+J_{L}\right)_{d t}^{d \omega}+T_{m}+T_{L}\right] \tag{2.4.3}
\end{align*}
$$

The input power to the motor, $\mathbf{P}_{\mathbf{i}}$, is given by

$$
\begin{align*}
P_{i}=V I_{a} & =I_{a}\left(R I_{a}+K_{E} \omega\right) \\
& =I_{a}^{2} R+K_{E} \omega I_{a} \tag{2.4.4}
\end{align*}
$$

In view of (2.4.3), $P_{i}$ becomes

$$
\begin{equation*}
P_{i}=I_{a}^{2} R+\frac{K_{E} \omega}{K_{T}}\left[T_{m}+T_{L}+\left(J_{m}+J_{L}\right) \frac{d \omega}{d t}\right] \tag{2.4.5}
\end{equation*}
$$

or

$$
\begin{align*}
P_{i}=I_{a}^{2} R & +\frac{K_{E} \omega}{K_{T}} T_{m}+\frac{K_{E} \omega}{K_{T}} T_{L}+ \\
& +\frac{K_{E}\left(J_{m}+J_{L}\right)}{K_{T}} \omega \frac{d \omega}{d t} \tag{2.4.6}
\end{align*}
$$

Note that the last term of (2.4.6) is equal to

$$
\begin{align*}
& \frac{K_{E}\left(J_{m}+J_{L}\right)}{K_{T}} \omega \frac{d \omega}{d t} \\
& =\frac{K_{E}\left(J_{m}+J_{L}\right)}{2 K_{T}} \frac{d\left(\omega^{2}\right)}{d t} \tag{2.4.7}
\end{align*}
$$

This quantity will have zero average value when the velocity is constant or periodic, or over any time interval where the final velocity is equal to the initial velocity. Consequently, the contribution of this term to the power dissipation is zero and it can be ignored. Of the remaining terms we can identify the last one as the output power which is delivered to the load,

$$
\begin{equation*}
P_{o}=\frac{K_{E} \omega T_{L}}{K_{T}} \tag{2.4.8}
\end{equation*}
$$

Note that the ratio of the voltage and torque constants, $\mathrm{K}_{\mathrm{E}} / \mathrm{K}_{\mathrm{T}}$, forms the conversion factor in (2.4.8), whose value depends on the units used in the equation. If all the quantities are expressed in basic or main SI units, i.e., $\left[P_{0}\right]=W,[\omega]=\operatorname{rad} \cdot \mathrm{s}^{-1}$, $\left[\mathrm{T}_{\mathrm{L}}\right]=\mathrm{Nm},\left[\mathrm{K}_{\mathrm{E}}\right]=\mathrm{V} / \mathrm{rad} \cdot \mathrm{s}^{-1},\left[\mathrm{~K}_{\mathrm{T}}\right]=$ $\mathrm{Nm} / \mathrm{A}$, then $\mathrm{K}_{\mathrm{E}} / \mathrm{K}_{\mathrm{T}}=1$ and (2.4.8) becomes

$$
\begin{equation*}
P_{o}=\omega T_{L} \tag{2.4.8a}
\end{equation*}
$$

The same applies for other quantities in this section.

The remaining two terms are the losses in the motor.

$$
P_{L}=I_{a}^{2} R+\frac{K_{E} \omega T_{m}}{K_{T}}
$$

Note that the first term, $I_{a}^{2} R$, is the winding resistive loss and the second term corresponds to the velocity sensitive losses. Moreover, by substituting equation (2.4.1) into (2.4.9), the losses become

$$
\begin{equation*}
P_{L}=I_{a}^{2} R+\frac{K_{E} \omega T_{f}}{K_{T}}+\frac{K_{E} D \omega^{2}}{K_{T}} \tag{2.4.10}
\end{equation*}
$$

Equation (2.4.10) describes the instantaneous power dissipation in general, and it can be used to determine the losses under specific velocity profiles. In the following discussion the power dissipation is evaluated for two common types of angular motion: constant speed and incremental motion.

### 2.4.3. DISSIPATION AT CONSTANT VELOCITY

When the motor runs at a constant velocity, $\omega_{m}$, the power dissipation may be found directly from (2.4.10). This is given by

$$
\begin{equation*}
P_{L}=I_{a}^{2} R+\frac{K_{E} T_{f} \omega_{m}}{K_{T}}+\frac{K_{E} D \omega_{m}^{2}}{K_{T}} \tag{2.4.11}
\end{equation*}
$$

This is illustrated by the following example.

## Example

Given are the following motor and load parameters:

$$
\begin{aligned}
\mathrm{R}= & 1 \Omega \\
\mathrm{~K}_{\mathrm{T}}= & 6 \mathrm{oz}-\mathrm{in} / \mathrm{A}=4.24 \cdot 10^{-2} \mathrm{Nm} / \mathrm{A} \\
\mathrm{~K}_{\mathrm{E}}= & 4.45 \mathrm{~V} / \mathrm{krpm} \\
& =4.24 \cdot 10^{-2} \mathrm{~V} / \mathrm{rad} \cdot \mathrm{~s}^{-1} \\
\mathrm{~J}_{\mathrm{m}}= & 0.5 \cdot 10^{-3} \mathrm{oz}-\mathrm{in}^{2} \mathrm{~s}^{2} \\
& =3.53 \cdot 10^{-6} \mathrm{~kg} \mathrm{~m}^{2} \\
\mathrm{~T}_{\mathrm{f}}= & 6 \mathrm{oz}-\mathrm{in}=4.24 \cdot 10^{-2} \mathrm{Nm} \\
\mathrm{D}= & 0.04 \frac{\mathrm{oz}-\mathrm{in}}{\mathrm{rad} / \mathrm{s}}=2.825 \cdot 10^{-4} \frac{\mathrm{Nm}}{\mathrm{rad} \cdot \mathrm{~s}^{-1}} \\
\mathrm{~J}_{\mathrm{L}}= & 2 \cdot 10^{-3} \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}}=1.412 \cdot 10^{-5} \mathrm{~kg} \mathrm{~m} \\
\mathrm{~T}_{\mathrm{L}} & =12 \mathrm{oz}-\mathrm{in}=8.474 \cdot 10^{-2} \mathrm{Nm}
\end{aligned}
$$

The motor will run at constant velocity $\omega_{\mathrm{m}}=300 \mathrm{rad} / \mathrm{s}(2860 \mathrm{rpm})$. The power dissipation is found as follows.

The armature current is found from (2.4.3) and (2.4.1)

$$
\begin{aligned}
I_{a} & =\frac{1}{K_{T}}\left(T_{f}+D \omega_{m}+T_{L}\right) \\
& =\frac{1}{6}[6+0.04 \cdot 300+12]=5 \mathrm{~A}
\end{aligned}
$$

The power dissipation can be found from (2.4.11). It can be calculated by using either the SI or the British system of units. In this example we will use the SI system.

Note that in this system $\mathrm{K}_{\mathrm{T}}$ and $\mathrm{K}_{\mathrm{E}}$ are numerically equal, when expressed in basic and main SI units.

$$
\begin{aligned}
P_{L}= & 5^{2} \cdot 1+\frac{4.24 \cdot 10^{-2} \cdot 300 \cdot 4.24 \cdot 10^{-2}}{4.24 \cdot 10^{-2}}+ \\
& +\frac{4.24 \cdot 10^{-2} \cdot 2.825 \cdot 10^{-4} \cdot 300^{2}}{4.24 \cdot 10^{-2}} \\
P_{L}= & 25+12.72+25.42=63.14 \mathrm{~W}
\end{aligned}
$$

### 2.4.4. DISSIPATION DURING INCREMENTAL MOTION

Equation (2.4.10) for power dissipation can be applied to the case of incremental motion where the motor steps periodically following a trapezoidal velocity profile, as shown in Fig. 2.4.4.


Fig. 2.4.4. Motor velocity profile.
The motor accelerates over $t_{1}$ to the velocity $\omega_{m}$ and runs at that speed for $\mathrm{t}_{\mathbf{2}}$. Later, it decelerates to zero velocity over $\mathrm{t}_{3}$. The stepping is periodic with a repetition rate of $f$ steps per second. Equation (2.4.10) describes the instantaneous power dissipation in the motor. However, it is of greater importance to determine the average power loss in the motor over one
cycle by averaging the power. The analysis was found long and laborious, and we present here the final resulcs for this case.

The average power dissipation, $\overline{\mathbf{P}}$, was found to have five terms.

$$
\begin{equation*}
\bar{P}=P_{1}+P_{2}+P_{3}+P_{4}+P_{5} \tag{2.4.12a}
\end{equation*}
$$

where

$$
\begin{align*}
& P_{1}=\frac{f R J^{2} \omega_{m}^{2}}{K_{T}^{2}}\left(\frac{1}{t_{1}}+\frac{1}{t_{3}}\right) \\
& P_{2}=\frac{f R}{K_{T}^{2}}\left(T_{f}+T_{L}+D \omega_{m}\right)^{2} t_{2}  \tag{2.4.12c}\\
& P_{3}=\frac{f R}{K_{T}^{2}}\left[\left(T_{f}+T_{L}\right)^{2}+\left(T_{f}+T_{L}\right) D \omega_{m}+\right. \\
& \left.+\frac{1}{3} D^{2} \omega_{m}^{2}\right]\left(t_{1}+t_{3}\right) \quad(2.4 .12 d  \tag{2.4.12d}\\
& P_{4}=\frac{f K_{E} T_{f} \omega_{m}}{2 K_{T}}\left(t_{1}+2 t_{2}+t_{3}\right)  \tag{2.4.12e}\\
& P_{5}=\frac{f K_{E} D \omega_{m}^{2}}{3 K_{T}}\left(t_{1}+3 t_{2}+t_{3}\right) \tag{2.4.12f}
\end{align*}
$$

Clearly, the various terms can be interpreted physically. $\mathrm{P}_{1}$ describes the power dissipation due to the acceleration and deceleration of the motor and load. $\mathrm{P}_{\mathbf{2}}$ and $\mathrm{P}_{\mathbf{3}}$ describe the winding losses due to the
opposing torques during the various intervals of motion. Finally, $P_{4}$ and $P_{5}$ represent the rotational losses due to the constant opposing torque, $\mathbf{T}_{\mathbf{f}}$, and the damping, $\mathbf{D}$, respectively.

The following example illustrates the use of the equations (2.4.12). Special care should be taken to use the right units.

## Example

Consider a loaded motor with the following parameters:

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{T}}= 25 \mathrm{oz} \cdot \mathrm{in} / \mathrm{A}=0.176 \mathrm{Nm} / \mathrm{A} \\
& \mathrm{~K}_{\mathrm{E}}= 18.4 \mathrm{~V} / \mathrm{krpm}=0.176 \mathrm{~V} / \mathrm{rad} \cdot \mathrm{~s}^{-1} \\
& \mathrm{R}= 1.2 \Omega \\
& \mathrm{~T}_{\mathrm{f}}= 6 \mathrm{oz}-\mathrm{in}=4.24 \cdot 10^{-2} \mathrm{Nm} \\
& \mathrm{D}= 4 \mathrm{oz}-\mathrm{in} / \mathrm{krpm}=2.7 \cdot 10^{-4} \\
& \mathrm{Nm} / \mathrm{rad} \cdot \mathrm{~s}^{-1} \\
& \mathrm{~T}_{\mathrm{L}}= 50 \mathrm{oz}-\mathrm{in}=0.353 \mathrm{Nm} \\
& \mathrm{~J}= 0.02 \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}=1412 \mathrm{~g} \mathrm{~cm}^{2}}_{=}^{=} \\
& 1.412 \cdot 10^{-4} \mathrm{~kg} \mathrm{~m}^{2}
\end{aligned}
$$

The motion requirements are given by

$$
\begin{aligned}
& \mathrm{t}_{1}=50 \mathrm{~ms}=0.05 \mathrm{~s} \\
& \mathrm{t}_{2}=200 \mathrm{~ms}=0.2 \mathrm{~s} \\
& \mathrm{t}_{3}=40 \mathrm{~ms}=0.04 \mathrm{~s} \\
& \omega_{\mathrm{m}}=3000 \mathrm{rpm}=314 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

and the repetition rate is

## $\mathrm{f}=2$ steps $/ \mathrm{s}$

We may use any system of units to evaluate the power. In this example, we use both systems to illustrate their use. To evaluate $\mathrm{P}_{1}$, we use the SI system, whereas the British system is used for the other terms.

$$
\begin{aligned}
P_{1}= & \frac{2 \cdot 1.2\left(1.412 \cdot 10^{-4}\right)^{2} \cdot 314^{2}}{0.176^{2}} \times \\
& \times\left(\frac{1}{0.05}+\frac{1}{0.04}\right)=6.82 \mathrm{~W} \\
P_{2}= & \frac{2 \cdot 1.2}{25^{2}}(6+50+4 \cdot 3)^{2} \cdot 0.2=3.55 \mathrm{~W} \\
P_{3}= & \frac{2 \cdot 1.2}{25^{2}}\left[56^{2}+56 \cdot 4 \cdot 3+\frac{1}{3} 4^{2} \cdot 3^{2}\right] \mathrm{x}
\end{aligned}
$$

$$
x(0.05+0.04)=1.33 \mathrm{~W}
$$

$$
P_{4}=\frac{2 \cdot 18.4 \cdot 6 \cdot 3}{2 \cdot 25}(0.05+0.4+0.04)=6.49 \mathrm{~W}
$$

$$
P_{5}=\frac{2 \cdot 18.4 \cdot 4 \cdot 3^{2}}{3 \cdot 25}(0.05+0.6+0.04)
$$

$$
=12.19 \mathrm{~W}
$$

and the total average power dissipation is

$$
\overline{\mathrm{P}}=30.38 \mathrm{~W}
$$

In the following section we will discuss the ability of a motor to handle armature power dissipation.

### 2.5. THERMAL CHARACTERISTICS OF DC MOTORS

As the motor is operated, power losses are dissipated in the armature, resulting in temperature rise. The increase of temperature is important since it may limit the motor performance and, therefore, it deserves a careful study.

In this section we consider the temperature rise in the armature resulting from the power losses. We start with the simple case of continuous operation and determine the boundaries for working conditions. Later, we study the case of intermittent operation and develop a thermal model for the motor in order to analyze the temperature response.

### 2.5.1. CONTINUOUS OPERATION

When the motor runs at a constant or near constant velocity and the operation time is long ( 30 min or more), the motor temperature reaches a steady-state level, depending on the power dissipation.

In order to limit the temperature rise, we have to limit the losses, $\mathrm{P}_{\mathrm{L}}$, in the motor. Since the losses depend on the velocity $\omega_{m}$, and the load torque $T_{L}$ [see (2.4.10)], we can limit the values of the speed and the torque so that the resulting temperature is acceptable.

This can be done by constructing a boundary in the speed-torque plane which defines the safe operation area. The boundary curve is called Safe Operation Area bound-
ary for Continuous operation (SOAC). An example for such SOAC curve is shown in Fig. 2.5.1. The area to the left of the curve is safe for operation, whereas points to the right of the SOAC will result in excessive losses and heating.


Fig. 2.5.1. Speed-torque diagram and the Safe Operation Area boundary for Continuous operation (SOAC).

The speed-torque diagram may include additional information on the voltage required to drive the motor under those conditions, or on rating points. This, however, does not affect the SOAC.

When a motor is forced-air cooled, the thermal resistance decreases with the cooling, allowing a larger amount of power losses to be dissipated. This can be described by two SOAC curves for the uncooled and the cooled motors. An example is shown in Fig. 2.5.2.

Area of safe continuous operation without air cooling.

Area of safe continuous operation with adequate air cooling.

Area beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent bases provided armature temperature rating is not exceeded.


Fig. 2.5.2. The SOAC for a motor with cooling capability.

### 2.5.2. INTERMITTENT OPERATION

When the motor is operated continuously for a short time, or when the operation is periodic, the power losses in the motor vary with the time and the analysis of the resulting temperature is more complex than for the continuous case.

In order to determine the temperature rise, a thermal model is developed for the motor. The model describes the relation between the power and the temperature, and makes a thermal analysis for various power dissipation functions possible.

### 2.5.3. THERMAL MODEL

The motor consists of two parts, the armature (rotor) and the housing (stator). It is assumed here that the temperature over the two bodies are uniform. In other words, the armature temperature, relative to am-
bient, $\Theta_{\mathrm{a}}$, is the same all over the armature. Similarly, the relative temperature of the housing, $\Theta_{h}$, is also uniform.

The heat capacity is the ratio between the heating energy and the temperature rise. For the given model the heat capacities of the armature and the housing are denoted by $\mathrm{C}_{\mathrm{a}}$ and $\mathrm{C}_{\mathrm{h}}$, respectively.

The power losses, $\mathbf{P}_{\mathbf{L}}$, are assumed to be dissipated in the armature, since this is the common case. However, the final results of the following analysis are valid also for the case where some of the losses are dissipated in the housing.

The third assumption about the motor is that the heat transfer is proportional to temperature difference. Therefore, the power transfer from the armature to the housing, $P_{a}$, equals

$$
\begin{equation*}
P_{a}=G_{a}\left(\Theta_{a}-\Theta_{h}\right) \tag{2.5.1}
\end{equation*}
$$

where $\mathbf{G}_{\mathbf{a}}$ is the thermal conductance between the two parts. Similarly, the heat transfer from the housing to the ambient, $\mathrm{P}_{\mathrm{h}}$, is given by

$$
\begin{equation*}
P_{h}=G_{h} \Theta_{h} \tag{2.5.2}
\end{equation*}
$$

with $\mathbf{G}_{\mathrm{h}}$ being the heat conductance between the housing and the ambient.

If the motor is cooled by forced air, with the temperature of the air inflow being equal to ambient temperature, the heat transfer by cooling, $\mathbf{P}_{\mathbf{c}}$, equals

$$
\begin{equation*}
P_{c}=G_{c} \Theta_{a} \tag{2.5.3}
\end{equation*}
$$

where $G_{c}$ is the heat conductance and depends on the air flow.

### 2.5.4. THERMAL EQUATIONS

The thermal equations for the motor armature and housing are:

$$
\left.\begin{array}{l}
C_{a} \frac{d \Theta_{a}}{d t}=P_{L}-P_{a}-P_{c}  \tag{2.5.4}\\
C_{h} \frac{d \Theta_{h}}{d t}=P_{a}-P_{h}
\end{array}\right\}
$$

In view of (2.5.1), (2.5.2), and (2.5.3) the thermal equations become

$$
\begin{equation*}
C_{a} \frac{d \Theta_{a}}{d t}=P_{L}-G_{a}\left(\Theta_{a}-\Theta_{h}\right)-G_{c} \Theta_{a} \tag{2.5.5a}
\end{equation*}
$$

$$
\begin{equation*}
c_{h} \frac{d \Theta_{h}}{d t}=G_{a}\left(\Theta_{a}-\Theta_{h}\right)-G_{h} \Theta_{h} \tag{2.5.5b}
\end{equation*}
$$

The heat flow in the motor is illustrated in Fig. 2.5.3.

The electrical analog of the thermal system can be constructed by noting the equivalence between temperature and power on one hand, with voltage and current on the other hand. The resulting equivalent network is shown in Fig. 2.5.4.

Since we are interested in $\Theta_{a}$, which is higher than the housing temperature, it is desired to develop the transfer function between $\mathbf{P}_{\mathbf{L}}$ and $\Theta_{\mathbf{a}}$.

Upon application of the Laplace transformation to (2.5.5), it becomes

$$
\begin{align*}
s C_{a} \Theta_{a}(s)= & P_{L}(s)- \\
& \left(G_{a}+G_{c}\right) \Theta_{a}(s)+  \tag{2.5.6a}\\
& +G_{a} \Theta_{h}(s)  \tag{2.5.6b}\\
s C_{h} \Theta_{h}(s)= & G_{a} \Theta_{a}(s)-\left(G_{a}+G_{h}\right) \Theta_{h}(s)
\end{align*}
$$



Fig. 2.5.4. Electrical analog of thermal system.

Equation (2.5.6b) may be used to substitute for $\Theta_{h}(\mathbf{s})$.

$$
\Theta_{h}(s)\left(s C_{h}+G_{a}+G_{h}\right)=G_{a} \Theta_{a}(s)
$$

Combining (2.5.7) with (2.5.6a) results in

$$
\begin{equation*}
\Theta_{a}(s)\left[s C_{a}+G_{a}+G_{c}-\frac{G_{a}^{2}}{s C_{h}+G_{a}+G_{h}}\right]=P_{L}(s) \tag{2.5.8}
\end{equation*}
$$

and the transfer function between $P_{L}$ and $\Theta_{a}$ is

$$
\begin{equation*}
\frac{\Theta_{a}(s)}{P_{L}(s)}=\frac{s C_{h}+G_{a}+G_{h}}{s^{2} C_{h} C_{a}+s\left[C_{a}\left(G_{a}+G_{h}\right)+C_{h}\left(G_{a} G_{c}\right)\right]+G_{a} G_{c}+G_{a} G_{h}+G_{c} G_{h}} \tag{2.5.9}
\end{equation*}
$$

Note that (2.5.9) may be written in the form
where

$$
\begin{equation*}
A=\frac{G_{a}+G_{h}}{G_{a} G_{c}+G_{a} G_{h}+G_{c} G_{h}} \tag{2.5.11}
\end{equation*}
$$

and

$$
\begin{equation*}
\tau_{\mathrm{th}_{3}}=\frac{\mathrm{C}_{\mathrm{h}}}{\mathrm{G}_{\mathrm{a}}+\mathrm{G}_{\mathrm{h}}} \tag{2.5.12}
\end{equation*}
$$

The time constants, $\tau_{\text {th }_{1}}$ and $\tau_{\text {th }_{2}}$, correspond to the roots of the thermal characteristic equation:

$$
\begin{align*}
s^{2} C_{h} C_{a} & +s\left[C_{a}\left(G_{a}+G_{h}\right)+C_{h}\left(G_{a}+G_{c}\right)\right]+ \\
& +G_{a} G_{c}+G_{a} G_{h}+G_{c} G_{h}=0 \tag{2.5.13}
\end{align*}
$$

Note that the characteristic roots of (2.5.13) depend on the air cooling conductance, $\mathbf{G}_{\mathbf{c}}$. Therefore, the thermal time constants, $\tau_{\text {th }_{1}}$ and $\tau_{\text {th }}^{2}$, will vary with the introduction of forced-air cooling.

The transfer function (2.5.10) may be written as the sum of partial fractions:

$$
\begin{equation*}
\frac{\Theta_{\mathrm{a}}}{\mathrm{P}_{\mathrm{L}}}(\mathrm{~s})=\frac{\mathrm{R}_{\mathrm{th}_{1}}}{\mathrm{~s} \tau_{\mathrm{th}_{1}}+1}+\frac{\mathrm{R}_{\mathrm{th}_{2}}}{\mathrm{~s} \tau_{\mathrm{th}_{2}}+1} \tag{2.5.14}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{R}_{\mathrm{th}_{1}}=\frac{\mathrm{A}\left(\tau_{\mathrm{th}_{3}}-\tau_{\mathrm{th}_{1}}\right)}{\tau_{\mathrm{th}_{2}}-\tau_{\mathrm{th}_{1}}} \tag{2.5.15a}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{R}_{\mathrm{th}_{2}}=\frac{\mathrm{A}\left(\tau_{\mathrm{th}_{2}}-\tau_{\mathrm{th}_{3}}\right)}{\tau_{\mathrm{th}_{2}}-\tau_{\mathrm{th}_{1}}} \tag{2.5.15b}
\end{equation*}
$$

Now define $\Theta_{1}(s)$ and $\Theta_{2}(s)$ as

$$
\begin{align*}
& \Theta_{1}(\mathrm{~s})=\frac{\mathrm{R}_{\mathrm{th}_{1}}}{\mathrm{~s} \tau_{\mathrm{th}_{1}}+1} \mathrm{P}_{\mathrm{L}}(\mathrm{~s})  \tag{2.5.16a}\\
& \Theta_{2}(\mathrm{~s})=\frac{\mathrm{R}_{\mathrm{th}_{2}}}{\mathrm{~s} \tau_{\mathrm{th}_{2}}+1} \mathrm{P}_{\mathrm{L}}(\mathrm{~s}) \tag{2.5.16b}
\end{align*}
$$

Then the armature temperature, $\Theta_{a}$, equals

$$
\begin{equation*}
\Theta_{a}(s)=\Theta_{1}(s)+\Theta_{2}(s) \tag{2.5.17}
\end{equation*}
$$

Equations (2.5.16) and (2.5.17) describe the transfer function as a parallel combination of two partial fractions, as shown in Fig. 2.5.5. This model is easier to analyze due to its simplicity; it is therefore preferred over (2.5.9).


Fig. 2.5.5. Parallel model for thermal system.

### 2.5.5. THERMAL ANALYSIS

The temperature rise in a motor due to power losses may be determined from the transfer function model of Eq. (2.5.9). However, this method may be unnecessarily complicated in most cases, and an easier procedure has been developed, based
on the parallel model of (2.5.16) and (2.5.17), and is presented below.

The basic idea of this approach is that the temperatures, $\Theta_{1}$ and $\Theta_{2}$, should be found separately. Later, they can be combined to form $\Theta_{a}$. Since the transfer function of either $\Theta_{1}$ or $\Theta_{2}$ is of the form

$$
\begin{equation*}
\frac{\Theta(s)}{P_{L}(s)}=\frac{R_{t h}}{s \tau_{t h}+1} \tag{2.5.18}
\end{equation*}
$$

the method for determining $\Theta_{1}$ and $\Theta_{2}$ is the same.

The engineer has the freedom to solve (2.5.18) in any way. However, in the following discussion, the solution for most common cases is presented.

## Case 1 - Constant Power

When a constant power loss, $\mathrm{P}_{\mathrm{L}}$, is dissipated in the motor from $\mathbf{t}=\mathbf{0}$, the resulting temperature rise is

$$
\begin{equation*}
\Theta=R_{t h} P_{L}\left(1-e^{-\frac{t}{\tau_{t h}}}\right) \tag{2.5.19}
\end{equation*}
$$

If the power is applied for a long time,

$$
\begin{equation*}
\mathrm{t} \geqslant 4 \tau_{\mathrm{th}} \tag{2.5.20}
\end{equation*}
$$

the temperature reaches the steady-state value of

$$
\begin{equation*}
\Theta=R_{t h} P_{L} \tag{2.5.21}
\end{equation*}
$$

Therefore, the steady-state temperature rise due to constant power dissipation is the product of the power and the thermal resistance.

## Case 2 - Short Term Power Variations

When the power loss variations are much faster than the thermal time constant, e.g., power variation time is less than $\tau_{\text {th }} / 5$, the power variations will be filtered and only the average power will affect the temperature. To determine the temperature rise due to the average power, use the results of Case 1.

## Case 3 - Low Frequency Power Variations

When the power dissipation varies periodically and the period is neither too short nor too long with respect to the thermal time constant, we can not use any of the results of Case 1 or 2 , and the temperature has to be found from the transfer function. In the following discussion the case where the applied power is of square wave form is analyzed and the temperature rise is determined.

## Pulsed Power Case

If $P_{L}$ is as shown in Fig. 2.5.6, with an "ON" time $t_{1}$ and a period $t_{c}$, the temperature can be found as follows:

At time $t=0$, temperature equals $\Theta_{0}$; then power is applied and the resulting temperature is found:


Fig. 2.5.6. An example of "pulsed power".

$$
\begin{equation*}
\Theta_{m}=\Theta_{0}+\left(R_{t h} P_{D}-\Theta_{0}\right)\left(1-e^{-t_{1} / \tau_{t h}}\right) \tag{2.5.22}
\end{equation*}
$$

At time $t_{1}$, power is removed and temperature drops according to:

$$
\begin{equation*}
\Theta_{0}=\Theta_{m} e^{-\left(\frac{t_{c}-t_{1}}{\tau_{t h}}\right)} \tag{2.5.23}
\end{equation*}
$$



Fig. 2.5.7. "Pulsed power" is applied for a long time period so that thermal equilibrium is attained.

Fig. 2.5.7 shows the time display of (2.5.22) and (2.5.23) as the temperature variation with respect to time.

Suppose the system is operated long enough to achieve a steady-state condition as shown in Fig. 2.5.7. Then for the maximum tem-
perature rise, we can substitute (2.5.23) into (2.5.22) to obtain

$$
\begin{equation*}
\Theta_{m}=\frac{R_{t h} P_{D}\left(1-e^{-t_{1} / \tau_{t h}}\right)}{1-e^{-t_{c} / \tau_{t h}}} \tag{2.5.24}
\end{equation*}
$$

In order to illustrate the method of thermal analysis, consider the following example.

## Example

The thermal behavior of a motor is described by the following parameters:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{th}_{1}}=1{ }^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{\mathrm{th}_{2}}=0.8^{\circ} \mathrm{C} / \mathrm{W} \\
& \tau_{\mathrm{th}_{1}}=15 \mathrm{~s} \\
& \tau_{\mathrm{th}_{2}}=15 \mathrm{~min}=900 \mathrm{~s}
\end{aligned}
$$

The power loss is of pulse type with

$$
\begin{aligned}
\mathrm{P}_{\mathrm{D}} & =150 \mathrm{~W} \\
\mathrm{t}_{1} & =5 \mathrm{~s} \\
\mathrm{t}_{\mathbf{c}} & =15 \mathrm{~s}
\end{aligned}
$$

and the operation lasts for two hours. First,
we will calculate $\Theta_{2}$, which corresponds to the larger time constant. Note that the period $t_{c}=15 \mathrm{~s}$ is much shorter than $\tau_{\text {th }}^{2} 2=900 \mathrm{~s}$, and only the average power will be considered (Case 2).

The average power is

$$
\mathrm{P}_{\mathrm{av}}=\frac{150 \cdot 5}{15}=50 \mathrm{~W}
$$

and the resulting steady-state temperature rise is

$$
\Theta_{2}=R_{\mathrm{th}_{2}} \mathrm{P}_{\mathrm{av}}=0.8 \cdot 50=40^{\circ} \mathrm{C}
$$

To determine $\Theta_{1}$, note that $\mathrm{t}_{\mathrm{c}}=\tau_{\text {th } 1}$, and therefore, neither Case 1 or 2 will apply. The temperature rise is found from (2.5.24), which becomes

$$
\Theta_{1}=\frac{1 \cdot 150 \cdot\left(1-e^{-\frac{5}{15}}\right)}{1-e^{-\frac{15}{15}}}=90^{\circ} \mathrm{C}
$$

Then the total temperature rise, $\Theta_{a}$, equals

$$
\Theta_{\mathrm{a}}=\Theta_{1}+\Theta_{2}=130{ }^{\circ} \mathrm{C}
$$

If the ambient temperature is $25^{\circ} \mathrm{C}$, the temperature of the armature will be $155^{\circ} \mathrm{C}$.

### 2.6. MOTOR CHARACTERISTICS AND TEMPERATURE

Motor constants such as $K_{\mathbf{T}}, \mathrm{K}_{\mathbf{E}}$, and R are parameters used to define motor characteristics. But it is well to remember that they are not true constants in the strictest sense, since these parameters will be temperature sensitive to some degree. The following describes the effects of temperature variations, and presents a method for calculating the magnitude of parameters' change.

## ARMATURE RESISTANCE

The majority of PM motor armatures are wound with copper conductors, and some of the new low inertia types use aluminum conductors. With either metal, electrical resistance increases with temperature, each at a different rate. There are several effects of an increase in armature resistance as the motor is heated.

1) Motor $\left.\right|_{a} ^{2} R$ losses will increase for a given value of current. If a servomotor is being driven with a constant value of current to achieve a constant acceleration, the winding loss will increase in proportion to the amount of resistance increase. For this reason the "hot" resistance of the motor armature winding should be used when assessing a motor's capability for performing a prescribed duty cycle.
2) Speed regulation constant will increase. This constant was defined as the slope of the speed versus torque curve and was found to be proportional to R. Thus a
motor loaded close to rated torque will slow up somewhat as it warms up due to the higher value of speed regulation constant.
3) The frequency response characteristic of the motor will be different from "cold" to "hot". This is a consideration in high-performance servo applications, where the motor is required to respond to rapidly varying input signals, and occurs since the mechanical time constant used in the transfer function is proportional to $\mathbf{R}$.

The change of $\mathbf{R}$ can be calculated if the anticipated temperature rise can be estimated. The resistance of copper as a function of temperature is shown in Fig. 2.6.1.


Fig. 2.6.1. Resistance versus temperature characteristic for copper conductor.

As can be seen from the curve, the characteristic is linear in the region which is of most interest, from $25^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$. If this straight portion is extended, as shown by the broken line, it intersects the temperature axis at $-234.5^{\circ} \mathrm{C}$. Then by similar triangles, we can say that

$$
\begin{equation*}
\frac{R_{2}}{R_{1}}=\frac{234.5+\Theta_{2}}{234.5+\Theta_{1}} \tag{2.6.1}
\end{equation*}
$$

or,

$$
\begin{equation*}
R_{2}=\frac{234.5+\Theta_{2}}{234.5+\Theta_{1}} R_{1} \tag{2.6.2}
\end{equation*}
$$

## Example

If an armature resistance $R=2.5 \Omega$ at a standard $25{ }^{\circ} \mathrm{C}$ temperature, calculate the expected resistance when the winding is at its rated $155^{\circ} \mathrm{C}$ operating temperature.

$$
\begin{aligned}
\Theta_{2} & =155^{\circ} \mathrm{C} \\
\Theta_{1} & =25^{\circ} \mathrm{C} \\
\mathrm{R}_{1} & =2.5 \Omega
\end{aligned}
$$

then,

$$
\begin{equation*}
R_{2}=\frac{(234.5+155)}{(234.5+25)} \times 2.5=3.75 \Omega \tag{2.6.3}
\end{equation*}
$$

It can be seen that over the operating temperature range a $50 \%$ variation in resistance is observed, a very significant change, indeed.

An alternate formula for calculating resistance with temperature change is:

$$
\begin{align*}
& \mathbf{R}_{2}=\mathrm{R}_{1}\left[1+\psi\left(\Theta_{2}-\Theta_{1}\right)\right] \\
& \quad\left[\Omega ; \Omega, \Omega /{ }^{\mathrm{O}} \mathrm{C},{ }^{\mathrm{o}} \mathrm{C}\right] \tag{2.6.4}
\end{align*}
$$

where

$$
\begin{aligned}
& R_{2} \text { is resistance at } \Theta_{2} \\
& R_{1} \text { is resistance } \Theta_{1}
\end{aligned}
$$

and

$$
\begin{array}{ll}
\psi_{\mathbf{c u}}=0.00393 & \text { for copper } \\
\psi_{\mathrm{al}}=0.00415 & \text { for aluminum }
\end{array}
$$

## TORQUE CONSTANT $K_{T}$ AND VOLTAGE CONSTANT K

The effects of temperature variation in $\mathrm{K}_{\boldsymbol{T}}$ and $\mathrm{K}_{\mathrm{E}}$ are:

1) An increase in $\left.\right|_{a} ^{2} R$ losses. If a given torque is required to accelerate a load, as the motor heats up, $\mathrm{K}_{\mathrm{T}}$ and $\mathrm{K}_{\mathrm{E}}$ decrease and more current will be required to provide the same torque.
2) Speed regulation constant will increase. The decrease in $\mathrm{K}_{\mathrm{T}}$ and $\mathrm{K}_{\mathrm{E}}$ would cause an additional fall off in speed of a motor carrying rated load torque.
3) Change in frequency response characteristics. Because of the reduced torque capability, the frequency response of the motor is reduced.

Of the two main types of magnets commonly used, the temperature coefficient of magnetic flux density for ceramics is -0.002 ; Alnico magnets have coefficients that range from -0.0001 to -0.0005 .

Using the above temperature coefficients of magnetic flux density, it is possible to develop an expression to calculate a derated torque constant to be used where the motor magnets will experience a significant temperature rise.

$$
\begin{equation*}
\mathrm{K}_{\mathrm{TD}}=\mathrm{K}_{\mathrm{T}}\left[1+\psi_{\mathrm{B}} \Theta_{\mathrm{rm}}\right] \tag{2.6.5}
\end{equation*}
$$

where
$\mathrm{K}_{\mathrm{TD}}$ is the derated torque constant
$\mathrm{K}_{\mathrm{T}}$ is the rated torque constant (at $25^{\circ} \mathrm{C}$ )
$\psi_{B}$ is the temperature coefficient of magnetic flux density
$\Theta_{\mathrm{rm}}$ is the temperature rise of the magnet material.

This equation points out a great advantage in using forced-air cooling. Air circulating through the motor will greatly reduce magnet temperature rise, and will minimize temperature variation.

## DERATING MOTOR TORQUE

The above discussion of temperature effects should make it very obvious that a motor's torque capability is degraded as operating temperature increases. An expression can be developed to compare the maximum torque that can be produced by "hot" and "cold" motors. If a motor at stall is excited with a current $I_{a}$ to its maximum dissipation level, the power will be dissipated as $I_{a}^{2} R$ loss. Then torque developed will be:

$$
\begin{equation*}
T_{g}=K_{T} I_{a} \tag{2.3.4}
\end{equation*}
$$

As the armature heats up the current must be reduced to avoid exceeding the rated armature dissipation. The "hot" current is found to be:

$$
\begin{equation*}
I_{a h}=\frac{I_{a}}{\sqrt{1+\psi_{c u} \Theta_{r w}}} \tag{2.6.6}
\end{equation*}
$$

where $\Theta_{r w}$ is the winding temperature rise
Torque can be calculated as the product $K_{T D}{ }^{\prime}{ }^{\text {a }}$ :

$$
\begin{equation*}
T_{g h}=\frac{K_{T}\left[1+\psi_{\mathrm{B}} \Theta_{\mathrm{rm}}\right] I_{\mathrm{a}}}{\sqrt{1+\psi_{\mathrm{cu}} \Theta_{\mathrm{rw}}}} \tag{2.6.7}
\end{equation*}
$$

The ratio of maximum "hot" torque to "cold" torque will be:

$$
\begin{equation*}
\frac{\mathrm{T}_{\mathrm{gh}}}{\mathrm{~T}_{\mathrm{g}}}=\frac{1+\psi_{\mathrm{B}} \Theta_{\mathrm{rm}}}{\sqrt{1+\psi_{\mathrm{cu}} \Theta_{\mathrm{rw}}}} \tag{2.6.8}
\end{equation*}
$$

## Example

A motor with copper armature winding and ceramic magnets has the following ratings:

$$
\begin{aligned}
& K_{T}=800 z-\mathrm{in} / \mathrm{A} \\
& R=2.5 \Omega \\
& \left(\mathrm{l}_{\mathrm{a}}^{2} \mathrm{R}\right)_{\max }=105 \mathrm{~W}
\end{aligned}
$$

If the motor is used with a controller that limits current so that the maximum armature dissipation cannot be exceeded, cal-
culate the torques at $25^{\circ} \mathrm{C}$ and at $155^{\circ} \mathrm{C}$ operating temperature, and torque derating factor.

Allowable armature current at moment of excitation (i.e., at $25^{\circ} \mathrm{C}$ ) is:

$$
\mathrm{I}_{\mathrm{a}}=\sqrt{\frac{105}{2.5}}=6.48 \mathrm{~A}
$$

"Cold" torque at $25^{\circ} \mathrm{C}$ is:

$$
\mathrm{T}_{\mathrm{g}}=6.48 \times 80=518 \mathrm{oz}-\mathrm{in}
$$

With armature at $155{ }^{\circ} \mathrm{C}$ and ambient of $25^{\circ} \mathrm{C}$, we estimate magnet temperature will be $90^{\circ} \mathrm{C}$. Then

$$
\begin{aligned}
& \Theta_{r m}=90-25=65^{\circ} \mathrm{C} \\
& \Theta_{\mathrm{rw}}=155-25=130^{\circ} \mathrm{C}
\end{aligned}
$$

Use (2.6.7) to calculate allowable "hot" torque.

$$
T_{g h}=\frac{80(1-0.002 \times 65) \times 6.48}{\sqrt{1+0.00393 \times 130}}
$$

$$
T_{g h}=367 \mathrm{oz}-\mathrm{in}
$$

The derating factor is found by use of (2.6.8):

$$
\frac{T_{g h}}{T_{g}}=\frac{1-0.002 \times 65}{\sqrt{1+0.00393 \times 130}}=0.708
$$ or, using the calculated $T_{g h}$ and $T_{g}$ :

$$
\frac{T_{g h}}{T_{g}}=\frac{367}{518}=0.708
$$

### 2.7. OTHER CHARACTERISTICS

Other mechanical motor characteristic besides those previously discussed (speed and torque output, power dissipation, and elasticity) are also important. These are: mounting accuracies; noise problems (such as mechanically generated audible noise, noise and dynamic balancing, noise caused by side loading, bearing and brush noise); environmental considerations; brush wear; and motor life. Each of these characteristics will be covered separately in the following discussion.

Further, we will discuss the problems of demagnetization.

## MOUNTING

As an illustration of the construction accuracies built into a servomotor, we will look at an Electro-Craft ET-4000 motor integrally mounted with a M-110 Tachometer, as shown in Fig. 2.7.1.


Fig. 2.7.1. Electro-Craft ET-4000 Motor and M-110 Tachometer, integrally mounted.

The outline drawing (Fig. 2.7.2) shows that the mounting surface is held perpendicular to the shaft within 0.005 in, which means that when the shaft is rotated about the mounting surface it can rise and fall no more than 0.005 in total when measured at a radius of 1.75 in .

Also note that shaft end play and radial play are closely controlled. Radial play is especially significant in applications which require a stable shaft position when radially loaded.

Projecting above the level of the mounting surface is a close-tolerance, circular pilot boss which, when matched with a pilot hole in the mating mounting structure, facilitates interchangeability and minimizes the need for machine calibration adjustments. The mounting surface in our illustration is fitted with four tapped holes to accept screws which clamp it to the mounting panel.

The mounting panel, in addition to containing the pilot hole and the attachment screw clearance holes, must have certain other characteristics. For example, it should be stiff enough so that it does not deflect significantly when radial loading is applied to the motor shaft. Also, it should have good thermal conductivity, which is particularly necessary if peak performance is being demanded of the motor.

All applications, of course, are not identical; but all must adequately consider the factors described above. Foot-mounted motors or


Notes:

1. Shaft runout: $0.001 \mathrm{in} / \mathrm{in}$.

2. Shaft endplay: 0.004 in max. 0.005 in min. when measured with a 20 lb reversing load.
3. Shaft radial play: 0.0015 in max. when measured with a 3 lb load.
4. Runout of mtg . surface $-\mathrm{B}-$ within 0.005 T.I.R. measured ot 1.75 in radius.

Fig. 2.7.2. Typcial outline drawing of a motor.
those mounted to a gearhead may utilize slotted mounting holes and/or flexible couplings, for example, instead of pilot mounting.

## NOISE

Mechanically generated audible noise is usually an unwelcome output from any machine, and DC servomotors are no exception. Unfortunately, however, acceptable noise limits are difficult to define and even more difficult to verify, since quantitative noise measurements are costly. Consequently, in the interest of economics, noise is seldom specified as a measurable parameter. In those applications which require it, noise is controlled qualitatively
by means of some readily measured output, such as a vibration level, which can be attributed to rotor imbalance, excessive side loading, excessive bearing noise, and improper bedding of brushes to the commutator.

## NOISE AND DYNAMIC BALANCING

Critical speed of a motor is the speed at which vibrational resonance occurs. This critical speed should be as far above the maximum operating speed of the motor as practical to avoid undue amplification of the vibration amplitude. For this reason, and since servomotors are intended for rotational services, the rotors should be dynamically balanced; i.e., balanced under running speeds.

The higher the operating speed, the more critical dynamic balancing becomes. For very high speed operation, the rotor should be balanced at the operating speed rather than at the normally used standard balancing speed. The amount of rotor imbalance which can be tolerated depends on the particular motor and applications. In general, however, it can be said that for satisfactorily, smooth and quiet operation, rotor vibration must not result in motor accelerations greater than a fraction of standard gravitational acceleration, $\mathbf{g}$.

## NOISE CAUSED BY SIDE LOADING

Excessive side loading of the output shaft can result in shaft bowing and internal misalignments, thus causing noisy operation due to overloading of the bearings. To avoid this, a particular motor design must be matched to the application.

## BEARING NOISE

Excessive bearing noise can be caused by a mismatched ball complement, dented races, ring distortion, misalignment, insufficient lubrication, or overloading. Bearings which exhibit rough, noisy operation should be replaced. Sleeve bearings, while they are usually quieter than ball bearings, are not as suitable as ball bearings for many applications. They require more length than ball bearings and use a lower viscosity lubricant, which is more likely to migrate to unwanted locations in the motor. Moreover, they are not as well suited for carrying axial loads.

BRUSH NOISE

Smooth operation of the brushes on the commutator enhance the quietness of the motor. This is often one of the most difficult factors to control, since it is dependent on the initial surface finish, cylindricity, concentricity, subsequent wear and filming of the commutator, bedding of the brushes to the commutator, and fitting of the brushes to their holders. Material content of the brushes will also contribute to the noise factor. Brushes having high metal content (which is introduced to reduce resistance) are far noisier than nonmetallic brushes. Therefore, it may be necessary to sacrifice low brush resistance in order to achieve quieter motor operation.

## ENVIRONMENTAL CONSIDERATIONS

Motors which operate in environments which are more severe than a standard room temperature environment usually require special design attention. For example, at high altitudes, motors must be provided with specially treated brushes; in elevated temperatures, motors must be either derated or provided with cooling; and when exposed to chemical liquids or fumes, dust or dirt, motors require additional sealing. All of these affect the useful life of the servomotor.

## BRUSH WEAR

The key to proper brush operation is the commutator surface film formation. The film prevents undue wear, and yet permits a low resistance current path between brush
and the commutator. A variety of conditions affect the film maintenance, but two basic factors are dominant: brush pressure and the current density of the brush.

The inter-relationship of brush pressure and brush wear is shown in Fig. 2.7.3, where the optimum wear region is shown as an operational area between the electrical wear region (insufficient pressure to insure proper conductance) and the mechanical wear region (high pressure which destroys the film). Thus, the brush spring must be designed so that the brush pressure will stay in the optimum wear region during its wear life.


Fig. 2.7.3. Generalized picture of brush wear at a constant speed under varying brush pressure.
The brush must be designed with sufficient cross-sectional area with respect to its conductance value to insure cool operation under worst-case current conditions.

Brush life, then, depends on load conditions, commutator surface velocity, operating temperature, humidity level, vibration,
reversing cycles, choice of brush and spring material, commutator design, commutator finish and runout, to mention the most important factors.

Today's DC permanent magnet motor has brush designs which exceed 5000 hours life in normal uses. Electro-Craft Corporation has Moving Coil Motors which have qualified for use in incremental motion systems, measuring the life span in hundreds of millions of start-stop cycles of pulse currents of 20-30 A. Brush selections in Electro-Craft motors have been made after many years of life testing under various conditions.

## MOTOR LIFE

In a typical application, the service-free life of a motor may exceed 10000 operating hours. The useful life of a DC servomotor is dependent on the severity of its application. Operating a motor continuously at rated output will result in shorter life in terms of operating hours when compared with operation at a lower average power level. Life is limited by brushes, bearings, and winding insulation.

Brush life is influenced by the brush material, geometry, brush spring pressure, commutator, motor duty cycle, and operating environment. Brush life is reduced by electrical erosion, which will cause rapid wear of the brushes and commutator, and also by excessive spring pressure, rough commutators, reduced atmospherical pressure, or by the presence of abrasive particles.

Bearing life is dependent on loading, speed, and lubrication. Most servomotors have their bearings loaded to only a small portion of their rated capacity, but are operating at a relatively high speed. Lubrication must then be considered.

Ball bearings utilize a grease to inhibit corrosion, transfer heat, and to prevent the entry of contaminating particles. For each bearing application there is an optimum fill of grease. Deviating from the optimum by using either too much or too little grease, or perhaps a grease of different composition, can shorten the life of the bearing.

Sleeve bearings are usually made from a porous material (typically sintered bronze) which contains oil to lubricate the bearings. Sometimes additional oil is stored in a reservoir.

The insulation of the motor winding can break down with time due to the influence of high temperature, rotational stresses, oil migration, or contamination with chemical fluids or fumes, thus limiting motor life.

## DEMAGNETIZATION OF PM MOTORS

It was shown that energy conversion is accomplished in a PM motor by means of the magnetic field provided by the permanent magnets. These magnets, which provide the field, must be activated or magnetized by placing them into a strong external field. This initial magnetization of the motor is usually accomplished by magnetizing equipment at a stage in the produc-
tion process prior to the assembly of the motor. Even as the magnets must be magnetized to make them functional, so they can be demagnetized to lose their torquedeveloping capability. Such a demagnetizing effect occurs whenever a current flows in the motor armature. The armature in effect becomes an electromagnet which tends to oppose the main stator magnetic flux.

In most cases this armature demagnetization, which is called armature reaction, has only a reversible effect; i.e., when the current goes to zero the magnets return to full strength. This is the effect that causes an apparent loss in the torque constant, $\mathrm{K}_{\mathrm{T}}$, of the motor under heavy load conditions. However, there is a definite value of current which will cause a permanent demagnetization of the motor. When this occurs the magnetic field intensity, developed by the armature reaction, is great enough to overcome the magnetic domains of the magnet structure.

Fig. 2.7.4. shows how armature reaction and main field relate to each other. It can be seen that if the armature reaction is represented by a resultant vector, A, lying along the neutral axis there will be successively weaker components acting radially outward as the angle $\theta$ increases. As those component vectors at angles $\theta>\theta_{\mathrm{n}}$ reach the intrinsic coercive strength of the magnet material, they will cause the portion of magnet opposite them to be permanently demagnetized.


Fig. 2.7.4. The armature demagnetization, or armature reaction effect causes an apparent loss in $K_{T}$.

It can be seen that demagnetization is not an "all or nothing" proposition. Rather, the amount of demagnetization is a function of the magnitude of armature current. For purpose of a standard definition, it is common to speak of demagnetizing current as the current that will cause a permanent decrease of $5 \%$ of the torque constant, $K_{T}$. Of course greater values of current will cause a greater decrease of $K_{T}$. The partially demagnetized motor will always be stabilized at the reduced value of $K_{T}$. That means that having been partially demagnetized, it can withstand subsequent applications of demagnetizing current without additional degradation of $\mathrm{K}_{\mathrm{T}}$. If a motor is demagnetized, it must be disassembled and remagnetized.

A major reason for the popularity of PM motors in recent years is the high intrinsic coercivity of the new ceramic magnets.

This characteristic permits relatively thin magnet segments (for economical motor design) to be used without experiencing bothersome demagnetizing problems.

It is common practice in motor design to provide the magnets strong enough to withstand armature reaction developed by up to seven or eight times the rated current. Thus if the rated current for a motor is 5 A , it is usually safe to assume that current pulses up to 35 A can be applied safely. If some applications create currents in excess of the demagnetizing level, it is necessary for the controller to incorporate some type of current limiter.

Example 1: A motor has the following ratings:

$$
\begin{aligned}
& \text { rated current } \mathrm{I}_{\mathrm{a}}=16 \mathrm{~A} \\
& \text { armature resistance (at } 25^{\circ} \mathrm{C} \text { ) } \mathrm{R}=0.25 \Omega
\end{aligned}
$$

What is the maximum voltage that can be applied suddenly with the motor at standstill without running the risk of demagnetization?

Using the $7: 1$ criterion to determine the demagnetizing current for the subject motor we obtain:

$$
I_{d}=7 \times 16=112 \mathrm{~A}
$$

At standstill the armature current is limited only by the armature resistance; so then,

$$
V=112 \times 0.25=28 \mathrm{~V}
$$

We can assume that applying 28 V is likely to cause some demagnetization. Then a safe voltage would be $10 \%$ less; or, a safe voltage is:

$$
V=28 \times 0.9=25.2 V
$$

Motor demagnetization commonly occurs when the motors are used in an application requiring a "plug reversal". A plug reversal means a sudden switching of the polarity of the applied voltage to achieve a rapid reversal of motor rotation, or perhaps a quick stop. When a motor is plug-reversed, it sees not only the voltage applied to its terminals, but also the armature generated voltage as voltage sources.

This condition is shown in Fig. 2.7.5. It can readily be seen that when this type of application is encountered a proper investigation of the motor demagnetization characteristics must be made.


Fig. 2.7.5. In (a) the motor is running in a forward manner. In (b) the motor is plug-reversed. The generated voltage, $\mathrm{E}_{\mathrm{g}}$, is that voltage at the instant of reversal.

Example 2: The same motor used above in example 1 is to be used in an application where it runs under a no-load condition at

1200 rpm and then must be stopped as quickly as possible. If the voltage constant of the motor $\mathrm{K}_{\mathrm{E}}=\mathbf{1 8} \mathrm{V} / \mathrm{krpm}$, what is the maximum plugging voltage that can be used to help stop the motor?

At 1200 rpm, the generated armature voltage will be:

$$
\mathrm{E}_{\mathrm{g}}=18 \times 1.2=21.6 \mathrm{~V}
$$

From example 1, demagnetizing current is 112 A . Then the plugging voltage can be calculated by the current formula shown in Fig. 2.7.5.

$$
\begin{aligned}
\mathrm{V} & =\mathrm{R} \mathrm{I}_{\mathrm{a}}+\mathrm{E}_{\mathrm{g}} \\
& =0.25(-112)+21.6 \\
\mathrm{~V} & =-6.4 \mathrm{~V}
\end{aligned}
$$

Again, since the demagnetizing current was calculated from an assumed 7:1 ratio of demagnetizing current to rated current, a prudent designer would downgrade the plugging voltage that will be used, or provide current limiting.

In PM motors with Alnico magnets, it is often necessary to provide some type of protection against demagnetization. The most common form of this protection is the use of a soft iron pole shoe between the armature and magnet. This arrangement is shown in Fig. 2.7.6. The high permeability of the pole show provides a low reluctance shunt for armature reaction flux around the permanent magnet, thus protecting it.


Fig. 2.7.6. A soft iron pole shoe is used for portection against demagnetization.

### 2.8. MOVING COIL MOTORS $\left(M C M{ }^{\circledR}\right)^{\star}$

A description of the "iron-less" or "moving coil" concept, covered in this section, will introduce the reader to the MCM* line of permanent magnet DC motors - motors with the highest torque to moment of inertia ratio of any motor type. Motor ratings and their relationship to this unique design is discussed, as well as thermal properties, resonance, and demagnetization.

## MOVING COIL MOTORS

In their efforts to improve the performance of servo mechanism actuators, designers have continually sought to produce motors with higher torque and lower moment of inertia. The trend in the design of AC servomotors in the late 1940's and early 1950's was to make motors with small armature diameter and maximum practicable magnetic flux density. A high-performance, low inertia AC servomotor of two decades past was capable of a maximum acceleration of $15000 \mathrm{rad} / \mathrm{s}^{2}$.

Dramatic improvements in permanent magnet motor characteristics accompanied the introduction of high-performance Alnico materials, and some basic trends in the design of DC low inertia motors appeared. One was the small diameter iron core armature, operating in very high magnetic flux levels. The more refined style of this design featured conductors bonded to the armature

[^1]core surface: the so-called slotless armature design. This design has great mechanical strength, high torsional rigidity and is reasonably efficient; but it suffers from two principal drawbacks: 1) inductance - hence, its electrical time constant is great; and 2) armature resistance and the torque constant are small - requiring very high armature currents and low power supply voltages, and making the transistor amplifier unnecessarily complex and expensive.

The other design trend, which has since gained almost universal acceptance, is the moving coil concept. This principle basically consists of a multiple d'Arsonval movement with a commutation arrangement. This working concept is not new; patents granted at the turn of the century described moving coil motors - not necessarily of the low inertia variety - but supposedly offering high efficiency and good commutation. The adhesive and polymer technology being what is was then, the inventor had the rotor structure held together with iron bands, continuing around the circumference, and undoubtedly contributing enormous eddy current losses.

The moving coil structure design of the present era has followed two general paths: the flat disc armature and the "shell" or "cup" armature. The two structures are shown in Fig. 2.8.1 and 2.8.2. Both units have a multitude of conductors which move in a magnetic field, connected in a manner explained in section 2.1. The armature structure is supported mainly by non-magnetic materials and the active conductors


Fig. 2.8.1. Disc armature motor ("printed" motor).
are therefore moving in an air gap with a high magnetic flux density.

The moving coil motor is characterized by the absence of rotating iron in the armature. Since the conductors are operating in an air gap, the armature features low inductance, hence low electrical time constant - typically less than 0.1 ms . The absence of iron in the armature also brings about another benefit: there is no reluctance torque effect, i.e., no magnetic cogging.

The disc armature motor shown in Fig. 2.8 .1 is often called the "printed motor". This is in reference to its early production process in which the armature was fabricated by means of photoetching - similar to the technique used in making printed circuits. Now printed motors are made from stamped segments, which are arranged and joined to form a continuous conductor pattern and a commutating surface. Fig. 2.8.1 shows an 8-pole configuration which, when assembled, provides flux across an air gap of about 0.1 in. Current flow is radial across the disc surface, with rotary forces acting on these conductors tangentially.


Fig. 2.8.2. Shell armature motor.

The end turns (the path from an active conductor under the "north" pole to the corresponding conductor under the adjoining "south" pole) are at a relatively large radius, and since the moment of inertia of a disc increases by the fourth power of its diameter, the outer end turns therefore contribute a large amount of inertia to this type of armature.


Fig. 2.8.3. Shell armature.

The shell type armature (Fig. 2.8.2) on the other hand consists of a cylindrical, hollow rotor which is fabricated to form a rigid shell structure by bonding copper or aluminum coils or skeins by the use of polymer resins and fiberglass and other structural members (Fig. 2.8.3). This method offers


Fig. 2.8.4. Typical Electro-Craft MCM motor armatures.
considerable flexibility in design since the manufacturer can offer a variety of wire sizes, turns per coil, and diameter and length options.

A sample of the variety of armatures manufactured by Electro-Craft is shown in Fig. 2.8.4. Because of the cylindrical shape of the shell-type armature, the end turns do not burden the armature by inequitable contributions of inertia.

Consequently, the shell-type armature has the highest torque-to-moment of inertia ratio, providing acceleration capabilities of up to $1000000 \mathrm{rad} / \mathrm{s}^{2}$. Fig. 2.8 .5 shows the complete Electro-Craft MCM motor, capstan, optical encoder and vacuum manifold for an incremental tape transport operating at speed of $200 \mathrm{in} / \mathrm{s}$, capable of 250 start-stop cycles per second.

Electro-Craft moving coil motors are used in incremental tape transports, line printers, optical character readers, incremental motion drives, phase-locked servos, computer printers, machine tool drives and video recorders.


Fig. 2.8.5. Entire motor, capstan, optical encoder and vacuum manifold for tape transport.

MOVING COIL MOTORS AND
mOTOR RATINGS

The moving coil motor has the highest acceleration capability of the DC motor family; it can handle a start-stop duty cycle of several thousand cycles per second. On the other hand, it can be destroyed in seconds if improperly handled. Therefore, a good understanding of the physical makeup of this exceedingly fast actuator is important to the user.

Moving coil servomotors obey all the fundamental concepts and equations which were developed in section 2.1. However, some of their features are unique. How moving coil motors differ from conventional servomotors is discussed in the following paragraphs.

One common misunderstanding arises from the use of conventional motor ratings among the makers of moving coil motors. For example, the specification sheet for a high performance moving coil motor may state: Rated Voltage $=24 \mathrm{~V}$; Rated Current $=8 \mathrm{~A}$. This may be a motor which the manufacturer has proposed to be used in a system having an amplifier supply voltage of 45 V and working at peak currents of 24 A . The designer may rightfully ask what meaning does the ratings have if there is no need to adhere to it.

The use of motor rating numbers originated with the specifications for fixed speed motors, illustrated in Fig. 2.8.6, where we see a typical induction motor torque-speed
characteristic with the motor rating point shown at its maximum output power point.


Fig. 2.8.6. Typical rating point on an AC induction motor.

In Fig. 2.8.7 we see similar characteristics for a conventional DC motor intended for use in a drive system. The maximum allowable internal power dissipation locus is shown as a line emerging from stall speed (zero speed) conditions vertically, then leaning toward the speed axis as speed increases. The rating point could then be chosen to be at the one location along this line which would give the maximum output power, thus giving the motor a most favorable rating (if the point would be at a reasonable, utilizable speed).

The moving coil motor speed torque characteristic shown in Fig. 2.8.8 has a maximum power dissipation locus which is essentially vertical for the useful speed range of the motor. One is therefore tempted, in setting


Fig. 2.8.7. Rating point on a typical DC motor.


Fig. 2.8.8. Rating point for a typical moving coil DC motor.
a rating point for such motors, to choose the highest possible speed in such a way as to obtain a high rated output power to place on the specification sheet.

Since moving coil motors are used primarily for their incrementing ability rather than ultra high speed performance, such a rating
would be unrealistic. It therefore happens that the moving coil motor designer may pick a point along the maximum uncooled power dissipation locus for the motor at a speed which represents a good compromise between speed and long life. Then why this discrepancy between ratings and actual intended use? The rating numbers for moving coil motors are really obsolete methods of describing the performance capability of a motor intended for use in incremental motion applications, but are still given in specifications merely as a concession to those who like to see these numbers in the rare cases where moving coil motors are used in velocity control applications.

The really meaningful parameters which properly describe the performance capability of a moving coil motor are the torque constant $K_{T}$, motor resistance $R$, motor moment of inertia $J_{m}$, thermal resistance, $\mathrm{R}_{\mathrm{th}}$ and maximum armature temperature, $\Theta_{a}$ max . From these parameters most all essential characteristics can be found or derived. Other factors of importance (but often not discussed until the customer discovers them) are shaft and armature torsional resonance and motor shaft critical speed vibrations.

Going back to the essential parameters we recall the following:

Speed regulation constant

$$
\begin{equation*}
R_{m}=\frac{R}{K_{E} K_{T}} \tag{2.8.1}
\end{equation*}
$$

[krpm/Nm; $\Omega, \mathrm{V} / \mathrm{krpm}, \mathrm{Nm} / \mathrm{A}]$
[krpm/oz-in; $\Omega, \mathrm{V} / \mathrm{krpm}$, oz-in/A]
where

$$
\begin{equation*}
\mathrm{K}_{\mathrm{T}}=9.5493 \times 10^{-3} \mathrm{~K}_{\mathrm{E}} \tag{2.1.23}
\end{equation*}
$$

[ Nm/A; V/krpm]
or

$$
\begin{equation*}
\mathrm{K}_{\mathrm{T}}=1.3524 \mathrm{~K}_{\mathrm{E}} \tag{2.1.24}
\end{equation*}
$$

[oz-in/A; V/krpm]
Motor mechanical time constant:

$$
\begin{equation*}
\tau_{\mathrm{m}}=\frac{\mathrm{RJ}_{\mathrm{m}}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}} \tag{2.8.2}
\end{equation*}
$$

$$
\left[\mathrm{s} ; \Omega, \mathrm{kg} \mathrm{~m}^{2}, \mathrm{~V} / \mathrm{rad} \mathrm{~s}^{-1}, \mathrm{Nm} / \mathrm{A}\right]
$$

or

$$
\begin{align*}
\tau_{\mathrm{m}}= & 104.72 \frac{\mathrm{RJ} \mathrm{~m}_{\mathrm{m}}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}}  \tag{2.8.3}\\
& {\left[\mathrm{~s} ; \Omega, \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}, \mathrm{~V} / \mathrm{krpm}, \mathrm{oz}-\mathrm{in} / \mathrm{A}\right] }
\end{align*}
$$

Maximum armature power dissipation (steady state):

$$
\begin{equation*}
P_{D \max }=\frac{\Theta_{a \max }-\Theta_{A}}{R_{t h}} \tag{2.8.4}
\end{equation*}
$$

$$
\left[\mathrm{W} ;{ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{C} / \mathrm{W}\right]
$$

where $\Theta_{A}$ is the ambient temperature.
As we see, all these essential parameters can be derived from the basic ratings given earlier.

## THERMAL PROPERTIES

The thermal resistance of a moving coil motor should not be looked on as a single number (as given above for simplicity's sake) but as at least two separate numbers: an armature-to-housing thermal resistance, and a housing-to-ambient thermal resistance. In conjunction with this, the thermal time constant of each case should be considered. This is essential when considering the transient temperature characteristics under severe load conditions, since typical moving coil motors may have a thermal time constant of 20-30 s for armature-to-housing and a thermal time constant of $30-60 \mathrm{~min}$ for housing-to-ambient. It is easy to see that the armature could be heated to destructive temperatures in less than a minute without it giving any warning because of the long thermal time constant between housing and ambient.

An analysis of this problem was given in section 2.5.

In order to prevent thermal destruction, air cooling can be provided for the motor. To show the effects of varying amounts of air cooling on motor power dissipation capability, a graphical illustration of cooling air flow aerodynamical impedance and total motor thermal resistance, as shown in Fig. 2.8.9, is usually provided.


Fig. 2.8.9. The effect of air cooling on thermal resistance.

We can see from this figure that a pressure in 15 in of $\mathrm{H}_{2} 0$ will supply 25 cubic feet of air per minute, giving a thermal resistance of $0.28^{\circ} \mathrm{C} / \mathrm{W}$. By such impedance charts a system designer can select the most economical cooling system to fulfill his needs, within the thermal resistance range of a high of $1.5^{\circ} \mathrm{C} / \mathrm{W}$ to an asymptotic limiting value of $0.25^{\circ} \mathrm{C} / \mathrm{W}$. Note that this chart shows total motor thermal resistance; it cannot be used for transient studies where the short term power dissipation may exceed the heat capacity of the motor.
Almost all moving coil motors are equipped with Alnico 5 or 5-7 magnets which have a magnetic flux temperature coefficient of $-0.05 \% /{ }^{\circ} \mathrm{C}$. This means that the armature
magnetic flux will decrease by that amount as the magnet temperature increases, giving a change in torque constant of $-0.05 \% /{ }^{\circ} \mathrm{C}$. The motor torque constant, $\mathrm{K}_{\mathbf{T}}$, is directly proportional to the air gap magnetic flux. A common error is to calculate the decline in the torque constant using the proper temperature coefficient, but using the armature temperature as parameter. This gives the wrong answer, since the magnet temperature never reaches the maximum armature temperature. For calculation purposes the housing-to-ambient thermal resistance can be used as a guideline for the magnet temperature. It generally amounts to a worstcase decline of torque constant by a few percent.

## RESONANT PHENOMENA IN MOVING COIL MOTORS

Due to the unique construction of moving coil motors and their use in high-performance (wide bandwidth) servo systems, the resonant characteristics may cause problems to the user. The most prominent resonant mode is the torsional resonance between the motor armature and the load, and between the motor armature and the tachometer. Theory of this torsional resonance is described in full detail in section 2.3.4. Fig. 2.8.10 shows a plot of tachometer output with the motor armature excited by a constant amplitude, variable frequency input signal. At lower frequencies, up to 10 Hz , the tachometer output is essentially constant. At the mechanical "break" frequency, 48 Hz , the tachometer response declines -6 dB per octave up to 3 kHz , then a sharp resonant peak occurs at 4.7 kHz , rapidly declines, and again peaks at 5.9 kHz .


Fig. 2.8.10. Bode plot of an unloaded motortachometer.

This example shows the behavior of the motor-tachometer unit without being connected to a load. When a typical inertial load is direct-coupled to the motor shaft, the plot takes the appearance shown in Fig. 2.8.11. Note that the resonant peak between armature and shaft with load has decreased to 3.8 kHz , and this is what the servo system designer should look at in designing servo system bandwidth and compensation. The amplitude of the resonances and respective frequencies will dominate the servo characteristics in high-performance systems, and it is very important to have these characteristics measured and established before the system design is completed. Thus, if a given motor model and shaft configuration would prove to be difficult for the designer to reconcile, the motor manufacturer has means of suggesting and providing alternatives to solve these problems.


Fig. 2.8.11. Bode plot of motor-tachometer with inertial load.


Fig. 2.8.13. Mechancial diagram of a moving coil motor armature and shaft.

Another phenomenon which has been puzzling to many motor users is critical speed resonance. The term critical speed is used here advisedly, since it is not the classical critical speed phenomenon described in various mechanical textbooks. The classical critical speed problem is generally defined as the speed at which a shaft will resonate in a bending mode equivalent to that of the "free bar" bending as shown in Fig. 2.8.12.

Note the similarity between the "free bar" and the moving coil shaft-armature arrangement in Fig. 2.8.13.

The true critical speed is that at which the time (period) of one revolution will be equivalent to the period of free bar resonance oscillations. For the machine sizes we are discussing, the true critical speed


Fig. 2.8.12. Classical example of a "free bar" bending.
would occur in the range of 20000 to 50000 rpm - much higher than any practical useful speed for devices under discussion. However, the bending resonance oscillations can be excited at shaft speeds lower than the true critical speed, such as, for instance, by the commutation frequency.

Thus it turns out that critical shaft speeds of a moving coil motor are defined by the following equation:

$$
\begin{equation*}
n_{c}=\frac{60 \mathrm{f}_{\mathrm{c}} \mathrm{~K}_{1}}{\mathrm{~N} \mathrm{~K}_{2}} \quad[\mathrm{rpm} ; \mathrm{Hz},-] \tag{2.8.5}
\end{equation*}
$$

where
> $f_{c}$ is basic "free bar" resonance frequency of armature shaft and load

N is number of commutator bars

$$
K_{1}=1,2,4,8
$$

$K_{2}=1,2$ and sometimes 4

At these critical speeds the problem may manifest itself as a peak in audio noise with tendency for the output shaft to exhibit "whip". The resonance is induced by the commutation current flowing in the winding, generating a torque vector which disturbs the shaft radially. This excites the free bar resonance oscillations of the shaft and armature assembly, and may transmit forces into end caps and other structural members, causing audible noise.

The factors $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ contribute harmonics of the fundamental resonant frequency with amplitudes dependent on specific internal component configurations.

Electro-Craft has developed several means of coping with this problem, and can supply moving coil motors which will meet the most rigid critical speed resonance standards.

## DEMAGNETIZING CURRENT

Moving coil motors are generally not as susceptible to demagnetizing peak currents as iron core motors because of their significant airgap and special pole shoe design. Experience at Electro-Craft with moving coil motors shows that peak current limits are set not by demagnetizing effects, but by structural and conductance limits in the armature structure and brush assembly.

### 2.9. SPECIALTY MOTORS - Permanent Magnet Motors With Variable $\mathrm{K}_{\mathrm{T}}$

### 2.9.1. INTRODUCTION

While the permanent magnet DC motor has many advantages over other motor types for speed control and servo applications, in some instances its inherent characteristics cause problems. Such applications are, typically, digital tape transport drive systems, where high torque forward drive and high speed rewind requirements exist; and numerically controlled production machinery, where close position control and rapid traverse conditions prevail. Typically, these applications require the motor to run intermittently at light load and high speed, and other times at heavy load and low speed.

The problem is caused by the inherent correlation between the torque constant $\mathrm{K}_{\mathrm{T}}$ and voltage constant $\mathrm{K}_{\mathrm{E}}$ (see section 2.1.). Thus, when the designer desires large torque constant to achieve large torque without unduly high current, he then underwrites the need for a high voltage from the amplifier for the high speed condition. On the other hand, if he limits the amplifier voltage, he needs a smaller voltage constant


Fig. 2.9.1. Schematic of wound-field motor showing separately excited field winding.
and torque constant, and he must now supply a higher current for the torque application. Since the ratio of optimum $\mathrm{K}_{\mathrm{T}}$ for the two modes can easily be 3:1, we can see that the servo controller cost can be a big factor in making design decisions.

### 2.9.2. THE WOUND-FIELD MOTOR

The shortcoming of the PM motor is due to its constant permanent magnet field; or, in terms of its transfer function characteristic, it is the invariance of the torque constant, $\mathrm{K}_{\mathrm{T}}$. A solution to the problem of prohibitive amplifier costs is to use a motor with an adjustable $\mathrm{K}_{\mathrm{T}}$. The small $\mathrm{K}_{\mathrm{T}}$ can then be employed when high speed is required and the large $\mathrm{K}_{\mathrm{T}}$ for the high torque mode. A variable $\mathrm{K}_{\mathrm{T}}$ motor permits an economical amplifier design.

As is often the case in going to the woundfield (WF) motor, a number of undesirable things also develop. The motor will be substantially larger and heavier (see Fig. 2.9.2) because of volume required by the field windings. This could cause problems with the installation and maintenance of the motors. The wound-field motor is usually more expensive than the PM motor of comparable rating. This is also due to the added costs of the field windings and larger housing. A third factor to consider is the loss of magnetic detent. While a PM motor can be designed so that it will require a measurable torque to turn it from a position of minimum reluctance, when power is removed from the field winding of the WF motor there is nothing to prevent the shaft
from turning. This can be very annoying if the application requires that a certain shaft position is to be held after power is removed. In some cases, it is necessary to add a failsafe brake to prevent shaft rotation, as in the case of a tape reel unwinding due to the weight of tape.


Fig. 2.9.2. A comparison of the relative sizes of wound-field and permanent magnet motor diameters.

### 2.9.3. NEW MOTOR TYPES

From the foregoing discussion it can readily be seen that the ideal DC servomotor would have cost and performance characteristics of the PM motor and also the versatility of speed control as exemplified by the woundfield motor. This ideal motor has been the goal of DC servomotor manufacturers for a number of years. The efforts in this direction have resulted in several new types of special purpose motors which offer some attractive characteristics. The motors described in the following paragraphs are the subjects of patent applications action by Electro-Craft Corporation.

## The Hybrid Motor

An early approach to the design of a motor that would incorporate desirable features of the "ideal motor" was the hybrid motor shown in cross-section in Fig. 2.9.3. The hybrid motor simply took a "wound" field and put it in series with a PM field with both encompassing a common armature.


Fig. 2.9.3. The hybrid motor showing permanent magnet and wound-field functions in one armature.

The portion of the armature rotating in the PM field generates a torque that is proportional to the magnetic flux density and the length of conductors therein. This part of the armature is designed also to provide magnetic detent. Another portion of the armature rotates within the "wound" field and generates a torque that can either add to or subtract from that due to the PM. By properly proportioning the relative lengths of WF and PM portions of the armature, the motor $\mathrm{K}_{\mathrm{T}}$ can be changed by a fixed ratio merely by switching the polarity of the voltage applied to the field winding.
variation in torque constant can be achieved very easily with a relatively small field winding power. The field winding can be equipped with a center tap so that the winding polarity can be simply switched by means of a flip-flop when triggered by an appropriate signal. The dual mode characteristics of a hybrid motor (the ElectroCraft 5260) are shown in Tab. 2.9.1. It can be seen that the $3: 1$ ratio is obtained with only 50 W of field power required.

|  | PM <br> field <br> only | + <br> voltage <br> on WF | - <br> voltage <br> on WF |
| :---: | :---: | :---: | ---: |
| Torque constant <br> [0z-in/A] | 28 | 42 | 14 |
| No-load speed at <br> 30 V [rpm] | 1450 | 965 | 2850 |
| Field winding <br> power - 50 W |  |  |  |

Tab. 2.9.1. The hybrid motor characteristics.
Although the hybrid approach offers some desirable application characteristics it does not achieve the full cost savings potential and efficiency of the PM motor. A portion of it does have the cost and bulkiness of the WF motor associated with it.

## The DAARC Motor

The not completely satisfactory costs and bulk of the hybrid motor led to the DAARC concept. DAARC is a handy acronym which derives from the words "Direct Axis Armature Reaction Control". The acro-
description of the principle of operation of the motor, which seems to offer the most potential at this time.

The principle of operation of the DAARC motor can most easily be understood by considering Fig. 2.9.4. It shows the arrangement of magnets and armature conductors on either side of the air gap in the PM motor. It can be seen that armature current developes a triangular mmf distribution around the air gap. The peak absolute values of the armature mmf (also called armature reaction) coincide with the placement of the brush axis. If the brush axis is shifted, the armature reaction axis will shift with it. Now in a normal motor the brush axis is usually placed close to the field quadrature axis since this is the most efficient area in developing torque. In contrast current introduced through brushes on the direct axis would produce zero torque. Getting back to Fig. 2.9.4, it can be seen that the armature mmf tends to increase magnet flux at one corner of the magnet and decrease it at the other corner.


Fig. 2.9.4. Armature mmf in a $D C$ permanent magnet motor.


Fig. 2.9.5. Armature mmf in a DC permanent magnet motor with brushes shifted $90^{\circ}$ el.

In a normal motor, the two effects will approximately offset and there is little effect on motor action. Now suppose that the brush axis in Fig. 2.9.4 is shifted $90^{\circ} \mathrm{el}$ so that the positive peak is aligned with the north pole axis. This arrangement is shown in Fig. 2.9.5. It can be seen that the armature mmf is now working in opposition to the permanent magnet, and will act to decrease the air gap flux. Thus, by controlling the current to the direct axis brushes, the air gap flux (and hence torque constant) can be reduced to a desired lower level. This is the principle of the DAARC motor operation.


Fig. 2.9.6. Location of brushes in a DAARC motor.

Of course, current going to brushes on the direct axis will not develop torque or motor rotation. It is therefore necessary to add another set of brushes on the quadrature axis for motor operation. The arrangement of brushes is shown for a two-pole configuration in Fig. 2.9.6. The direct axis brushes are used to control air gap flux. They can be excited by a predetermined fixed voltage to drop the motor $K_{T}$ to the desired level for high speed operation. The quadrature axis brushes are excited by the servo amplifier in the normal manner to develop torque and achieve the desired speed.

The DAARC approach has several advantages over other approaches. Since a zero net voltage is generated across the directaxis brushes, armature resistance is theoretically the only limiting parameter on direct axis current. This means that a low voltage source is sufficient to supply the current to these brushes. By actually reducing the magnetic flux in the air gap the rotational losses of the motor due to iron loss and circulating currents are made small. These losses are otherwise proportional to speed and can become excessive at high speeds.

Perhaps the greatest advantage of the DAARC motor approach is the potential for cost savings. The basic cost structure is that of a PM motor with the addition of an extra set of brushes. In the practical case some additional design changes to optimize DAARC operation do add back cost but overall cost is very favorable compared to wound-field motors.

|  | Without <br> direct axis <br> current | With direct <br> axis current |
| :---: | :---: | :---: |
| Torque constant <br> [oz-in/A] | 43 | 13 |
| No-load speed at <br> $30 ~ V ~[r p m] ~$ | 940 | 2600 |
| Direct axis input <br> power -120 W |  |  |

Tab. 2.9.2. The DAARC motor characteristics.

Test data from an actual motor is shown in Tab. 2.9.2. It is seen that the motor charteristics are comparable to those of the hybrid motor as shown in Tab. 2.9.1.

The DAARC motor is a response to a requirement for a very special kind of motor. It is truly a good example of the saying that "necessity is the mother of invention".

### 2.10. MOTOR TESTING

This section will enlighten the reader as to how a motor manufacturer checks motor parameters. Incoming inspection and troubleshooting charts are included to completely describe tests performed on motors. The following tests are described:

## End play

Radial play
Shaft runout
Moment of inertia
Resistance
Inductance
Friction torque and starting current
No-load current and rotational losses, no-load voltage, no-load speed
Demagnetization current
Torque constant
Voltage constant
Electrical time constant
Mechanical time constant
Torque ripple
Speed regulation constant
Efficiency
Frequency response
Thermal resistance
Thermal time constant
Air flow impedance

## END PLAY

A movement of a motor shaft in the axial direction, caused when an axial force is applied on either end of the motor shaft, is termed end play. To measure the amount of end play, a hub is placed on the motor shaft, the motor is locked in a test fixture, and an indicator point is placed touching
the hub. A predetermined force is applied on one end of the shaft such as $\mathrm{F}_{1}$ in Fig. 2.10.1, and the deflection is noted on the indicator. The indicator displays the end play directly.


Fig. 2.10.1. End play test setup.

## RADIAL PLAY

The amount of deflection of the motor shaft in response to a radial force in the radial direction is termed radial play. The amount of radial play is determined by placing a hub on the motor shaft at a specific distance from the mounting surface, locking the motor in a test fixture and placing an indicator point on the hub. A predetermined force, such as $F_{2}$ in Fig. 2.10.2, is applied at a specific distance from the mounting surface, first in one direction, then in the other. The total displacement measured is the radial play.


Fig. 2.10.2. Radial play test setup.

## SHAFT RUNOUT

As a motor shaft rotates it may exhibit non-concentric characteristics. The amount of wobble is termed runout. To measure runout, an indicator point is placed on the motor shaft at a specific distance from the mounting surface, see Fig. 2.10.3. The shaft is rotated through one revolution and deflection is measured on the indicator which reads shaft runout directly.


Fig. 2.10.3. Runout test setup.

## MOMENT OF INERTIA

Inertia is the inherent property of bodies which resist any change in their state. A measure of this property is the moment of inertia. There are two means of measuring moment of inertia. The first involves an inertial measuring device, while the second method makes use of the torsional pendulum technique. Both methods relate moment of inertia through a mathematical proportionality to the square of the period of oscillations.

An inertial measuring device as shown in Fig. 2.10.4 will quickly and accurately provide the desired result.


Fig. 2.10.4. Inertial measuring device.

First attach a collet to the shaft, then mount this into the test fixture, turn on the oscillation lever, and read the period of oscillations. Remove the armature from the collet and read the period of oscillations of the collet only. Moment of inertia, J, can be calculated from:

$$
\begin{equation*}
\mathrm{J}=\mathrm{C}\left[\mathrm{t}_{1}^{2}-\mathrm{t}_{2}^{2}\right] \tag{2.10.1}
\end{equation*}
$$

where

C is the calibrated constant of the instrument
$t_{1}$ is the period of oscillations of armature and collet
$t_{2}$ is the period of oscillations of collet only

The second method of measurement requires a collet and master cylinder of known moments of inertia and approximately the same size as the armature under test.

Attach the collet to the armature under test and connect as shown in Fig. 2.10.5.


Fig. 2.10.5. Alternate method of measuring the moment of inertia.

Give the armature a half twist and allow it to oscillate freely while recording the time for 20 oscillations. Attach the collet to the master cylinder and repeat the above procedure. Moment of inertia of the armature under test can be calculated from:

$$
\begin{equation*}
J=J_{1}\left(\frac{t_{2}}{t_{1}}\right)^{2}-J_{2} \tag{2.10.2}
\end{equation*}
$$

where
$J_{1}$ is moment of inertia of master cylinder and collet
$J_{2}$ is moment of inertia of collet

[^2]$\mathrm{t}_{2}$ is time of 20 oscillations of armature under test and collet

## RESISTANCE

In this procedure, the motor under test is driven and the resistance of brushes, commutator, and armature winding is measured dynamically. Resistance values are specified at $25^{\circ} \mathrm{C}$; therefore, readings should be made quickly to avoid heating effects; the current should be about $1 / 4$ of the rated current.

A power supply is connected across the terminals of the motor under test while it is being driven with a low-speed motor, as shown in Fig. 2.10.6.

Current is adjusted for a predetermined level and voltage is measured. Resistance is calculated from:

$$
\begin{equation*}
R=\frac{V}{I_{a}} \tag{2.10.3}
\end{equation*}
$$

inductance

In measuring inductance, the motor under test is connected to the impedance bridge as shown in Fig. 2.10.7. The $\mathbf{Q}$ and L settings are alternately adjusted for minimum voltmeter readings. When the minimum reading is obtained, the $L$ setting indicates the motor inductance. Repeat above procedure for three other shaft positions $90^{\circ}$ apart, and take the average of the four readings to determine the motor inductance, $\mathrm{L}_{\mathrm{a}}$.


Fig. 2.10.6. Motor resistance test.


Fig. 2.10.7. Impedance bridge measurements of inductance.

## FRICTION TORQUE AND STARTING CURRENT

When current becomes sufficient to overcome torque caused by static friction, motor rotation will start. The minimum current is called starting current, and the static friction termed friction torque.

Friction torque can be measured by two methods. The first utilizes a torque watch, and the second involves current and voltage measurements. In using a torque watch, attach the watch to the motor shaft as indicated in Fig. 2.10.8, and start to rotate the torque watch slowly. Torque as indicated on the watch will be friction torque.

Repeat this procedure at three other positions 90 degrees apart in both clockwise and counterclockwise rotation directions. The highest reading is the maximum friction


Fig. 2.10.8. Torque watch method of measuring friction torque.

The second method is based on starting current measurements as an indicator of starting torque.

To measure the minimum current required to start rotation, the motor under test is connected as illustrated in Fig. 2.10.9. Voltage is slowly increased until the motor shaft barely turns. This current value is the starting current, $\mathrm{I}_{\mathrm{as}}$.


Fig. 2.10.9. Test setup for measuring minimum current required to overcome friction in order to start motor rotation.

The friction torque may be calculated by using the starting current value which was just measured, and the formula:

$$
\begin{equation*}
T_{f}=K_{T} I_{a s} \tag{2.10.4}
\end{equation*}
$$

## NO-LOAD CURRENT AND ROTATIONAL LOSSES, NO-LOAD VOLTAGE, NO-LOAD SPEED

Not all input power supplied to a motor is converted into mechanical power. There are both mechanical and electrical losses. If a motor is rotating with no load on its shaft, a small current will still be drawn. This is no-load current, $\mathrm{I}_{\mathrm{ao}}$, directly attributed to rotational losses.

With the motor connected as in Fig. 2.10.10 , a predetermined (rated) voltage is applied. The measured current is termed the no-load current, $\mathrm{I}_{\mathrm{ao}}$; motor speed is termed no-load speed, $\mathbf{n}_{\mathbf{0}}$.

A plot is actually made for determining rotational losses. Voltage is adjusted to correspond to motor speed from 500 rpm up to the maximum allowable safe motor speed, in increments of 500 rpm , while recording currents and voltages. If losses in units of [oz-in/krpm] are desired, the $\mathrm{K}_{\mathrm{T}} \mathrm{I}_{\text {ao }}$ values versus speed are plotted. If losses in units of [W/krpm] are desired, the current $\mathrm{I}_{\mathrm{ao}}$ timies the input voltage V versus speed is plotted. The slope of the line will give rotational loss in appropriate units.


Fig. 2.10.10. Measuring no-load current and no-load speed.

## DEMAGNETIZATION CURRENT

When the motor is subjected to high current pulses, the magnets may become demagnetized. This results in a lowered $\mathrm{K}_{\mathrm{T}}$, and requires more current to produce the same torque as before demagnetization.

The motor under test is actually demagnetized to obtain the demagnetization current. The procedure is:

1. Note voltage required for motor speed of 1000 rpm .
2. Insert motor in test circuit of Fig. 2.10.11, lock its shaft, apply a step voltage for a short period of time and measure the current.

The above procedure is repeated, always increasing the magnitude of step voltage
of test \#2, until test \#1 requires $2 \%$ less voltage to maintain 1000 rpm . The current which causes the $2 \%$ change is the demagnetization current.

## TORQUE CONSTANT

The measurement of the motor torque constant, $\mathrm{K}_{\mathrm{T}}$, will require a dynamometer or a torque indicating instrument. This test method is based on the setup shown in Fig. 2.10.18. Keep the motor running at a constant speed, while adjusting the load of the dynamometer up to the rated load of the motor. The results of the measurements can be plotted, as shown in Fig. 2.10.12, and an average slope established which will give

$$
\begin{equation*}
K_{T}=\frac{\Delta T}{\Delta I} \tag{2.10.5}
\end{equation*}
$$



Fig. 2.10.11. Demagnetization current test.


$$
\begin{equation*}
\mathrm{K}_{\mathrm{T}}=1.3524 \mathrm{~K}_{\mathrm{E}} \quad[\mathrm{oz}-\mathrm{in} / \mathrm{A} ; \mathrm{V} / \mathrm{krpm}] \tag{2.1.24}
\end{equation*}
$$

Fig. 2.10.12. Minimizing measurement errors in establishing $\mathrm{K}_{\mathrm{T}}$.

Another way of determining $\mathrm{K}_{\mathrm{T}}$ is to calculate the value based on the measurement of the voltage constant $K_{E}$ (see following test). The torque constant is always related to the voltage constant in the following way:

$$
\mathrm{K}_{\mathrm{T}}=\mathrm{K}_{\mathrm{E}} \quad\left[\mathrm{Nm} / \mathrm{A} ; \mathrm{V} / \mathrm{rad} \mathrm{~s}^{-1}\right]
$$

$$
\begin{equation*}
K_{T}=9.5493 \times 10^{-3} K_{E} \tag{2.1.23}
\end{equation*}
$$

[Nm/A; V/krpm]

## VOLTAGE CONSTANT

The voltage constant, $K_{E}$, can best be tested by running the motor as a generator (driven by another motor) and measuring the generated voltage, $\mathbf{E}_{\mathbf{g}}$, while measuring the shaft speed, $\mathbf{n}$. The voltage constant is then obtained by the following relationship:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{E}}=\frac{\mathrm{E}_{\mathrm{g}}}{\mathrm{n}} \quad[\mathrm{~V} / \mathrm{krpm} ; \mathrm{V}, \mathrm{krpm}](2.1 .18) \tag{2.1.22}
\end{equation*}
$$

A typical setup is shown in Fig. 2.10.13.


Fig. 2.10.13. Arrangement for motor back-emf test to determine $K_{E}$ (voltage constant) and $K_{T}$ (torque constant).

## ELECTRICAL TIME CONSTANT

The electrical time constant of a motor may be calculated from the measured values of motor inductance, $L_{a}$, and armature resistance, $\mathbf{R}$, as follows:

$$
\begin{equation*}
\tau_{e}=\frac{L_{a}}{R} \quad[s ; H, \Omega] \tag{2.3.30}
\end{equation*}
$$

In an alternative method using a direct measurement, a step input voltage is applied to the motor armature with the motor shaft locked, and the exponential rise of the motor current is measured on an oscilloscope. The time required for the current to rise to $63.2 \%$ of its final value is equal to the electrical time constant, $\tau_{\mathrm{e}}$, of Fig. 2.10.14a.

The motor is connected as in Fig. 2.10.14b. A step input voltage is applied and current through a resistor of low value ( $\mathrm{R}_{1}<0.1 \mathrm{R}$, where $\mathbf{R}$ is the armature resistance) is monitored on the scope in the form of the voltage $\mathbf{v}_{\mathbf{2}}$. The time required for current to reach $63.2 \%$ of its final value is measured and the value of $\tau_{\mathrm{e}}$ determined.

The present discussion is based on the assumption that the armature impedance is a series combination of a resistance and an inductance. Hence, it can be written as

$$
\begin{equation*}
Z(s)=R+s L_{a} \tag{2.10.6}
\end{equation*}
$$



Fig. 2.10.14a. Measuring electrical time constant.


Fig. 2.10.14b. Electrical time constant test setup.
While this description is valid for "conventional" iron core motors, it is found that the impedance of moving coil motors is approximately [see Fig. 2.3.1. and Eq. (2.3.1)] :

$$
\begin{equation*}
Z(s)=R+\frac{s L_{a} R_{L}}{s L_{a}+R_{L}} \tag{2.10.7}
\end{equation*}
$$

where $\mathbf{R}_{\mathbf{L}}$ is an additional resistance in parallel with $\mathbf{L}_{\mathbf{a}}$, making the inductance
term not quite orthogonal to $\mathbf{R}$. The parallel resistance, $\mathbf{R}_{\mathbf{L}}$, is very large for iron core rotors; for these the approximation of the resistance-inductance series combination is valid.

## MECHANICAL TIME CONSTANT

The mechanical time constant, $\tau_{m}$, defined by (2.3.29) can be measured in several ways. For example, if a motor is given a step voltage, the time required for a motor to reach $63.2 \%$ of its final speed is the mechanical time constant, provided the electrical time constant of the armature does not affect the measurement.

Moving coil motors usually have $\tau_{\mathrm{e}}<0.1 \tau_{\mathrm{m}}$, and such measurement is possible if a tachometer is provided for speed measurements.

The mechanical time constant can also be measured by noting the mechanical break frequency, $f_{b}$, in the frequency response test (to be described under "Frequency Response" in this section). The mechanical time constant can be derived from the measured break frequency as follows:

$$
\begin{equation*}
\tau_{\mathrm{m}}=\frac{1}{2 \pi \mathrm{f}_{\mathrm{b}}} \quad[\mathrm{~s} ; \mathrm{Hz}] \tag{2.10.8}
\end{equation*}
$$

In cases where the electrical time constant is of the same order as the mechanical time constant, the measurement is a bit more complex and is beyond the scope of this brief treatment. However, $\tau_{\mathrm{m}}$ can be calculated [see Eqs. (2.3.29), (2.8.2), (2.8.3),
and (4.1.26)] from known values of moment of inertia, armature resistance and torque constant as follows:

$$
\begin{align*}
\tau_{m}= & \frac{\mathrm{JR}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}}  \tag{2.10.9}\\
& {\left[\mathrm{~s} ; \mathrm{kg} \mathrm{~m}^{2}, \Omega, \mathrm{~V} / \mathrm{rad} \mathrm{~s}^{-1}, \mathrm{Nm} / \mathrm{A}\right] }
\end{align*}
$$

## TORQUE RIPPLE

The output torque of a DC motor at low speeds appears to be constant, but under closer examination it has in fact a cyclic component as illustrated in Fig. 2.10.15. This cyclic action is referred to as torque ripple, and is caused by the switching action of the commutator (and sometimes by the armature reluctance torque).


Fig. 2.10.15. Torque ripple
Although torque ripple usually is a very small percentage of the rated output torque, and in most applications can be neglected, there are times when it may be critical; it therefore becomes necessary to have a means of checking torque ripple.

By connecting the motor under test to the apparatus as illustrated in Fig. 2.10.16, measurement of torque ripple can be made,
if the moment of inertia of the measuring device is much smaller than the motor moment of inertia (otherwise inertia filtering takes place and the ripple measurement will be invalid).

The percent peak-to-peak ripple torque can be calculated from:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{r}}=\frac{\text { Peak-to-Peak Torque Ripple }}{\text { Average Output Torque }} 100[\% \text { ] } \tag{2.10.10}
\end{equation*}
$$

## SPEED REGULATION CONSTANT

The motor under test is operated at rated voltage from no load to 1.5 times the maximum continuous torque, while speed and torque are recorded. Readings must be taken quickly to avoid overheating. The data is plotted in a speed versus torque curve as shown in Fig. 2.10.17.

Speed regulation constant is the slope of the line and may be determined from:

$$
\begin{equation*}
R_{m}=\frac{\Delta n}{\Delta T} \tag{2.10.11}
\end{equation*}
$$

This value can be also calculated using the formula:

$$
\begin{equation*}
R_{m}=\frac{R}{K_{E} K_{T}} \tag{2.8.1}
\end{equation*}
$$

## EFFICIENCY

The motor is connected in the test apparatus as shown in Fig. 2.10.18. The speed $\mathbf{n}$ and output torque $\mathrm{T}_{0}$, at which efficiency will be checked, should correspond closely to actual conditions under which motor will be used. Input voltage $\mathbf{V}$ and current $\mathbf{I}_{\mathbf{a}}$ are measured. Efficiency can be calculated from:


Fig. 2.10.16. Torque ripple test setup.


Fig. 2.10.17. Speed-torque curve of a motor at rated voltage.
$\eta=1.0472 \times 10^{4} \frac{\mathrm{~T}_{\mathrm{o}} \mathrm{n}}{\mathrm{V} \mathrm{I}_{\mathrm{a}}}$
[\%; Nm, krpm, V, A]
or

$$
\begin{equation*}
\eta=73.948 \frac{\mathrm{~T}_{\mathrm{o}} \mathrm{n}}{\mathrm{~V} \mathrm{I}_{\mathrm{a}}} \tag{2.10.13}
\end{equation*}
$$

[\%; oz-in, krpm, V, A]


Fig. 2.10.18. Motor efficiency test setup. Dynamometer

## FREQUENCY RESPONSE

The determination of motor-generator frequency response and resonant frequencies points can be performed in a simple test setup shown in Fig. 2.10.19. The amplifier shown is Electro-Craft's Motomatic Control System Laboratory amplifier, but any amplifier with a response from 5 Hz to 10 kHz will do. The MCSL unit is convenient in that it has adjustable gain and a current limiting output stage, preventing accidental overload of the motor or amplifier.

The test is done by first establishing a convenient reference level at a low frequency. Then the frequency region of interest is scanned and pertinent points recorded. A typical plot of frequency response of a motor-tachometer was shown earlier in Figs. 2.8.10 and 2.8.11.


Fig. 2.10.19. Frequency response test setup using a Motomatic Control System Laboratory amplifier.

## THERMAL RESISTANCE

Measurement of thermal resistance of a motor is generally done by the manufacturer; it is not usually possible for a customer to disassemble a motor and make the necessary alterations to perform this test. However, it may be of interest to the reader to understand how thermal tests are performed at Electro-Craft Corporation, and we will therefore briefly describe these tests.

A small calibrated thermistor is inserted in the expected "hot spot" as in Fig. 2.10.20 (in some motors several such thermistors are used, where the worst-case hot spot is not known). In most cases, the thermistor output leads are brought out through slip rings so that dynamic testing can be performed.
(Fig. 2.10.21). Once the motor has reached thermal equilibrium, record input voltage V , input current $\mathrm{I}_{\mathrm{a}}$, motor speed n , output torque $\mathrm{T}_{\mathrm{o}}$, armature temperature $\Theta_{\mathrm{a}^{\prime}}$ and ambient temperature $\Theta_{\mathbf{A}}$.

Thermal resistance (armature-to-ambient) is calculated from:

$$
\begin{equation*}
R_{t h}=\frac{\Theta_{a}-\Theta_{A}}{P_{L}} \tag{2.10.14}
\end{equation*}
$$

$$
\left[{ }^{\circ} \mathrm{C} / \mathrm{W} ;{ }^{\circ} \mathrm{C}, \mathrm{~W}\right]
$$

where the power loss in the motor is given by

$$
\begin{align*}
& P_{L}=P_{i}-P_{o} \\
& P_{L}=V I_{a}-104.72 T_{o} n \tag{2.10.15}
\end{align*}
$$

[W; V, A, Nm, krpm]

The motor is run at a power level comparable to that in an actual application


Fig. 2.10.20. Inserting a thermistor in motor armature for thermal resistance test.


Fig. 2.10.21. Thermal resistance test setup.
or

$$
\begin{equation*}
P_{L}=V I_{a}-0.73948 T_{o} n \tag{2.10.16}
\end{equation*}
$$

[W; V, A, oz-in, krpm]

## THERMAL TIME CONSTANT

Thermal time constant is the time required for temperature to attain $63.2 \%$ of its final value, as $\tau_{\text {th }}$ in Fig. 2.10.22.


Fig. 2.10.22. Armature temperature versus time curve.

To measure the thermal time constant, the thermistor arrangement need in the thermal resistance test is used and the motor is connected as shown in Figs. 2.10.23 and 2.10.24. The motor is energized with a step function of power, which is held constant during the test. The strip chart recorder measures the armature temperature rise, and after thermal equilibrium is attained, the chart will appear as in Fig. 2.10.25.

The armature thermal time constant can be determined from the chart paper by measuring the record, as shown in Fig. 2.10.25. The thermistor calibration characteristic in circuit connection of Fig. 2.10.24 must be considered.

This example is based on a single thermal time constant for the sake of simplicity; as shown earlier in section 2.5., a motor has at least two independent thermal time constants.


Fig. 2.10.23. Thermal time constant test setup.


Note: The $1 \mathrm{k} \Omega$ resistor is used as a source of voltage signal for strip chart recorder and also as a means of limiting the current in the thermistor to avoid self-heating.

Fig. 2.10.24. Circuit used in thermal time constant test.

## AIR FLOW IMPEDANCE

By increasing the air flow through a motor, the heat removal rate is increased, thereby substantially increasing the output power capability of the motor. When air is forced through a motor, the internal parts and
design of the air cooling channels will influence the air flow in such a way so as to resist or impede that flow. Air flow impedance is a measure of this influence, and is given as air flow rate (volume per unit of time) divided by given pressure, e.g. in units of

$$
\frac{\mathrm{m}^{3} / \mathrm{min}}{\text { at }} \text { or } \frac{\mathrm{ft}^{3} / \mathrm{min}}{\text { in of } \mathrm{H}_{2} \mathrm{O}}=\frac{\text { CFM }}{\text { in of } \mathrm{H}_{2} \mathrm{O}}
$$



Fig. 2.10.25. Record of strip chart recorder.

Following are two alternate tests. One utilizes an air flow meter, while the other requires air pressure gages, a plastic bag, and a container of known size. The latter test is perhaps a little less accurate, but it is handy for occasional tests with a minimum of equipment outlay.

In the first test method, the motor under test is connected as in Fig. 2.10.26. With the blower on, the air flow rate on the air flow meter and the air pressure are recorded and plotted. The damper is adjusted increasing the air flow rate, while data is plotted, until a curve similar to that in Fig. 2.10.27 is obtained.


Fig. 2.10.26. Air flow meter method of measuring air flow impedance.

For the second method, the motor is connected as in Fig. 2.10.28, with air inside the plastic bag evacuated. The blower is turned on and the damper adjusted for a predetermined pressure, then the air hose is connected to the plastic bag and a stop watch is started. Time to fill the bag and air pressure are recorded. Air flow rate is calculated, and the above procedure re-
peated until a graph similar to Fig. 2.10.27
can be plotted.


Fig. 2.10.27. Example of a motor air flow impedance.


Fig. 2.10.28. Alternate scheme of measuring air flow impedance.
Example of calculating air flow rate:

$$
\text { Container Volume }=4.7 \mathrm{ft}^{3}
$$

Air pressure $=1$ in of $\mathrm{H}_{2} \mathrm{O}$
Average time to fill bag $=\mathbf{2 3} \mathrm{s}$

$$
\begin{aligned}
& \text { Air flow rate }=\frac{4.7 \mathrm{ft}^{3}}{23 \mathrm{~s}} \\
& =\frac{4.7}{23} \cdot 60=12.3 \mathrm{CFM} @ 1 \text { in of } \mathrm{H}_{2} 0
\end{aligned}
$$

## INCOMING INSPECTION OF MOTORS

The following list of motor parameters is intended to serve as a basic guide for incoming inspection tests. Whether this list is expanded or shortened would depend on the testing program (for example, 100\% testing or sampling techniques) and on individual customer needs, as well as the type of motor under consideration.
mOTOR TESTING PARAMETERS

1. Visual Inspection
A. LEADWIRES - LOOK FOR:
nicked, cut or cracked insulation WHICH EXPOSES BARE WIRE.

PROPER COLOR IDENTIFICATION.
PROPER STRIPPING OF LEADWIRE ENDS.
CONNECTORS AND TERMINALS (IF USED) are properly attached
B. OUTPUT SHAFT(S)

RUST OR FOREIGN SCALE.
NICKS OR UP.SETS, WHICH MAY CAUSE an over-size condition.

IF PINIONS OR GEARS ARE USED, CHECK FOR BURRS WHICH MAY CAUSE IMPROPER MATING.
C. BALL BEARINGS

CHECK FOR DAMAGED SHIELDS.
PROPER INSTALLATION OF RETAINING RINGS.
D. PILOTS AND LOCATING SURFACES NICKS, BURRS, OR UPSETS WHICH MAY CAUSE IMPROPER MATING OR ALIGNMENT.

PROPER FINISH (IRRIDITE, ANODIZE, ETC., IF APPLICABLE).
E. BRUSH HOLDERS

CHECK FOR LOOSE OR CRACKED BRUSH holders.

LOOSE OR CRACKED BRUSH HOLDER CAPS.
F. NAMEPLATES

PROPER LABELING AND IDENTIFICATION.
2. Electrical and Mechanical Checks
A. END PLAY
B. RADIAL PLAY
C. SHAFT RUNOUT
D. RESISTANCE
E. NO-LOAD CURRENT
F. NO-LOAD SPEED
G. TORQUE CONSTANT
H. VOLTAGE CONSTANT
I. ABNORMAL NOISE

## Problem

Motor Runs Hot 1. Excessive load.
NOTE:
Don't judge the motor
temperature by feel.
Use an appropriate
temperature measuring
device.
2. Maximum speed of motor exceeded for long periods of time.
3. High ambient temperature.
4. Worn bearings.
5. Short-circuited coils in armature.
6. Armature rubbing

Motor Burns Out

High No-Load Speed

High No-Load Current

1. Worn bearings.
2. Worn or sticking brushes.
3. Magnets demagnetized.

## Corrective Action

1. Determine if motor rating is correct for given load. Inspect coupling between motor and load for excess drag.
2. Re-check motor maximum speed rating.
3. Re-check motor rating for given ambient temperature. Reduce ambient temperature by providing ventilation.
4. Replace bearings.
5. Repaị or replace armature.
6. Note cause of rubbing. Remove obstruction or replace with new armature.
7. Any of the conditions listed in "Motor Runs Hot" will cause a motor to burn up if not detected in time. Replace motor and correct problem when possible to avoid recurrences.
8. Remagnetize magnets.
9. Replace bearings.
10. Replace brushes. Check brush holder slots for obstructions.
11. Remagnetize magnets.

|  | 4. Armature rubbing. | 4. Note cause of rubbing. Remove obstruction or replace with new armature. |
| :---: | :---: | :---: |
|  | 5. Excessive pre-load on bearings. | 5. Remove excess shims. |
|  | 6. Misaligned or cocked bearings. | 6. Remove armature and bearing assembly and re-insert same with proper alignment. |
| Low Output | 1. Magnets demagnetized. | 1. Remagnetize magnets. |
| Torque at Rated Input Power | 2. Open or shorted armature winding. | 2. Repair or replace armature. |
|  | 3. Excessive motor drag. | 3. Check for any added friction spots such as worn bearings, rubbing armature, misalignment between motor and load, etc. |
| High Starting Current | 1. Worn bearings. | 1. Replace bearings. |
|  | 2. Worn or sticking brushes. | 2. Replace brushes. Check holder slots for obstructions. |
|  | 3. Obstruction in air gap. | 3. Remove obstruction. |
|  | 4. Armature rubbing. | 4. Note cause of rubbing, remove obstruction or replace with new armature. |
|  | 5. Magnets demagnetized. | 5. Remagnetize magnets. |
|  | 6. Open or shorted armature winding. | 6. Repair or replace armature. |
|  | 7. Misalignment between motor and load. | 7. Re-align coupling to reduce drag. |
| Erratic Speed | 1. Varying load. | 1. Re-adjust load. |
|  | 2. Worn or sticking brushes. | 2. Replace brushes. Check brush holder slots for obstructions. |
|  | 3. Obstruction in air gap. | 3. Remove obstruction. |
|  | 4. Worn bearings. | 4. Replace bearings. |


| Problem | Probable Causes | Corrective Action |
| :---: | :---: | :---: |
|  | 5. Open or shorted armature winding. | 5. Repair or replace armature |
| Reversed Rotation | 1. Motor leadwires to power supply reversed. | 1. Reverse leadwires. |
|  | 2. Magnets have reversed polarity. | 2. Rotate magnet and housing assembly to proper orientation or remagnetize. |
| Excessive Brush Wear | 1. Incorrect spring tension. | 1. Springs may lose temper due to excessive heat. Replace brush with proper spring material and tension. |
|  | 2. Dirty or rough commutator. | 2. Clean or re-machine commutator. Insure a good surface finish. |
|  | 3. Incorrect brush seating (off center neutral) | 3. Check to see that motor is properly neutralized. |
|  | 4. Excessive load | 4. Check the value of rated current vs. the actual. If over the mfg. rating, the excessive heat and current would speed brush wear. |
|  | 5. Short circuited coils in armature. | 5. Repair or replace armature. |
|  | 6. Loose-fitting brushes. | 6. Check brushes and brush holder slots for proper size. Replace as needed. |
|  | 7. Vibration | 7. The armature should be dynamically rebalanced. The vibration causes brush bounce, hence, arcing and excessive wear. |
|  | 8. Lack of moisture. | 8. Operating in a near vacuum condition causes rapid brush wear. Use a special treated or "high altitude" brush. |
| Excessive Bearing Wear | 1. Belt tension too great. Misalignment of belt, coupling, or drive gears. Unbalanced coupling or too closely meshed gears. Excessively heavy flywheel or other load hung on motor shaft. | 1. Correct mechanical condition. Limit radial load to mfg . specification. |

2. Dirty bearings.
3. Insufficient or inadequate
Iubrication.
4. Excessive thrust load.
5. Bent output shaft causing excessive vibration.

Excessive Noise

1. Unbalanced armature.
2. Worn bearings.
3. Excessive end play
4. Misalignment between motor and load.
5. Motor not fastened firmly to mounting.
6. Dirt in air gap.
7. Amplified motor noises.
8. Rough commutator.
9. Clean or replace bearings. If condition is bad, provide means for shielding motor from the dirt.
10. Review bearing mfr. recommendation for type and amount of lubricant to use.
11. Reduce the load or obtain a motor designed to handle the required thrust load.
12. Remove armature and check shaft with mechanical indicator. Straighten if possible or replace armature.
13. An unbalanced armature can set up vibrations that can be easily felt. The armature should be rebalanced dynamically.
14. Replace bearings.
15. Add washers (shims) to the motor to take up end play. Re-check friction level and end play as excessive pre-load will cause early bearing failure.
16. Correct mechanical conditions of misalignment.
17. Correct mechanical condition of mounting.
18. Noise is irregular, intermittent and scratchy: dismantel and clean motor.
19. Uncouple motor from load and allow it to run. If noise persists, loosen the mounting bolts and lift motor while it is still running. If motor is quiet, the mounting was acting as an amplifier. If possible, replace with a cushion type mounting to reduce noise.
20. Re-machine commutator.

Excessive Radial Play



Shaft Will Not Turn

1. No input voltage
2. Motor bearings tight or seized.
3. Load failure.
4. Dirt or foreign matter in air gap.
5. Excessive load.
6. Open motor winding.
7. Check fits, the I.D. of the bearing should be a light press fit.
8. Replace bearings.
9. The bearings should be free to slide in the bearing bore. However, the clearance should not be excessive. If excessive, replace end cap.
10. Add shims as needed. Re-check motor for proper end play.
11. Re-balance armature dynamically to proper level.
12. Replace bearings.
13. See corrective action for "excessive radial play'.
14. Repair or replace armature.
15. Check at motor leads for proper input voltage.
16. Replace bearings.
17. Disconnect motor from load and then determine if motor will run without load. (Check load to see if it turns freely.)
18. Remove the obstruction.
19. Check the load. It may be enough to stall the motor.
20. Check for open windings. Repair or replace armature.
21. Worn or sticking brushes.
22. Brushes may be worn enough so that they do not touch commutator. Clean brushes and brush holders so that brushes move freely. Replace if necessary.

### 2.11. DC GENERATORS

## INTRODUCTION

The DC tachometer-generator is a rotating electromagnetic device which, when mechanically driven, generates an output voltage proportional to speed. This characteristic can be used in applications requiring either shaft speed readout or closed-loop speed control or stabilization.

Fig. 2.11.1 illustrates an application in which a tachometer-generator, or a tachometer as it is often called, is used in conjunction with a voltmeter calibrated directly in speed units (rpm) to provide speed readout - in this case at a remote control station. However, the most important application of tachometers is in servo systems, where they are employed to provide shaft speed feed-
back signal in speed control systems, as shown in Fig. 2.11.2. A tachometer providing rate feedback signal to improve stability of a position servo system is shown in Fig. 2.11.3.

## THEORY

The basic theory of the DC motor in Chapter 2. applies to the DC tachometer, but while the DC tachometer can be viewed essentially as a small DC motor, it has some important distinguishing requirements:

1. The tachometer shall provide a DC voltage proportional to the shaft speed, with a controlled linearity;
2. The output voltage shall be relatively free from rotation dependent voltage ripple or random voltage



Fig. 2.11.1. Tachometer providing shaft speed readout.
changes in the frequency range in which the tachometer is to be operating;
3. The voltage gradient $K_{E}$ should be stable with ambient and device temperature variations;
4. There is no requirement that the tachometer provide any significant power; hence, the design considerations are mainly directed at solving problems related to requirements 1 , 2 and 3 above.


Fig. 2.11.2. Block diagram of speed servo with tachometer feedback.


Fig. 2.11.3. Block diagram of position servo with tachometer damping.

The tachometer develops an output voltage proportional to shaft speed, with voltage polarity dependent on direction of rotation. Fig. 2.11.4 shows a relationship between tachometer speed and output voltage, which can be expressed mathematically as:

$$
\begin{equation*}
E_{g}=K_{E} n \tag{2.1.18}
\end{equation*}
$$

where $\mathrm{E}_{\mathrm{g}}$ is the tachometer output voltage and $\mathbf{n}$ is the tachometer speed. $\mathrm{K}_{\mathrm{E}}$ is the proportionality constant between output voltage and speed, i.e. the voltage constant of the tachometer.


Fig. 2.11.4. Tachometer output voltage vs. shaft speed.

## TYPES OF DC GENERATORS

There are basically two types of DC generators: the shunt-wound generator and the permanent magnet generator. The shuntwound tachometer for feedback purpose is obsolete, however, and the permanent magnet units are universally accepted for their compactness, efficiency and reliability.

The majority of DC tachometers are of the conventional iron-copper armature type.

In many cases Electro-Craft provides the tachometer with coin silver commutators, which gives highly reliable commutation for cases where the tachometer is run for extended periods of time at very low speeds - a difficult operational condition for copper commutators. A variety of windings can easily be provided on iron core armatures; thus, a wide range of $\mathrm{K}_{\mathrm{E}}$ values can be supplied.

Moving coil tachometers are, in principle, identical to moving coil motors, as described in section 2.8. with one important difference: a high number of coils per pole is desirable to minimize ripple voltage.

Essentially, the moving coil concept is characterized by a lack of iron in the armature design. The armature windings consist of a cylindrical, hollow rotor, composed of wires and held together with fiber-glass and polymer resins. Because the armature is not burdened by iron in its design, this hollow or shell-type armature has the lowest moment of inertia of any DC tachometer type. Since coil inductance is low, due to the absence of iron, moving coil tachometers have an extremely low ripple amplitude, approximately $1 \%$ peak-to-peak for Electro-Craft tachometers. This makes moving coil tachometers excellent, not only in low inertia applications, but in precision speed controls as well. The ripple content is sometimes difficult to measure, since most test methods produce inherent ripple
contributions of the same order of magnitude. Suitable test procedures are discussed in detail in section 2.12.

## MOUNTING FEATURES

Tachometers can, of course, be used as separate devices; but in a majority of servo applications the tachometer is closely connected to a motor.

A unique solution to the motor-tachometer combination is represented by ElectroCraft's patented Motomatic ${ }^{\circledR}$ motor-tachometer (Figs. 2.11.5 and 2.11.6). In this device the tachometer winding is wound on the motor armature alongside the motor


Fig. 2.11.5. Cross-sectional view of Motomatic motor-tachometer.
winding. Due to the use of magnets with low permeability and high coercivity, there is no significant interaction between the two windings over a wide range of operating conditions. A thermistor-resistor network minimizes the effect of the temperature dependent magnetic flux on the tachometer voltage gradient. The "Motomatic" system - the motor-tachometer used with a transistor amplifier - has been preferred for speed control applications for many years, and its applications range from blood pumps, crystal-pulling machines and semiconductor spinners to textile machines, motion picture editing machines, and automatic welding machines.

While the Motomatic motor-generator performs excellently for wide range speed controls, it has a limitation in its ability to respond rapidly in high-performance servo systems, due to the magnetic coupling between the two windings. Thus, for servo bandwidths beyond $15-20 \mathrm{~Hz}$ the tachometer must be physically separated from the motor armature.


Fig. 2.11.6. Electro-Craft's Motomatic armature. Motor and tachometer windings simultaneously wound on the same armature.


Fig. 2.11.7. Motor and tachometer on a common shaft (Electro-Craft E-576).

Fig. 2.11.7 shows a cross-sectional view of a patented Electro-Craft design of a motortachometer ( $\mathrm{E}-576$ ) on a common shaft. This design combines the advantage of rig-
idly coupled motor-tachometer armatures, which are magnetically separated, with independently adjustable magnet mounting features. The device represents a good combination of high-performance features with economical manufacturing attributes.

A photograph of the armature assembly is shown in Fig. 2.11.8, and the assembled unit is shown in Fig. 2.11.9. The physical separation of the armature enables servo bandwidths of up to 100 Hz to be achieved.


Fig. 2.11.8. The Electro-Craft E-576 is an example of motor and tachometer mounted on the same shaft in one housing.


Fig. 2.11.9. The assembled unit E-576.


Fig. 2.11.10. Cross-section of an Electro-Craft Moving Coil Motor-Tachometer.

When moving coil motors are equipped with tachometers, it is necessary to mount the tachometer external to the motor housing, but still with the moving coil tachometer


Fig. 2.11.11. Electro-Craft M1030-110 Moving Coil Motor-Tachometer. Motor and tachometer armatures are rigidly mounted on the same shaft.
armature rigidly coupled to the motor shaft. Fig. 2.11.10 illustrates a typical moving coil motor-tachometer combination. Because of the low armature moments of inertia and rigid shaft connections, the torsional resonance frequency between motor armature and generator armature is over 3.5 kHz - well beyond most servo requirements. The assembled unit is shown in Fig. 2.11.11.

In some cases a customer may want to mount a tachometer to a motor on his own, perhaps as shown in Fig. 2.11.12. Since firm alignment of two independent sets of bearings presents problems, the two shafts are usually joined by a flexible coupling. This coupling, in turn, may limit the performance of the servo system due to the torsional resonance introduced by the coupling. It is therefore advisable to use the factory-assembled, direct-coupled motorgenerators whenever possible.

## TEMPERATURE EFFECTS

Tachometers can be grouped into four main classes according to the accuracy of their output:

1. Ultrastable generators
2. Stable generators
3. Compensated generators
4. Uncompensated generators

The desired accuracy of each individual system design determines which generator to employ.


Fig. 2.11.12. Assembly of a separate motor and tachometer, using a flexible shaft coupling.

## Ultrastable Generators

This class of generators will show temperature errors of less than $0.01 \%$ per ${ }^{\circ} \mathrm{C}$ of temperature change. To achieve this low level of temperature sensitivity, a stable magnet material is used which has a low temperature coefficient, such as Alnico 8 or Alnico 5, in combination with a compensator alloy in the magnetic circuit. Additional design features minimize heating effects in the area of the magnet. This type of unit is the most costly of the four.

## Stable Generators

This group of tachometer-generators will exhibit temperature errors ranging up to $0.02 \%$ per ${ }^{\circ} \mathrm{C}$ of temperature change. They are usually based on uncompensated Alnico 5 magnets. This class of generators is a bit less costly to build, and will usually be quite satisfactory for many applications.

## Compensated Generators

This group of tachometer-generators is characterized by temperature errors in the area
of $0.05 \%$ per ${ }^{\circ} \mathrm{C}$ of temperature change over a restricted temperature range. This type of unit utilizes lower cost ceramic magnets in combination with thermistor temperaturecompensating networks to achieve a level of temperature sensitivity that is adequate for several applications. A minor disadvantage of this type of unit is that its output impedance is generally higher than that of the previously mentioned ones. Because of the compensating circuitry and the effectiveness of the compensation, this group is limited to a temperature range of up to $75{ }^{\circ} \mathrm{C}$.

## Uncompensated Generators

This group has the lowest cost of the four types mentioned here. It is characterized by temperature errors ranging up to $0.2 \%$ per ${ }^{\circ} \mathrm{C}$ of magnet temperature. This type of unit is often adequate for damping purposes in position servos, where adequate phase margin is provided in the initial design.

EXAMPLE: It is desired to use a motor-tachometer combination in a constant speed drive where the speed must not vary by more than $2 \%$ of set speed over a temperature range of $20^{\circ} \mathrm{C}$ to $70^{\circ} \mathrm{C}$. What type of tachometer-generator should be used?

The specified temperature range means that the tachometer-generator will see a temperature change of

$$
\Delta \Theta=70-20=50^{\circ} \mathrm{C}
$$

The allowable temperature coefficient of output voltage must be:

$$
\psi_{\mathrm{V}} \leqslant \frac{2 \%}{50^{\circ} \mathrm{C}}=0.04 \% /{ }^{\circ} \mathrm{C}
$$

The stable group of tachometer-generators would be the proper choice here to give this desired performance at the lowest cost.

## LINEARITY AND LOAD EFFECTS

The ideal tachometer-generator would have an output voltage vs. speed characteristic as shown earlier in Fig. 2.11.4, a perfectly straight line. If a generator is run within its rated speed range (usually below 6000 rpm) one can expect such linear behavior. However, if a generator designed for low speed operation, is run at very high speed (above 10000 rpm ), aerodynamic lift or commutator runout may cause a drop in the voltage-speed characteristic such as shown in Fig. 2.11.13. This, of course, represents abnormal operating conditions, and is not recommended.


Fig. 2.11.13. Abnormal tachometer characteristic.


Fig. 2.11.14. Schematic of tachometer connected to resistive load $\mathrm{R}_{\mathrm{L}}$.
The effect of load impedance can be derived from the schematic in Fig. 2.11.14.

The loop equation can be written as:

$$
\begin{equation*}
E_{g}=R I_{a}+V=K_{E} n \tag{2.11.1}
\end{equation*}
$$

Since:

$$
V=R_{L} I_{a}
$$

we obtain after substituting into (2.11.1) and rearranging

$$
\begin{equation*}
V=\frac{R_{L}}{R+R_{L}} K_{E} n \tag{2.11.2}
\end{equation*}
$$

This shows output voltage to be a function of load resistance, $\mathbf{R}_{\mathbf{L}}$, armature resistance, R , tachometer voltage constant, $\mathrm{K}_{\mathrm{E}}$, and speed, $\mathbf{n}$. $\mathbf{R}_{\mathbf{L}}$ does not affect linearity, but does affect the gradient $\mathrm{V} / \mathrm{n}$. The tendency is, then, to make $\mathbf{R}_{\mathbf{L}}$ as high as possible to get high voltage gradient. However, in order to keep a good commutation surface, the value of $\mathrm{R}_{\mathrm{L}}$ should be in the 5 to $10 \mathrm{k} \Omega$ region. In cases where the tachometergenerator is run for long periods at very low speeds (less than 100 rpm ), it may be necessary to use values of $\mathbf{R}_{\mathbf{L}}$ even lower than that in order to maintain good commutation. A rule of thumb is that in such cases a current $I_{a}=0.1$ to 1 mA should be maintained.

Linearity can be specified by limiting the maximum deviation of the voltage-speed characteristic from a straight line drawn between zero speed point and the output voltage at a specified calibration speed. This maximum deviation is usually specified as a percentage of the output voltage at the calibration speed.

The percent linearity error can be expressed mathematically as:

$$
\begin{equation*}
\Delta=100 \frac{V-\frac{n}{n_{c}} V_{c}}{V_{c}} \quad[\%] \tag{2.11.3}
\end{equation*}
$$

where
$\Delta$ is the linearity error
V is output voltage
$\mathbf{n}$ is tachometer speed
$\mathrm{n}_{\mathrm{c}}$ is a specified calibration speed
$\mathrm{V}_{\mathbf{c}}$ is output voltage at $\mathbf{n}_{\mathbf{c}}$
The calibration speed, $\mathrm{n}_{\mathrm{c}}$, that is specified for a particular generator should be established at $83.3 \%$ of the maximum operating speed of the application.

EXAMPLE: If a tachometer-generator has the following characteristics:

$$
\mathrm{K}_{\mathrm{E}}=7 \mathrm{~V} / \mathrm{krpm}
$$

and its linearity error is
$\Delta \leqslant 0.2 \%$ over speed range 0 to 4000 rpm,
what is the maximum deviation from a perfect straight line characteristic at 1500 rpm ?

For the maximum speed of 4000 rpm the calibration speed should be:

$$
n_{c}=0.833 \times 4000=3332 \mathrm{rpm}
$$

The output voltage at the calibration speed is:

$$
V_{c}=K_{E} n_{c}=7 \times 3.332=23.324 \mathrm{~V}
$$

The output voltage at 1500 rpm should be

$$
V=K_{E} n=7 \times 1.5=10.5 V
$$

We can calculate the maximum allowable deviation at 1500 rpm by solving (2.11.3) for V and subtracting the theoretical output voltage of 10.5 V .

Then:

$$
\begin{equation*}
v=v_{c}\left(\frac{\Delta}{100}+\frac{n}{n_{c}}\right) \tag{2.11.4}
\end{equation*}
$$

### 2.12. GENERATOR TESTING

Because of the many inherent similarities between motors and generators, a good share of the device testing procedures and setups are identical. This is the case for the following characteristics and performance parameters, and the reader is directed to the discussion in section 2.10. for detailed procedure.

## End play

Radial play
Shaft runout
Moment of inertia
Inductance
Friction torque
The following discussions present suggested tests and measurements which, in addition to those above, provide a complete repertoire of generator inspection routines.

## OUTPUT IMPEDANCE

With a motor driven generator connected as in Fig. 2.12.1, generator no-load output voltage is recorded. Switch SW1 is then closed, applying a load to the generator, then the coarse (R1) and fine (R2) resistors are adjusted until the output voltage equals


Fig. 2.12.1. Generator output impedance test.
one-half of the no-load output voltage, as recorded above. The resistors' values at this point equal the generator output impedance.

## VOLTAGE GRADIENT

The generator output voltage is proportional to the speed of armature. In testing for generator voltage gradient, the generator is driven at 1000 rpm with a voltmeter monitoring output voltage. Voltage at 1000 rpm is equal to generator voltage gradient, $K_{E}$, in units of [ $\mathrm{V} / \mathrm{krpm}$ ].

## VOLTAGE POLARITY

Generator output is checked to verify that a specific voltage polarity exists for a given direction of armature rotation.

GENERATOR RIPPLE

## Iron-Core Generator Ripple

DC tachometer-generator ripple is generally specified in two alternate ways. In noncritical applications, it is given as the ratio of RMS value of AC components of the output voltage to its average DC value; in cases where good control of ripple is essential, it is the ratio of peak-to-peak value of $A C$ components to average DC value. The latter measurement requires an oscilloscope, and the former requires true RMS and DC voltmeters.

The fundamental ripple frequency is dictated by the number of commutator seg-
ments, and can be calculated by the expression shown below:

$$
\begin{equation*}
f_{r}=\frac{100}{6} k u n \quad[\mathrm{~Hz} ;-, \mathrm{krpm}] \tag{2.12.1}
\end{equation*}
$$

where
$u$ is number of commutator segments
$k=1$ if $u$ is even
$k=2$ if $u$ is odd
$n$ is shaft speed


Fig. 2.12.2. Tachometer-generator test setup.

The RMS ripple measurement is simply made by connecting a DC voltmeter and a true RMS voltmeter with an AC input network to the tachometer terminals, while the tachometer is driven at a constant speed in the region of interest. The RMS percentage ripple is determined as follows:

$$
\begin{equation*}
\left(\overline{\mathrm{V}}_{\mathrm{r}}\right)_{\mathrm{RMS}}=100 \frac{\mathrm{~V}_{\mathrm{RMS}}}{\mathrm{~V}_{\mathrm{AV}}} \quad \text { [\%] } \tag{2.12.2}
\end{equation*}
$$

The peak-to-peak measurement is discussed in the following section, and illustrated in Fig. 2.12.2.

## Moving Coil Tachometer Ripple

Moving coil tachometers having peak-topeak ripple amplitude of the order of $1 \%$ are difficult to test, because most test methods contribute inherent shaft velocity ripple of the same order of magnitude. The following test method was developed at Electro-Craft Corporation to test motorgenerators designed for use as capstan motors for digital tape transports, but can as well be applied to any motor-tachometer or single tachometer having low ripple specifications.

The Electro-Craft test is based on the principle that the motor is run "open loop" with a heavy flywheel attached directly to the output shaft. No compliant coupling is used. A suitable collet or nonmarring chuck can be used to couple the flywheel to the shaft. The flywheel, in conjunction with the motor, effectively filters out any mechanical ripple due to commutation or to motor torque ripple.

Considerable care is necessary in performing this test to insure that the motor-tachometer under test is suitably isolated from all sources of external mechanical and electrical interference.

This test should be performed only on samples which have been properly run-in with a tachometer current drain of at least 1 mA . The motor-tachometer under test is connected into a test arrangement as shown in Fig. 2.12.2.

The motor is driven from a regulated power supply, and the speed is adjusted so that the tachometer DC output voltage is 0.5 V $\pm 50 \mathrm{mV}$, as measured on the oscilloscope. This is a typical voltage for capstan motors; other applications may require different values.

Adjust the oscilloscope time base to 20 or $50 \mathrm{~ms} / \mathrm{cm}$ and observe tachometer-generator output voltage with the oscilloscope amplifier switched to "DC". The vertical gain should be set to $0.1 \mathrm{~V} / \mathrm{cm}$. Estimate and record the AC component of generator output voltage plus output "noise".

Switch the oscilloscope amplifier to " $A C$ " and adjust gain to suitable level (5, 10, 20 or $50 \mathrm{mV} / \mathrm{cm}$ ). Adjust the waveform to the center of screen. Fig. 2.12 .3 shows a typical pattern. Examine and record:
a) Amplitude (peak-to-peak) of cyclic AC component (an RC filter, typically $1 \mu \mathrm{~F}$ and $10 \mathrm{k} \Omega$, will remove all high frequency components);
b) Amplitude (peak-to-peak) of hf noise or "hash";
c) Amplitude (peak-to-peak) of commutator ripple;
d) Total (peak-to-peak) AC component expressed as a percentage of DC output voltage, to obtain peak-to-peak percentage ripple.

Percentage (peak-to-peak) ripple can be calculated as follows:

$$
\begin{align*}
\left(\overline{\mathrm{V}}_{\mathrm{r}}\right)_{\mathrm{p}-\mathrm{p}}= & 100 \frac{\mathrm{AX}}{\mathrm{~V}_{\mathrm{AV}}}  \tag{2.12.3}\\
& {[\% ; \mathrm{V} / \mathrm{cm}, \mathrm{~cm}, \mathrm{~V}] }
\end{align*}
$$

where
A is oscilloscope amplifier gain
X is amplitude (peak-to-peak) of ripple (see Fig. 2.12.3) observed on the scope screen
$V_{A V}$ is tachometer DC average output voltage

## TEMPERATURE COEFFICIENT

Temperature coefficient relates a change in generator output voltage with temperature. It is expressed as a percent change of output voltage per ${ }^{\circ} \mathrm{C}$.

In testing for the temperature coefficient, the generated output voltage at a predetermined speed is recorded. The generator


Fig. 2.12.3. Typical ripple pattern.
is placed in a temperature chamber, allowed to reach thermal stability, then driven at the same speed while voltage is recorded. This can be done for any number of temperatures. Temperature coefficient is calculated by:

$$
\begin{array}{r}
\psi_{V}=100 \frac{V_{1}-V_{A}}{V_{A}\left(\Theta_{1}-\Theta_{A}\right)}  \tag{2.12.4}\\
{\left[\% /{ }^{\circ} \mathrm{C} ; \mathrm{V}_{,}{ }^{\circ} \mathrm{C}\right]}
\end{array}
$$

where
$\mathrm{V}_{1}$ is output voltage at temperature $\Theta_{1}$
$\mathrm{V}_{\mathrm{A}}$ is output voltage at ambient temperature $\Theta_{\mathrm{A}}$
$\Theta_{1}$ is adjusted chamber temperature
$\Theta_{A}$ is ambient temperature

## DIELECTRIC TEST

This is an electrical check to see whether there will be a breakdown of dielectric materials when subjected to high voltage. The test simply calls for application of a specified high voltage between generator winding and housing while monitoring leakage current. If specified leakage is not exceeded, the unit passes successfully.

## LINEARITY

The purpose of a tachometer is to generate a voltage proportional to speed. The linearity of a tachometer is defined as the maximum deviation of generated voltage vs. speed characteristic from the straight line, as described in section 2.11.

The generator is attached in a test fixture as illustrated in Fig. 2.12.4, and is driven at several different speeds while speed and output voltage is recorded on a chart similar to that in Tab. 2.12.1.

| SPEED | MEASURED <br> OUTPUT <br> [rpm] <br> VOLTAGE <br> [V] | CALCULATED <br> THEORETICAL <br> OUTPUT <br> VOLTAGE [V] | DIFFERENCE <br> $\mathbf{~} \Delta \mathrm{V}$ <br> [V] | DEVIATION <br> V <br> [\%] |
| :---: | :---: | :---: | :---: | :---: |
| 1000 | 3.05 | 3.0273 | 0.0227 | 0.0074 |
| 2000 | 6.20 | 6.0546 | 0.1454 | 0.0240 |
| 3000 | 8.99 | 9.0819 | 0.0919 | 0.0100 |
| 4000 | 12.02 | 12.1092 | 0.0892 | 0.0073 |
| 5000 | 15.15 | 15.1365 | 0.0135 | 0.00089 |

Tab. 2.12.1. Example of generator linearity test results.
The true mean value of the tachometer voltage constant, $\mathrm{K}_{\mathrm{E}}$, is obtained from data in Tab. 2.12.1 by dividing the sum of measured output voltages by the sum of speeds:

$$
\mathrm{K}_{\mathrm{E}}=\frac{45.41}{15}=3.0273 \mathrm{~V} / \mathrm{krpm}
$$

This is used to calculate theoretical output voltage (speed in [krpm] times $\mathrm{K}_{\mathrm{E}}$ ), as


Fig. 2.12.4. Generator linearity test setup.
recorded in column 3 of Tab. 2.12.1. Differences between calculated theoretical output voltages and measured output voltages are recorded in column 4. The deviations from linearity are calculated by:

$$
\begin{equation*}
\Delta=100 \frac{\Delta V}{V_{\text {theor }}} \tag{2.12.5}
\end{equation*}
$$

## STABILITY

Output voltage variations can occur in short term and/or long term periods. Short term variations may be of a random nature. Long term voltage drift may take place over a period of hours or days. Both changes are recorded as percentage voltage variations at a specified load.

Test equipment used should have the following capabilities:

1. A recorder with stability at least four times better than the voltage
stability to be measured. Full-scale range of the recorder shall be 5 to 10 times the expected voltage variations. Recorder input current should be no more than $5 \%$ of the specified generator current. The recorder may have appropriate filtering to prevent voltage ripple from recording.
2. A precision DC power supply with voltage stability at least four times better than the voltage stability to be measured.
3. Precision decade resistors, voltage dividers, and potentiometers as may be required to adjust the recorder sensitivity, zero the recorder, and match the power supply voltage to the generator voltage.
4. A room or enclosure with temperature control to maintain ambient temperature within $\pm 5^{\circ} \mathrm{C}$.


Fig. 2.12.5. Generator stability test setup.

The generator shall be loaded with a noninductive load which will remain unchanged throughout the test. Voltage dividing networks are used to adjust the recorder sensitivity, and to match the generator and stabilized power supply voltages, as in Fig. 2.12.5.

After thermal equilibrium of the generator is attained, the stabilized DC power supply is adjusted so that the recorder is at a midscale position.

The recorder can be operated at intervals, or for any continuous period of time desired. Short term operation is approximately five hours. Long term drift usually is read at one-hour intervals for eight hours, then at eight-hour intervals for seventy-two hours.

The largest peak-to-peak drift amplitude in any one-hour period expressed as a percentage of average DC voltage during this hour is the voltage stability figure for short term variations. Refer to Fig. 2.12.6.


Fig. 2.12.6. Strip chart recorder results for short term generator stability testing.

For long term drift, the percentage voltage change per hour is calculated for each time interval between voltage readings. Long term voltage stability is the largest of these percentages.

## INCOMING INSPECTION

The following list of electrical and mechanical tests can be employed as guidelines for incoming inspection:

GENERATOR INSPECTION

1. Visual Inspection

## A. LEADWIRES

Nicked, cut or cracked insulation TO EXPOSE BARE WIRE

PROPER COLOR IDENTIFICATION
PROPER STRIPPING OF LEADWIRE ENDS
if terminals or connectors are USED, CHECK THAT THE RIGHT ONE IS being used and that it is attached PROPERLY.
B. OUTPUT SHAFT(S)

RUST OR FOREIGN SCALE
NICKS OR UPSETS, WHICH MAY CAUSE an over-size condition
C. BALL BEARINGS

CHECK FOR DAMAGED SHIELDS
PROPER INSTALLATION OF RETAINING RINGS
D. PILOT AND LOCATING SURFACES

NICKS, BURRS, OR UPSETS WHICH MAY CAUSE IMPROPER MATING OR ALIGNMENT
E. BRUSH HOLDERS

CHECK FOR LOOSE OR CRACKED BRUSH holders

LOOSE OR CRACKED BRUSH HOLDER CAPS
F. NAMEPLATES

PROPER LABELING AND IDENTIFICATION
2. Electrical and Mechanical Checks
A. END PLAY
B. RADIAL PLAY
C. RUNOUT
D. OUTPUT IMPEDANCE
E. FRICTION TORQUE
F. VOLTAGE GRADIENT
G. VOLTAGE POLARITY
H. GENERATOR RIPPLE

## GENERATOR TROUBLESHOOTING CHART

Problem

High Generator Ripple

No Output Voltage

Intermittent Output

Low Output
Voltage

Problem Causes

1. Open or shorted winding
2. Improper filter
3. Improper test speed
4. Speed variations in drive motor
5. Brush bounce
6. Excessive axial or radial armature movement
7. Brushes out of electrical neutral zone
8. Faulty connection
9. Open wires
10. Hung brushes
11. Coupling between drive motor and generator
12. Intermittent opens
13. Hung brushes
14. Worn brushes
15. Brush bounce
16. Incorrect resistive load value

## Corrective Action

1. An open or shorted winding will show a very high spike in the normal ripple pattern. Repair or replace armature.
2. If filter is used, check components for proper values.
3. Check armature for proper speed. Generator ripple is speed sensitive. Also, filters are generally selected according to commutating frequency which is a function of speed.
4. Check stability of drive motor. Any variations in speed will be reflected in the generator output.
5. If brush bounce occurs, it's generally at high speeds. Check for rough or non-concentric commutator, weak brush springs, or worn brushes.
6. Check coupling between drive motor and generator for misalignment.
7. Readjust generator for minimum ripple. See test procedure for "Generator Ripple".
8. Check connections from generator leadwires to metering device.
9. Check leadwires and brush flex wires for breaks.
10. The brushes should slide freely inside the holder, otherwise should be replaced.
11. Be sure generator is rotating, check for slippage in coupling.
12. Check leadwires and brush flex wires for breaks.
13. Replace brush or brush holder.
14. Replace brushes.
15. Check for rough commutator and remachine if necessary. Check brush springs for proper tension.
16. Check resistive load for proper value.

|  | 2. Magnets partially demagnetized | 2. Remagnetize magnets. |
| :---: | :---: | :---: |
|  | 3. Brush out of neutral zone | 3. Readjust generator for proper output voltage. See generator test procedure for "Voltage Gradient." |
|  | 4. Incorrect checking speed | 4. Voltage output is proportional to speed. Be sure armature is rotating at the proper speed setting. |
| Reversed Polarity | 1. Generator lead wires to load reversed. | 1. Reverse leadwires. |
|  | 2. Magnets have reversed polarity. | 2. Rotate magnet and housing assembly to proper orientation or remagnetize. |
| Shaft Will Not Turn | 1. Generator bearings tight or seized | 1. Replace bearings. |
|  | 2. Dirt or foreign matter in air gap | 2. Remove the obstruction. |
| Excessive <br> Vibration | 1. Armature unbalanced | 1. Rebalance armature dynamically to proper level. |
|  | 2. Worn bearings | 2. Replace bearings |
|  | 3. Excessive radial play | 3. See corrective action for "Excessive Radial Play" |
| Excessive <br> Radial Play | 1. Shaft loose in bearing I.D. | 1. Check fits, the I.D. of the bearing should be a light press fit. |
|  | 2. Worn bearings | 2. Replace bearings. |
|  | 3. Bearings loose in end cap bearing bore | 3. The bearings should be free to slide in the bearing bore. However, the clearance should not be excessive. If excessive, replace end cap. |
| Excessive End Play | 1. Improper shimming | 1. Add shims as needed. Recheck generator for proper end play. |
| Excessive <br> Noise | 1. Armature unbalanced | 1. An unbalanced armature can set up vibration that can easily be felt. The armature should be rebalanced dynamically. |
|  | 2. Worn bearings | 2. Replace bearings. |
|  | 3. Excessive End Play | 3. Add shims to the generator to take up end play. Recheck friction level and end play as excessive preload will cause early bearing failure. |


|  | 4. Misalignment between generator and drive mechanism | 4. Correct mechanical condition of misalignment. |
| :---: | :---: | :---: |
|  | 5. Generator not firmly fastened to mounting | 5. Correct mechanical condition of mounting. |
|  | 6. Dirt in air gap | 6. Noise is irregular, intermittent and scratchy. Dismantle and clean generator. |
|  | 7. Rough commutator | 7. Remachine commutator. |
| Excessive <br> Bearing Wear | 1. Dirty bearings | 1. Clean or replace bearings. If condition is bad, provide means for shielding generator from the dirt. |
|  | 2. Insufficient or inadequate lubrication | 2. Review bearing mfg. recommendations for type and amount of lubricant to use. |
|  | 3. Excessive thrust load | 3. Reduce the load or obtain a generator designed to handle the required thrust load. |
|  | 4. Excessive vibration | 4. Check coupling between drive motor and generator for misalignment. |
| High <br> Friction Torque | 1. Worn bearings | 1. Replace bearings. |
|  | 2. Worn or sticking brushes | 2. Replace brushes. Check brush holder slots for obstructions. |
|  | 3. Obstruction in air gap | 3. Remove obstruction. |
|  | 4. Armature rubbing | 4. Note cause of rubbing, remove obstruction or replace with new armature. |
| Excessive Brush Wear | 1. Incorrect spring tension | 1. Replace brush with proper spring material and tension. |
|  | 2. Dirty or rough commutator | 2. Clean or remachine commutator. Insure a good micro-finish. |
|  | 3. Incorrect brush seating (off center neutral) | 3. Check to see that generator is properly neutralized. See test procedure for "Voltage Gradient". |
|  | 4. Short circuit coils in armature. | 4. Repair or replace armature. |
|  | 5. Loose-fitting brushes | 5. Check brushes and brush holder slots for proper size. Replace as needed. |

6. Vibration
7. Lack of moisture
8. The armature should be dynamically rebalanced. Vibration causes brush wear.
9. Operating in a new vacuum condition causes rapid brush wear. Use a special treated or "high altitude" brush.


Fig. 2.8.4. Typical Electro-Craft MCM motor armatures.
considerable flexibility in design since the manufacturer can offer a variety of wire sizes, turns per coil, and diameter and length options.

A sample of the variety of armatures manufactured by Electro-Craft is shown in Fig. 2.8.4. Because of the cylindrical shape of the shell-type armature, the end turns do not burden the armature by inequitable contributions of inertia.

Consequently, the shell-type armature has the highest torque-to-moment of inertia ratio, providing acceleration capabilities of up to $1000000 \mathrm{rad} / \mathrm{s}^{2}$. Fig. 2.8 .5 shows the complete Electro-Craft MCM motor, capstan, optical encoder and vacuum manifold for an incremental tape transport operating at speed of $200 \mathrm{in} / \mathrm{s}$, capable of 250 start-stop cycles per second.

Electro-Craft moving coil motors are used in incremental tape transports, line printers, optical character readers, incremental motion drives, phase-locked servos, computer printers, machine tool drives and video recorders.


Fig. 2.8.5. Entire motor, capstan, optical encoder and vacuum manifold for tape transport.

## Chapter 3

## Unidirectional Speed Controls

### 3.1. BASIC CONTROL METHODS

## SPEED CONTROLLERS AN INTRODUCTION

In this chapter we will discuss various speed control methods for DC motors in some detail. But first let us briefly review the various methods and their control range.

Rheostat controls for DC motors, the earliest and simplest of the control methods, have an effective controlled speed range of about $4: 1$ with poor regulation of motor speed against changes in load torque and line voltage. This control method is very inefficient because of the power dissipated in the rheostat.

The variable transformer, or "Variac", with rectifiers, can run DC motors over a wider speed range (up to 10:1) at an improved regulation compared to the rheostat control. It is also more efficient than the former.

SCR (thyristor) controls with half wave operation have characteristics similar to the variable transformer control. However, SCR systems with full wave rectification can achieve a 20:1 speed range when used with "IR" compensation techniques (a pseudo-closed loop current sensing technique). Using such speed compensation methods, full wave SCR controls can achieve
quoted regulation figures such as $3 \%$ from zero to full torque load. This regulation figure is understood to mean that the motor speed will not deviate from the set speed more than $3 \%$ of the rated speed at any speed setting. To illustrate this, say that a motor has a rated speed of 1800 rpm , and the control has a regulation of $3 \%$ of full speed; this means that the motor speed may vary 54 rpm due to load variations. Now, assume the motor is running at 180 rpm, at no load. When given rated load torque, the motor speed may decline to $180-54=126 \mathrm{rpm}$ - a regulation, based on set speed, of $30 \%$ !

The foregoing examples of speed controls were based on "open loop" features, meaning that there is no velocity feedback element in the control loop which tells the controller what the motor speed is, so that the controller can make appropriate adjustments to keep the speed essentially constant.

True closed-loopfeedback control is possible with SCR circuits, using a tachometer for feedback; with such devices a full wave or three-phase SCR control may achieve speed ranges of up to $100: 1$. Due to the pulsating nature of SCR control techniques, the speed stability of motors below 1 hp may not always be good in the lower speed range, due to the low moment of inertia of the
motor, and due to the load. However, in integral horsepower machines excellent speed range can be achieved.

Transistor controllers such as those supplied by Electro-Craft Corporation, on the other hand, can handle closed-loop speed control with a speed range of over 1000:1, and with regulation better than 1 or $2 \%$, based on set speed. For example, if an 1800 rpm, 1\% regulation rated speed control system is set at 180 rpm and the load torque changes from zero to rated value, the speed will not change more than 1.8 rpm . The transistor controls have their best operational advantage for motor sizes up to 1 hp , and will presently handle high-performance servomotors up to 5 hp . For the smaller motor sizes a continuous control method is used, but for motors above $1 / 3$ to $1 / 2 \mathrm{hp}$ a pulsewidth modulation technique is the most efficient control method.

In the following discussions we will more fully examine the various aspects of speed controllers, old and new.

## OPEN LOOP CONTROLS - <br> THE TRADITIONAL APPROACH

Speed controls for DC motors have a long history. Perhaps the oldest and most widely used control for small DC motors is the series resistor speed control for series motors pictured in Fig. 3.1.1. The variable resistor is inserted in series with both the armature and field circuit, yielding performance as shown in the accompanying speed-torque diagram. This control has good starting


Fig. 3.1.1. Electrical diagram and speed-torque curves for series motor resistor speed control.
characteristics (large torque available at low speed), but has a runaway speed tendency at small load torque conditions, making the control useful only for control applications with somewhat fixed friction conditions, such as sewing machine motors, food mixers, and the like. It is also evident that the speed regulation characteristics of the motor decline with decreasing speed, making it
difficult for the operator to achieve good overall speed control.

A totally different characteristic is obtained in the shunt motor variable resistor speed control circuit shown in Fig. 3.1.2. Here, a series resistor is inserted in the armature circuit, and the field winding is excited with a constant voltage (a permanent magnet field can also be used). The resulting speedtorque characteristics show the regular shunt motor regulation for resistor setting \#1 (no resistance in the circuit). With increasing insertion of resistance in the armature cir-


Torque


Fig. 3.1.2. Electrical diagram and speed-torque curve for shunt motor resistor speed control.
cuit, speed regulation degenerates. This type of control will work well for constant load torque, rather than for a widely varying torque situation.


Fig. 3.1.3. Shunt motor with variable transformer to control armature voltage.

Fig. 3.1.3 shows a control utilizing a variable transformer controlling the armature voltage of a shunt motor with constant field excitation. The resulting speed-torque characteristics are much. improved over variable resistor control characteristics, with a more uniform speed regulation over a wider speed range.

In Fig. 3.1.4 we see a shunt motor connected to a variable field resistor control. The action of this control circuit is unique in that the motor speed is variable only above the speed it would have without the field resistor control. This has an undesirable effect in that the torque constant of the motor will decrease with increasing


Fig. 3.1.4. Shunt motor with variable field resistor control.
resistance insertion (field weakening), the net effect is that the armature current for a given torque will increase with higher speed. The motor can easily be overloaded, and this control circuit is used only in unique cases where load conditions are both predictable and well controlled.

In some sophisticated open-loop control methods, such as the motor-generator arma-


Fig. 3.1.5. An open-loop control utilizing an armature control method.
ture control method shown in Fig. 3.1.5, a constant speed motor drives a generator with an adjustable control field voltage. The generator will produce an adjustable voltage which is delivered to the armature of the motor, and the resulting torque-speed characteristics are improved over the ones shown in Fig. 3.1.2, since in this case the regulation is essentially independent of the speed setting.

This results in superior motor speed control performance over any of the previously shown methods. However, due to the cost of the motor-generator set and associated field control, this method has not been practical for the small motor speed control
applications most commonly used in home and industry. Therefore, the motor-generator control method has been mainly confined to industrial uses of large motor speed controls sets of 1 hp and above. Because of the power amplifying characteristics of the motor-generator set, this was the easiest entry into the closed-loop control systems when power vacuum tubes and magnetic amplifiers became available for control purposes.

The control methods shown above are by no means all that have been developed and used; rather they exemplify the kind of problems encountered in the open-loop, manually operated systems of the past. We can see that in each case there was some undesirable side effect that limited its use to certain applications.

## CLOSED-LOOP CONTROLS

Although many of the "open-loop" controls are adequate for uses today, the trend
toward better speed regulation, such as in meeting servo systems requirements, have necessitated closed-loop control.

In its elementary form, a closed-loop control consists of an actuator (motor), a comparator, an amplifier, and a sensor (generator). Fig. 3.1.6 illustrates these elements and how the "closed-loop" expression is derived: the amplifier drives the motor, which is coupled to the generator; the generator sends a signal to the comparator, which measures the feedback signal against the command signal; the comparator keeps the two signals in balance by giving the amplifier the proper command. Thus, the "loop" consists of the components which are shown in Fig. 3.1.6. Signal flow is indicated by the arrow.

This example is just one of many feedback techniques. Usually, a modern, high-performance servo system has several separate feedback loops, such as voltage, current, velocity and position feedback.


Fig. 3.1.6. A closed-loop speed control system.

In the open loop controls described previously, the human operator was the sensor or feedback element, so in a sense these systems could also qualify under certain circumstances as closed-loop controls; but human control is not an automatic operation, and that is what closed-loop controls are all about.


Fig. 3.1.7. Unidirectional or regulator type speed control system operates in the first quadrant.

## SPEED CONTROLS VS. SERVO SYSTEMS

It is important to recognize the basic differences between the two types of motor control systems because of their individual unique advantages, and since in one case (the regulator control) linear control theory applies only in a restricted sense.

Fig. 3.1.7 shows four quadrants of shaft velocity versus torque plot.

The regulator (speed control) system operates only in the first quadrant of this dia-
gram. The system cannot produce a negative torque; nor can it reverse direction without a reversing switch, adding operation in the third quadrant. This is generally done by manual control, which makes servo considerations invalid. With the addition of a dynamic brake, we can achieve limited negative torque in the second quadrant, but since this is a passive control area, we choose to ignore it for the moment and call the regulator speed control a single quadrant control.

The bidirectional servo control system, on the other hand, can provide motor speed and torque in both negative and positive directions. Fig. 3.1.8 shows the areas within which servo control is possible, and explains why we call the bidirectional servo system a four quadrant control.


Fig. 3.1.8. A servo system operates in four guadrants.

### 3.2. VELOCITY CONTROL - SINGLE QUADRANT CONTROLLER



## VOLTAGE AND CURRENT OF A SINGLE QUADRANT CONTROLLER

A single quadrant controller is a control system which employs a unidirectional amplifier, as seen in Fig. 3.2.1.

As such, it can deliver to the motor only positive (or only negative) voltage and current, and therefore, control velocity only in one direction with the system load torque as an opposing torque. In Fig. 3.1.7, where the motor velocity $\omega$ corresponds to the vertical axis and the torque T to the horizontal axis, we find that the controllable velocity and torque range of a single quadrant controller is in the first quadrant of the plane.

The amplifier characteristic of the single quadrant controller will be described by:

$$
\begin{array}{ll}
\mathbf{V}=0 & \text { if } \epsilon<0 \\
\mathbf{V}=\mathbf{A} \epsilon & \text { if } 0 \leqslant \epsilon \leqslant \frac{\mathbf{V}_{\text {max }}}{\mathbf{A}}  \tag{3.2.1}\\
\mathbf{V}=\mathbf{V}_{\max } & \text { if } \epsilon>\frac{\mathbf{V}_{\max }}{\mathbf{A}}
\end{array}
$$

where $\mathbf{A}$ is the amplifier gain, or graphically à shown in Fig. 3.2.2.

Current delivered from the amplifier to the motor can be only positive, therefore:


Fig. 3.2.2. Amplifier output voltage vs. error signal for a single quadrant controller.

$$
\left.\begin{array}{ll}
I_{a}=\frac{V-E_{g}}{R} & \text { if } V>E_{g}  \tag{3.2.2}\\
I_{a}=0 & \text { if } V \leqslant E_{g}
\end{array}\right\}
$$

where

## $R$ is the motor resistance

$E_{g}$ is internally generated voltage (counter emf) in the motor

## SYSTEM OPERATION

The simplified dynamic equation of a motor is:

$$
\begin{equation*}
K_{T} I_{a}=J \frac{d \omega}{d t}+T_{L} \tag{3.2.3}
\end{equation*}
$$

When the motor is connected in the single quadrant controller system, the system will control the velocity and maintain it close to the desired value $\omega_{\mathrm{i}}$ in the following way.

Suppose that due to disturbances or change of the command signal the actual velocity $\omega_{0}$
is larger than the desired one $\omega_{\mathbf{i}}$. In this case, the velocity error is negative:

$$
\begin{equation*}
\epsilon=\mathbf{k}\left(\omega_{i}-\omega_{0}\right)<0 \tag{3.2.4}
\end{equation*}
$$

This results in zero current and voltage as can be seen from (3.2.1) and (3.2.2); consequently, the velocity will be reduced as can be found from (3.2.3). When the current is zero, the rate of change of the velocity will be:

$$
\begin{equation*}
a=\frac{\mathrm{d} \omega}{\mathrm{dt}}=-\frac{\mathrm{T}_{\mathrm{L}}}{\mathrm{~J}} \tag{3.2.5}
\end{equation*}
$$

The deceleration rate equals the ratio between the opposing load torque (usually friction) and the combined moments of inertia of the motor and the load. The deceleration will continue as long as the velocity $\omega_{0}$ is too high and the velocity error $\epsilon$ is negative.

On the other hand, if the motor velocity is too low, causing an error $\epsilon$ which is larger than $\frac{V_{\text {max }}}{A}$, this results in the output voltage $\mathbf{V}=\mathbf{V}_{\text {max }}$ and current

$$
\begin{equation*}
I_{a}=\frac{V_{\max }-E_{g}}{R} \tag{3.2.6}
\end{equation*}
$$

Consequently, the motor will accelerate at the rate

$$
\begin{align*}
& a=\frac{\mathrm{d} \omega}{\mathrm{dt}}=\frac{1}{\mathrm{~J}}\left(\mathrm{~K}_{\mathrm{T}, \mathrm{a}}-\mathrm{T}_{\mathrm{L}}\right) \\
& a=\frac{1}{\mathrm{~J}}\left(\mathrm{~K}_{\mathrm{T}} \frac{\mathrm{~V}_{\max }-\mathrm{E}_{\mathrm{g}}}{\mathrm{R}}-\mathrm{T}_{\mathrm{L}}\right) \tag{3.2.7}
\end{align*}
$$

The acceleration at this rate will continue until the velocity error $\epsilon$ becomes smaller than $V_{\text {max }} / \mathrm{A}$ and it moves to the linear region.

This discussion shows that in case of large deviations from the desired velocity the system tends to correct the error. It remains to analyze system performance under steady-state conditions and small deviations that keep the amplifier output voltage within the linear range, i.e. $\mathbf{0} \leqslant \mathrm{V}<\mathrm{V}_{\text {max }}$.

Note that in the steady state the velocity is constant. Therefore, the steady-state current, as found from (3.2.3) is

$$
\begin{equation*}
I_{a}=\frac{T_{L}}{K_{T}} \tag{3.2.8}
\end{equation*}
$$

The required voltage V and the velocity error $\epsilon$ can be found from (3.2.1), (3.2.2) and (3.2.8) as follows:

$$
\begin{equation*}
V=E_{g}+I_{a} R=E_{g}+\frac{T_{L} R}{K_{T}} \tag{3.2.9}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon=\frac{V}{A}=\frac{1}{A}\left(E_{g}+\frac{T_{L} R}{K_{T}}\right) \tag{3.2.10}
\end{equation*}
$$

Thus, the steady-state velocity error is positive, which indicates that the motor velocity is lower than the set velocity. This result is identical to that obtained in any velocity control system.

When the performance of the system around the steady-statepoint is analyzed, it is found
that the system is identical to that of velocity control. Thus, the system will be overdamped when the gain is low, and will tend to overshoot when the amplifier gain, $A$, is increased. And, considering the lag in the amplifier, the system can become unstable if the gain is increased beyond a certain limit.

To illustrate these results, suppose that the motor is running at a given velocity $\omega_{1}$ and the command is increased to $\omega_{2}$ for some time $\mathrm{t}_{1}$, then decreased again to $\omega_{1}$. The corresponding motor velocity profile is shown in Fig. 3.2.3.

It can be seen that when the command is increased, the motor will accelerate at the rate

$$
\begin{equation*}
a_{1}=\frac{1}{\mathrm{~J}}\left(\mathrm{~K}_{\mathrm{T}} \frac{\mathrm{~V}_{\max }-\mathrm{E}_{\mathrm{g}}}{\mathrm{R}}-\mathrm{T}_{\mathrm{L}}\right) \tag{3.2.7}
\end{equation*}
$$

The deceleration rate in $t>t_{1}$ is

$$
\begin{equation*}
a_{2}=-\frac{\mathrm{T}_{\mathrm{L}}}{\mathrm{~J}} \tag{3.2.5}
\end{equation*}
$$

When the deceleration rate is too small, due to small friction or large moment of inertia, it is possible to increase the deceleration rate by the use of dynamic braking.

## DYNAMIC BRAKING

The amplifier used in the single quadrant controller system has the feature that its output impedance is low as long as $\mathrm{V}>\mathrm{E}_{\mathrm{g}}$ and the current $\mathrm{I}_{\mathrm{a}}$ is positive. When this is


Fig. 3.2.3. Example of a stepwise changed command and motor velocity profile.
changed, output impedance becomes very large, and does not allow negative current. This can be modified by introducing a circuit which senses current $I_{a}$. As long as the current is positive, the circuit is inactive; but if $I_{a}$ becomes zero, the circuit shorts the motor terminals, allowing a negative current to circulate in the armature, thus stopping the motor. This method is called dynamic braking.

The voltage equation (3.2.1) for this case
is not changed, but the current equation (3.2.2) and the motor dynamic equation (3.2.5) will change.

The current equation becomes

$$
\left.\begin{array}{ll}
I_{a}=\frac{V-E_{g}}{R} & \text { if } V>E_{g} \\
I_{a}=-\frac{E_{g}}{R} & \text { if } V \leqslant E_{g} \tag{3.2.11}
\end{array}\right\}
$$

If the velocity error $\epsilon$ is positive and large, the amplifier output voltage is $\mathrm{V}=\mathrm{V}_{\text {max }}$, the current is given by (3.2.6) and the acceleration by (3.2.7), so that there is no change in system function.

However, if the velocity error $\epsilon$ becomes negative, which indicates that the motor runs too fast, the amplifier output voltage becomes $\mathrm{V}=\mathbf{0}$ and the current is negative:

$$
\begin{equation*}
I_{a}=-\frac{E_{g}}{R} \tag{3.2.12}
\end{equation*}
$$

This results in the deceleration of:

$$
a=\frac{\mathrm{d} \omega}{\mathrm{dt}}=-\frac{1}{\mathrm{~J}}\left(\frac{\mathrm{E}_{\mathrm{g}}}{\mathrm{R}} \mathrm{~K}_{\mathrm{T}}+\mathrm{T}_{\mathrm{L}}\right) \text { (3.2.13) }
$$

This is a larger rate than the one given by (3.2.5) and, hence, enables the system to respond faster.

### 3.3. AMPLIFIERS

Transistor amplifiers used in speed control systems fall into two broad categories; linear amplifiers (also called class $A$ amplifiers) and switching amplifiers. Linear amplifiers are almost exclusively transistor devices, whereas the switching amplifier can be designed using either transistors or SCR's (silicon controlled rectifiers, also known as thyristors).

## THE LINEAR AMPLIFIER

The linear amplifier is very desirable from a control analysis standpoint, since it has linear control characteristics and no significant control lag within the operating bandwidth. It is easily compensated, and is capable of providing speed control over a wide range. It is the least troublesome in terms of transient problems in adjacent circuits.

The simplest closed-loop control system is typified by the single transistor amplifier (Fig. 3.3.1). It has been used widely in control applications where the loads are essentially constant and a limited speed range was acceptable in view of the extreme simplicity and consequent low cost. The circuit consists simply of a 5 W potentiometer and a power transistor, mounted on a heat sink.

Circuit operation can be explained in the following way: the potentiometer is set at a point such that a voltage $\mathrm{V}_{1}$ appears be-


Fig. 3.3.1. A typical single-transistor speed control system.
tween the wiper and the negative terminal (ground) of the circuit. This voltage $\mathrm{V}_{1}$ will cause a current $I_{b}$ to flow through the base (B) and the emitter (E) of the transistor. The base current causes the transistor to conduct, and a current $I_{m}$ will. flow from the positive terminal through the motor winding ( M ) to the collector ( C ) and emitter (E) of the transistor to the negative terminal. The motor current $I_{m}$ will cause the motor to rotate and, if the tachometer is properly connected with respect to its polarity, an opposite voltage $\mathrm{V}_{2}$ will appear at the tachometer terminals. Since any transistor in conduction has a given associated base-to-emitter voltage for each conduction level, a voltage $\mathrm{V}_{3}$ will appear across the transistor with the polarity shown in the figure.

Equilibrium (speed control) will appear when the voltage $\mathrm{V}_{2}$ reaches its steadystate value. The motor cannot run faster than the speed at which it produces $\mathbf{V}_{2}$, since if it did, $I_{b}$ would cease flowing, thus
making the transistor non-conductive, hence slowing the motor down to the "equilibrium" speed. Conversely, if a load change would cause the motor to tend to slow down, the voltage $\mathrm{V}_{2}$ would be smaller, allowing an increase in $\mathbf{I}_{\mathbf{b}}$. This would tend to make the transistor to increase its conductivity, thus allowing an increase in $I_{m}$, which in turn would overcome the increased load. We can see that this circuit has a self-regulating quality, and therefore qualifies as a closed-loop regulator system. This explanation is somewhat simplified, but it brings out the general principles involved.
The single transistor amplifier has the basic limitation of low gain. In other words, it requires a significant error $\left(\Delta I_{\mathbf{b}}\right)$ to correct a speed disturbance. Providing more gain is a simple task, however, and Fig. 3.3.2 shows a four-transistor amplifier, employing a two-stage voltage amplifier and a dual emitter-follower current amplifier.


Fig. 3.3.3. Speed control system utilizing an integrated circuit.

Error signals on the order of a few millivolts are able to control the amplifier from "full off" to "full on" conditions, which makes the closed loop control more sensitive to disturbances. This yields a more precise speed control system. The availability of integrated circuits (IC) has simplified circuit design; in Fig. 3.3.3 the voltage amplifier of Fig. 3.3.2 was replaced by a single IC. This technology can be expanded to include the power stages (hybrid amplifiers).


Fig. 3.3.2. Elementary multi-transistor amplifier.

If the amplifier offers high gain, then one would expect an almost error-free speed control.

In reviewing the possible sources of errors, we find that the command stage (in speed controls usually the speed setting potentiometer) must be stabilized so that the reference voltage does not vary with changes in load current, line voltage and ambient temperature. This is done either by the use of Zener diodes, or by recently available IC regulators. A simple example of a regulated reference supply employing the Zener diode is shown in Fig. 3.3.4.

The Zener diode stabilizes the voltage across the potentiometer $\mathbf{R}_{\mathbf{2}}$ against changes in supply voltage. This circuit is sufficient for many applications, but for a higher degree of reference voltage stability a two-stage


Fig. 3.3.4. Example of regulated reference voltage supply.

Zener regulator is used. Sometimes series stacks of Zener diodes with low temperature coefficients are employed to minimize effects of changing ambient temperature.

A high-performance transistor speed control system will basically consist of a regulated reference supply, an error comparator stage, a voltage amplifier, and a power amplifier.

An example of the steady-state operating conditions of such a system will demonstrate the performance advantage of a high gain system compared with the earlier mentioned single-transistor control.

The system has the following parameters:
Motor:

$$
\begin{aligned}
\mathrm{K}_{\mathrm{T}} & =8 \mathrm{oz}-\mathrm{in} / \mathrm{A} \\
& =5.65 \times 10^{-2} \mathrm{Nm} / \mathrm{A}
\end{aligned}
$$

$$
\mathrm{K}_{\mathrm{E}}=5.916 \mathrm{~V} / \mathrm{krpm}
$$

$$
\mathrm{R}=2 \Omega
$$

Generator:

$$
\mathrm{K}_{\mathrm{Eg}}=5.916 \mathrm{~V} / \mathrm{krpm}
$$

Load:

$$
\begin{aligned}
\mathrm{T} & =\mathrm{T}_{\mathrm{L}}+\mathrm{T}_{\mathrm{f}}=6 \mathrm{oz}-\mathrm{in} \\
& =4.237 \times 10^{-2} \mathrm{Nm} \\
\mathrm{n} & =2000 \mathrm{rpm}
\end{aligned}
$$

Amplifier:
$D C$ gain $A=1000 \mathrm{~V} / \mathrm{V}$


Fig. 3.3.5. Example of steady-state operating conditions of a high performance speed control system.

Fig. 3.3.5 shows the system connection and the input and output signal polarities.

Following the input signal path:

$$
\begin{align*}
& \epsilon=R_{i} I_{c}  \tag{3.3.1}\\
& -V_{c}+V_{g}+\epsilon=0 \tag{3.3.2}
\end{align*}
$$

In a properly operating closed-loop speed control system the input error

$$
\begin{equation*}
\epsilon \ll V_{\mathbf{c}} \tag{3.3.3}
\end{equation*}
$$

meaning that the generator voltage $\mathrm{V}_{\mathrm{g}}$ is always nearly equal to the command voltage $\mathrm{V}_{\mathbf{c}}$.

The steady-state equation of motor is:

$$
\begin{equation*}
V=R I_{a}+K_{E} n \tag{3.3.4}
\end{equation*}
$$

Since

$$
\mathrm{I}_{\mathrm{a}}=\frac{\mathrm{T}}{\mathrm{~K}_{\mathrm{T}}}=\frac{6 \mathrm{oz}-\mathrm{in}}{8 \mathrm{oz}-\mathrm{in} / \mathrm{A}}=0.75 \mathrm{~A}
$$

we have:

$$
V=2 \times 0.75+5.916 \times 2
$$

$$
=13.332 \mathrm{~V}
$$

and since

$$
\begin{equation*}
V=A \epsilon \tag{3.3.5}
\end{equation*}
$$

the required input error signal to produce this voltage is:

$$
\epsilon=\frac{V}{A}=\frac{13.332}{1000} \cong 13.3 \mathrm{mV}
$$

Assume that the load torque is now changed from 6 to 10 oz-in. We now have new equilibrium conditions and the following changes occur.

First, combine (3.3.4) and (3.3.5) we obtain:

$$
\begin{equation*}
A \epsilon=R I_{a}+K_{E} n \tag{3.3.6}
\end{equation*}
$$

The armature current $I_{a}$ must change from 0.75 A to $\mathrm{I}_{\mathrm{a}}=\frac{10 \mathrm{oz}-\mathrm{in}}{8 \mathrm{oz}-\mathrm{in} / \mathrm{A}}=1.25 \mathrm{~A}$

The armature voltage drop is now $\mathrm{RI}_{\mathrm{a}}=$ 2.5 V , which is 1.0 V higher than at the 6 oz-in load condition. This must be made up for by the amplifier, thus by a change in error signal of

$$
\Delta \epsilon=\frac{\Delta V}{A}=\frac{1}{1000}=1 \mathrm{mV}
$$

To accommodate the load change, the error signal had to change from 13.3 to 14.3 mV . From (3.3.2) we have:

$$
\begin{align*}
& \epsilon=V_{c}-V_{g}=V_{c}-K_{E} n  \tag{3.3.7}\\
& V_{c}=\epsilon+K_{E} n \tag{3.3.8}
\end{align*}
$$

Thus in the first case ( 6 oz -in load torque)

$$
V_{c}=13.3 \times 10^{-3}+5.916 \times 2=11.8453 \mathrm{~V}
$$

Since $\mathbf{V}_{\mathbf{c}}$ was constant during the event, we can find the change in the motor speed $\mathbf{n}$ after the load torque became 10 oz -in from (3.3.7):

$$
\begin{aligned}
n=\frac{V_{c}-\epsilon}{K_{E}}= & \frac{11.8453-14.3 \times 10^{-3}}{5.916} \times 10^{3} \\
& \cong 1999.8 \mathrm{rpm}
\end{aligned}
$$

$$
\Delta \mathrm{n} \cong 0.2 \mathrm{rpm}
$$

The resulting change in speed is so small (approx. $0.01 \%$ ) that it is not worth pursuing the exact number of the final speed.

In actual practice, the effects from such sources as amplifier input summing errors, generator ripple voltage changes, and temperature coefficients of the generator voltage gradient may cause larger errors than this example shows.

A linear amplifier can minimize errors in velocity in inverse proportion to its gain due to external sources such as load changes, temperature changes of $K_{T}, R_{m}, \mathbf{R}$, but it cannot correct feedback errors due to changes in generator voltage constant $\mathrm{K}_{\mathrm{Eg}}$, amplifier input drift errors and reference voltage errors. The three last mentioned error sources can, of course, be minimized to a point where they are not significant to the user.

One may also want to increase amplifier gain to lessen the effects of the errors correctable by the feedback loop, but for each system there is a point where an increase in gain will cause instability due to the mechanical or electrical characteristics of the feedback loop.

Any speed control system has performance characteristics which are results of a balance between speed range, gain-bandwidth product, long and short-term stability and selling price. The Motomatic ${ }^{\circledR}$ speed controls by Electro-Craft are used worldwide, and have a reputation for great versatility in a variety of applications. Fig. 3.3.6 shows a typical ECC master control and motor, model E-650.

One general limitation of the linear transistor amplifier is that the output stage must


Fig. 3.3.6. Electro-Craft E-650 master control and motor-generator.
dissipate heat energy not absorbed by the motor. Thus, the worst amplifier power dissipation condition occurs when the motor is run at very low speed at maximum torque. The output stage will then have to absorb the power proportional to the product of the full armature current and almost all available DC voltage. Thus, ample provisions for heat dissipation must be made. Fig. 3.3.7 shows a typical open chassis


Fig. 3.3.7. Open chassis amplifier configuration.
amplifier configuration, capable of dissipating 160 W at $40^{\circ} \mathrm{C}$ environment temperature.

The speed control systems described above are generally augmented by accessories which improve operation in a variety of applications.

## TORQUE LIMITING

To prevent inadvertent overloading of the system, Electro-Craft supplies a current sensor which turns off the amplifier at adjustable current levels. Since torque is proportional to current in a permanent magnet DC motor, the current limiter becomes in effect a torque limiter. The torque-speed characteristics will then exhibit two distinct regions: the speed control region and the torque control region. Fig. 3.3.8 illustrates typical characteristics of the adjustable speed control system with current limiting. The system assumes an almost constant torque mode when the


Fig. 3.3.8. Speed torque curves of a high gain speed control system with torque limiting.
preset current limit has been exceeded. As soon as the overload is removed, the system will resume its preset speed.

In some cases, it is important to provide an acceleration torque which is many times higher than the torque limit necessary to protect the system from overload. This requirement is met by providing a time delay in function of the torque limiter. The time delay is usually of the order of 150 ms for iron-core armature motors, and somewhat less for low inertia motors. This provides ample time for rapid acceleration with virtually unrestricted current, and a subsequent introduction of current limit after acceleration has been completed.

Some users have found that the current limit works to advantage when very high inertia loads are to be accelerated. The current limit circuit in such a case permits controlled acceleration to the required speed without damage to the control circuit or motor.

## DYNAMIC BRAKING DEVICES

When the speed setting in a basic speed control circuit is rapidly reduced, the motor coasts to the lower speed setting. During this time the motor is basically "out of control" until the circuit is again in equilibrium.

The dynamic braking circuit shown in Fig. 3.3.9 minimizes this effect by providing a controlled "short circuit" around the motor whenever the control system is not


Fig. 3.3.9. Dynamic braking circuit.
providing a current through the motor a condition which is prevalent during a "coasting" situation.

The dynamic braking depends on the armature counter emf to provide the needed braking current; the braking torque is the greatest at high speeds and less effective at lower speeds. For example, the dynamic braking brings an Electro-Craft E-650 control system to a stop in about 400 ms .

## REVERSIBLE SPEED CONTROLS

Fig. 3.3.10 illustrates a switch arrangement which gives a single polarity speed control reversible features.

This reversing switch arrangement provides for a middle position (B) which gives a controlled dynamic braking condition, minimizing the possibility of quickly switching a reverse voltage to the motor still coasting in previous direction, which can cause transistor breakdown. The reversing circuit


Fig. 3.3.10. Reversing switch arrangement.
shown is used on all Electro-Craft Motomatic standard and master controls.

The reversing circuit can be operated by relays instead of by a manually operated switch, giving the speed control system a bidirectional characteristic for remote control manual operations. It is not practical, in most cases, to attempt to make a closedloop position or velocity servo by this method, however. A servo system is usually subject to endless reversals around the null point, and relay contacts would rapidly wear. Furthermore, such a system would be subject to time delays and nonlinearities due to relay switching action.

## CONTROLLED ACCELERATION-

 DECELERATION SYSTEMWhile torque limit circuits to some extent can control the acceleration and deceleration characteristics of a speed control sys-


Fig. 3.3.11. Simple ramp command circuit.
tem, it is clear that they are sensitive to variations in both load and friction torque.

In cases where it is desirable to have the acceleration and/or deceleration under velocity control, several methods can achieve the desired results.

The simple RC circuit of Fig. 3.3.11 can be employed to control acceleration. First, assume that the switch is initially in the $A$ position so that the command voltage $\mathrm{V}_{\mathbf{c}}$ is zero. The speed control is, therefore, holding the motor at rest. With the switch set in the $\mathbf{B}$ position, capacitor $\mathbf{C}$ is charged through voltage divider $\mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{2}}$. The voltage $\mathrm{V}_{\mathrm{r}}$ will follow an exponential rise (Fig. 3.3.12) until the Zener voltage $\mathrm{V}_{\mathbf{Z}}$ is achieved, at which point the reference voltage $\mathrm{V}_{\mathbf{r}}$ (as also the command voltage $\mathbf{V}_{\mathbf{c}}$ ) is stabilized.

As the switch is returned to $A$, the voltage will decline in an inverse exponential fashion.

It should be noted that while the speed control system will follow the acceleration


Fig. 3.3.12. Turn-on and turn-off characteristics of the RC circuit in Fig. 3.3.11.
curve (if acceleration rate is within the torque capacity of the system), it may not follow the deceleration curve unless the load conditions and the dynamic braking capacity allow. If this is not the case, then a bidirectional system (servo system) must be used where continuous velocity tracking is necessary.

Another circuit with linear features is shown in Fig. 3.3.13. This is a ramp generator which produces linear, independently adjustable slopes.

The circuits discussed above have been shown as manually operated devices. In many applications, they can be operated by various logic circuits, thereby taking their place in modern process controls.

## SWITCHING AMPLIFIERS

While the linear amplifier discussed above performs excellently in high-performance speed controls, it has the problem of heat generation in the output stage, requiring forced-air cooling in amplifiers over 100 to 200 W (depending on ambient temperature and heat sink design). The switching amplifiers overcome this problem by letting its output stage rapidly switch from a nonconductive state to a fully conductive state, thereby minimizing operation of the output stage in the high dissipation region.

Three basic methods are used to control power in switching amplifiers: pulse-width modulation (PWM), pulse-frequency modulation (PFM), and silicon controlled rectifier


Fig. 3.3.13. Ramp generator with independently adjustable slopes.

a, pulse-width modulated (PWM) system

b, pulse-frequency modulated (PFM) system

c, SCR control system
Fig. 3.3.14. Voltage waveforms in switching amplifiers.
(SCR) controls. Their principal differences are shown in Fig. 3.3.14.

The PWM system usually utilizes a DC supply, and the amplifier switches the supply voltage on and off at a fixed frequency and at a variable "firing angle" $a$ (see Fig. 3.3.14a) so that an adjustable average voltage across the load is established. The amount of power transferred to the load (motor) will depend on switching rate and the load inductance. In many of the PWM circuits, the pulse frequency is allowed to shift over a given range - in some cases for a good purpose.

The PFM system has a fixed firing angle and a variable repetition rate (Fig. 3.3.14b), achieving essentially the same results as the PWM, but when used in motor control circuits, the widely variable pulse frequency required causes dissipation problems which makes the PWM more attractive. Incidentally, a PWM circuit with a variable pulse rate is really a hybrid between the two.

The SCR circuit for DC control is usually used with a rectified AC supply voltage, although a rectification circuit can be placed before or after the control section of the amplifier. Fig. 3.3.14c shows a full-wave rectified supply voltage of a fixed frequency. The firing angle can be varied to cover a portion from 0 to $180{ }^{\circ} \mathrm{el}$ of a half-wave. The average output voltage is not proportional to the firing angle, and this part requires special attention in the control design.

In the following discussion only PWM and SCR circuits will be covered, since the pure PFM is not particularly suited to motor control applications.

## PULSE-WIDTH MODULATED <br> AMPLIFIERS

As already mentioned, the PWM amplifier generally is powered by a DC supply. It
can be designed using either transistors or SCR's as switches. Actually, the SCR amplifier is a PWM in spirit; but the functional differences, the output voltage frequency and wave shapes are different enough to warrant a separate discussion.

Then what about SCR's powering a PWM? Since PWM's are based on a continuously available DC power source, and since SCR's cannot "turn off" current conduction by themselves as transistors can, they must be provided with a separate "turn-off" circuit. The problem associated with such circuitry has limited the use of SCR's in PWM systems to high current, low switching rate applications, such as lift truck and vehicle traction controls. Due to the unique nature


Fig. 3.3.15. Elementary block diagram of a PWM speed control system.
of these controls, they are not discussed here.

Today, then, transistor-operated PWM switching amplifiers are used in most highperformance, high power speed control systems and servo sytems.

A typical block diagram of a PWM speed control system is shown in Fig. 3.3.15.

The pulse repetition rate is usually above 1 kHz (often about 10 kHz ), and this frequency is mainly dictated by the required system response bandwidth, motor inductance and motor high frequency loss characteristics. At times audio noise emissions through wiring, heat sinks and motor frame components can be loud enough to be disturbing, and in such applications one can raise the pulse frequency to a point where the noise is not audible.

In examining voltage and current characteristics of a PWM system, we can first look at the ideal motor and its behavior in a PWM system. This motor equivalent circuit is shown in Fig. 3.3.16.


Fig. 3.3.16. Equivalent circuit of a DC motor in a PWM control system.


Fig. 3.3.17. Current and voltage relationship in a PWM control system.

The waveform of the motor current during the switching mode is dependent not only on the switching rate, but on the motor speed $\mathbf{n}$, the total inductance $\mathbf{L}$, motor resistance $\mathbf{R}$ and the current level in the last cycle. Fig. 3.3.17 shows a steady-state relationship of current and voltage.

Since the supply voltage is being switched on and off at a high frequency, we should evaluate the power losses in the motor. The power losses in such a system may be due to a host of factors, depending on the design of the motor: eddy current losses, hysteresis losses, armature commutation losses, viscous friction losses and armature resistance losses. But considering modern permanent magnet servomotors with small electrical time constants and minimal high frequency losses, a first approximation to evaluate motor power losses due to the armature resistance can be expressed by:

$$
\begin{equation*}
P_{L}=R I_{R M S}^{2} \tag{3.3.9}
\end{equation*}
$$

In order to relate RMS current to average current in such cases, an expression relating
the two has been established, called the form factor (k):

$$
\begin{equation*}
\mathrm{k}=\frac{\mathrm{I}_{\mathrm{RMS}}}{\mathrm{I}_{\mathrm{av}}} \tag{3.3.10}
\end{equation*}
$$

The average motor current will determine the motor torque produced:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{g}}=\mathrm{K}_{\mathrm{T}} \mathrm{I}_{\mathrm{av}} \tag{3.3.11}
\end{equation*}
$$

Substituting (3.3.10) into (3.3.9), the motor losses under PWM conditions are:

$$
\begin{equation*}
P_{L}=R k^{2} I_{a v}^{2} \tag{3.3.12}
\end{equation*}
$$

and we see that the armature losses depend on average current, form factor, and armature resistance. Another way of looking at the sources of the losses is to substitute (3.3.11) into (3.3.12). Then

$$
\begin{equation*}
P_{L}=\frac{R}{K_{T}^{2}} k^{2} T_{g}^{2} \tag{3.3.13}
\end{equation*}
$$

Here it is clear that losses are due to several factors; the motor constants R and $\mathrm{K}_{\mathrm{T}}$ inherent in a given design, the form factor, and the output torque of the motor. Thus, the form factor has a strong influence on motor heating. In the case of a $\mathbf{k}=\mathbf{1}$, the heating effect of a linear amplifier from the previous section is:

$$
\begin{equation*}
P_{L}=R I_{a v}^{2} \tag{3.3.14}
\end{equation*}
$$

But for a form factor $\mathbf{k}=\mathbf{2}$, we have an armature power loss of four times the


Fig. 3.3.18. Relationship of additional armature losses in a servomotor with form factor.
amount shown above. Thus, we can construct a graphical solution of the increase in armature loss due to form factor. This is shown in Fig. 3.3.18.

The form factor greatly influences armature losses, as for example a $\mathbf{k}=\mathbf{1 . 2}$ will increase losses by $44 \%$.

Since servomotor performance in speed control systems is often limited mainly by power dissipation, it is important to investigate the form factor of a given amplifier source. In the case of a PWM amplifier the form factor will depend on the pulse frequency, on the electrical time constant of the motor and on any associated series inductances.

The net result of using a PWM circuit is usually that the power dissipation in the amplifier is vastly decreased, and the total
system dissipation is improved; but in some cases the motor power dissipation may be higher than if a linear (class A) amplifier is used. One should also check the application to see if the audio noise (if any) will be of consequence. This may be particularly irritating if the pulse frequency is shifting greatly with load variations. Another factor to consider in using a PWM system is the electrical noise generation, which can be transmitted into low level circuitry if care is not taken to properly shield and ground the high current portion of the system.

## SCR CONTROLS

SCR speed control systems are nearly always based on power line frequency (50-60 Hz ). This limits the control bandwidth of the system from $2-3 \mathrm{~Hz}$ for a half-wave, single-phase system to about $25-30 \mathrm{~Hz}$ for a three-phase, full-wave system. This translates into a speed control range of $5: 1$ for a half-wave control to $20: 1$ for a full-wave control with tachometer feedback.

While the SCR control system is not capable of controlling as wide a speed range as is the linear transistor amplifier, it has a high power conversion efficiency due to its switching mode and the low forward conduction voltage drop. The relative simplicity of its circuitry further adds to the positive features of the SCR control. The popularity of the SCR control for limited speed range DC motor control in the fractional and integral horsepower sizes has been largely due to these features, and to the fact that SCR's can operate directly
from the power line without AC-to-DC power conversion.

One serious problem of operating motors from SCR controls is the high form factor, which may cause serious derating of the motor. The form factor depends on the type of control used (half-wave, full-wave, three-phase), on the conduction angle and on motor inductance; therefore, it is difficult to obtain form factor values without a detailed knowledge of the application.

In the case of high-performance servomotors with low inductance, an appropriate determination of the form factor can be made on the assumption that the load is resistive. The graph in Fig. 3.3.19 shows the form factor versus conduction angle for resistive loads. The power dissipation in a motor is


Fig. 3.3.19. Form factor vs. conduction angle for resistive loads in SCR control circuits.
the highest at very low speeds and high torque conditions.

The addition of an inductance in the motor circuit will minimize the RMS current, and therefore make the form factor smaller. However, this may have the effect of decreasing system bandwidth which could affect the speed range or system stability.

Another important problem in SCR speed control systems is the so-called transportation lag in the system, caused by the discrete firing angle conduction. In attempting to control fractional horsepower motors over a wide speed range with SCR controls, the result is often momentary instability, especially at low speeds. In such situations, the system is operating at very small "firing angles" (Fig. 3.3.20) and if a sudden torque disturbance occurs, the resulting change in speed causes the error signal to switch on the output stage, sometimes at an "early" firing angle, as shown in the diagram.

The result will then be a heavy surge of
power for the remaining part of the half cycle (since the SCR cannot turn off until forward current goes to zero), even if the error signal in the meantime has returned to normal. The motor now has a large overspeed error, and will coast down to the predetermined controlled speed again, in which case the former firing pattern will resume. This behavior will be observed as a jerky, cogging action.

The most obvious remedy for this type of system behavior is to make the system transient response slower, using lag networks to overcome the effects of instability. This tends to make the system more sluggish in its response to sudden changes in load and speed command changes. This is not usually important in some applications with steady loads or loads with large moments of inertia, but for wide range, high-performance controls it is not desirable.

A properly compensated SCR control in a suitable application can be a fine speed con-


Fig. 3.3.20. Result of a sudden torque disturbance in a SCR speed control system.
trol system. Each unique application has an equally unique solution where a host of qualities have to be considered, such as per-
formance, price, versatility and utility. The selection of the "best" control is, indeed, a complex responsibility.

## Chapter 4

## Speed Controls and Servo Systems

A servo system in its most elementary form consists of an amplifier, actuator and feedback element. Paramount to the system function is stability, and a fair amount of a system designer's time is spent analyzing system parameters and arranging for conditions of servo stability and proper response time.

We will in this chapter analyze system fundamental equations, establish "block diagram" concepts and formulate transfer functions. We will cover Laplace transformation and use it to develop the input-output relationships (transfer functions) for a motor, amplifier and feedback elements. Also, in our discussion, we will cover system stability and describe the root locus method for determining stability.

The closed-loop servo system mentioned above is an example of just one of the many various types of control systems. We shall discuss the most common types, namely: velocity control, position control, torque control, and hybrid control systems. We will then describe the feedback components, and present their transfer functions. Various types of servo amplifiers will also be covered.

Besides describing the most common types of controls, we will also cover system char-
acteristics, such as system step response and system bandwidth.

We will also discuss phase-locked servo systems, "how to make systems work", and how to optimize your system (velocity profile, coupling, and capstan size).

### 4.1. SERVO THEORY

LAPLACE TRANSFORMATION

In engineering analysis and design, a linear system is, first of all, described mathematically, then the system is studied to determine response to a variety of input excitation signals. The differential or integrodifferential equation or equations which mathematically describe the system must be manipulated to obtain a desired equation characterizing the input-output relationships of the system.

Manipulation of these equations which contain exponential, transcendental or nonsinusoidal functions can be simplified when they are "transformed" from differential form to an easier-to-handle algebraic form by applying the Laplace transformation. The algebraic mathematics are equated into input-output relationships, then the result is transformed back into the original differential or integro-differential form by applying an inverse Laplace transformation.

In the first two sections, the Laplace transformation will be explained.

The defining equations and symbols of Laplace transformation are:

If
$f(t)$ is any function of time such that $f(t)$ $=0$ for $\mathrm{t}<0$ and the integral (4.1.2) has a finite value, then
$f(s)$ is the Laplace transform of $f(t)$.
The Laplace operator $s$ is defined as the complex variable:

$$
\begin{equation*}
\mathbf{s}=\sigma+\mathrm{j} \omega \tag{4.1.1}
\end{equation*}
$$

and the Laplace transformation is defined as:

$$
\begin{equation*}
f(s)=\int_{0}^{\infty} f(t) e^{-s t} d t \tag{4.1.2}
\end{equation*}
$$

Thus, to find the Laplace transform of a function $f(t)$, the function $f(t)$ is multiplied by $\mathrm{e}^{-\mathrm{st}}$, then integrated from $\mathrm{t}=\mathbf{0}$ to $\mathrm{t}=\infty$.

## Examples:

1. Let $f(t)=A$ for $t>0$ (a constant); then the Laplace transform will be:

$$
\begin{align*}
f(s) & =\int_{0}^{\infty} A e^{-s t} d t=\left.A \frac{-1}{s} e^{-s t}\right|_{0} ^{\infty} \\
& =-\frac{A}{s}\left(e^{-\infty}-e^{0}\right) \tag{4.1.3}
\end{align*}
$$

$$
\begin{equation*}
f(s)=\frac{A}{s} \tag{4.1.4}
\end{equation*}
$$

2. Let $f(t)=A e^{-a t}$. Then:

$$
f(s)=\int_{0}^{\infty} A e^{-a t} e^{-s t} d t
$$

$$
\begin{equation*}
=A \int_{0}^{\infty} e^{-a t} e^{-s t} d t \tag{4.1.5}
\end{equation*}
$$

$$
\begin{equation*}
f(s)=\frac{A}{a+s} \tag{4.1.6}
\end{equation*}
$$

## INVERSE LAPLACE TRANSFORMATION

Once an equation has been transformed, the solution for the unknown variable may be determined through algebraic manipulations. The solution expressed in terms of the complex variable, s, may be sufficient in some cases; if not, the solution as a function of time can be obtained by either taking the inverse Laplace transformation

$$
\begin{equation*}
f(t)=\frac{1}{2 \pi j} \int_{\sigma-j \infty}^{\sigma+j \infty} f(s) e^{s t} d s \tag{4.1.7}
\end{equation*}
$$

or by using standard tables such as Tab. 4.1.1.

As an example, assume that through algebraic manipulation, the Laplace transform is:

$$
\begin{equation*}
f(s)=\frac{2}{s(s+1)} \tag{4.1.8}
\end{equation*}
$$

| f(s) | $f(t)$ |
| :---: | :---: |
| $\begin{gathered} \frac{1}{s} \\ \frac{1}{s^{n}} \\ \frac{1}{s-a} \\ \frac{1}{(s-a)^{n}} \\ \frac{1}{s^{2}-a^{2}} \\ \frac{s}{s^{2}-a^{2}} \\ \frac{1}{(s-a)(s-b)}(a \neq b) \\ \frac{s}{(s-a)(s-b)} \quad(a \neq b) \\ \frac{1}{s^{2}+a^{2}} \\ \frac{s}{s^{2}+a^{2}} \\ \frac{1}{(s-a)^{2}+b^{2}} \\ \frac{s-a}{(s-a)^{2}+b^{2}} \\ \frac{s}{\left(s^{2}+a^{2}\right)^{2}} \\ \frac{s^{2}-a^{2}}{\left(s^{2}+a^{2}\right)^{2}} \\ \frac{1}{\left(s^{2}+a^{2}\right)^{2}} \end{gathered}$ | $\begin{gathered} \frac{1}{(n-1)!} t^{n-1} \quad(n=1,2,3 \ldots) \\ e^{a t} \\ \frac{1}{(n-1)!} t^{n-1} e^{a t \quad(n=1,2,3, \ldots)} \\ \frac{1}{a} \sinh \text { at } \\ \cosh \text { at } \\ \frac{1}{a-b}\left[e^{a t}-e^{b t}\right] \\ \frac{1}{a-b}\left[e^{a t}-b e^{b t}\right] \\ \frac{1}{a} \sin a t \\ \cos \text { at } \\ \frac{1}{b} e^{a t} \sin b t \\ e^{a t} \cos b t \\ \frac{1}{2 a} t \sin a t \\ t \cos a t \\ \left.\sin ^{3} a t-a t \cos a t\right] \end{gathered}$ |

Tab. 4.1.1. Laplace transforms
To determine the inverse transform, $f(t)$, the function $f(s)$ will be factored into:

$$
\begin{equation*}
f(s)=\frac{2}{s}-\frac{2}{s+1} \tag{4.1.9}
\end{equation*}
$$

From this section, or the table, we know that the inverse transform of $\frac{2}{s}$ is a constant, 2 , and the inverse transform of $2 /(s+1)$ is $2 \mathrm{e}^{-\mathrm{t}}$. Therefore the inverse transformation of the above $f(s)$ gives

$$
\begin{equation*}
f(t)=2-2 e^{-t} \tag{4.1.10}
\end{equation*}
$$

## TRANSFER FUNCTIONS

In control theory, the relationship between input driving signal and output response is expressed by means of a transfer function. The transfer function is defined as a ratio of the Laplace transforms of the output and input signals, with the assumption that all initial conditions are zero.

As an example, we will examine a circuit in Fig. 4.1.1., write the network equations and, utilizing the Laplace transformation, manipulate the equations so as to obtain a ratio of the output and input transforms.


Fig. 4.1.1. RC integratirig circuit.

In writing the network equations we obtain:

$$
\begin{align*}
v_{i} & =i R+\frac{1}{C} \int i d t \\
& =R \frac{d q}{d t}+\frac{q}{C}  \tag{4.1.11}\\
v_{0} & =\frac{q}{C} \tag{4.1.12}
\end{align*}
$$

where $\mathbf{q}=\int$ idt. These transform to:

$$
\begin{align*}
& v_{i}(s)=\left(s R+\frac{1}{C}\right) q(s)  \tag{4.1.13}\\
& v_{0}(s)=\frac{1}{C} q(s) \tag{4.1.14}
\end{align*}
$$

Dividing, we arrive at a ratio of input to output transforms:

$$
\begin{equation*}
\frac{v_{0}(s)}{v_{i}(s)}=\frac{\frac{1}{C} q(s)}{\left(s R+\frac{1}{C}\right) q(s)}=\frac{1}{s R C+1} \tag{4.1.15}
\end{equation*}
$$

Since this expression is the ratio of the Laplace transforms of output and input voltages, it represents the transfer function of the RC network of Fig. 4.1.1.

## BLOCK DIAGRAMS

To show the relationship between the various components and the flow of signals between them, a diagram is drawn in block form. Each block describes a component by means of its transfer function and is associated with arrows to illustrate direction of signal flow. Such a block is shown in Fig. 4.1.2.


Fig. 4.1.2. A block diagram showing input signal, block transfer function, and output signal.

The arrow indicates that $V_{1}(s)$ is the input, $\mathrm{V}_{2}(\mathrm{~s})$ is the output, and $\mathrm{G}(\mathrm{s})$ is the transfer function [the ratio of $V_{2}(s)$ to $V_{1}(s)$ ].

Several signals are joined by summing points such as shown in Fig. 4.1.3. A summing point has numerous inputs, but only one output which equals the sum of the inputs.


Fig. 4.1.3. A summing point may have several inputs. The output is the sum of the input signals.

The output of the summing point in Fig. 4.1.3 is:

$$
\begin{equation*}
E(s)=A(s)-B(s)+C(s) \tag{4.1.16}
\end{equation*}
$$

A block diagram representation of a closedloop system, where the output signal is fed back and compared to the input, is illustrated in Fig. 4.1.4.


Fig. 4.1.4. Block diagram of a closed-loop system.

An input signal $\mathrm{R}(\mathrm{s})$ is compared to feedback signal $B(s)$ at the summing point, the difference $E(s)$ is input to the block $G(s)$, and an output $\mathbf{C}(s)$ is obtained. The relation between the variables are given by the equations:

$$
\begin{align*}
C(s) & =G(s) E(s) \\
B(S) & =H(s) C(s)  \tag{4.1.17}\\
E(s) & =R(s)-B(s) \\
& =R(s)-H(s) C(s)
\end{align*}
$$

By eliminating $E(s)$ from the equations, we obtain:

$$
\mathrm{C}(\mathrm{~s})=\mathrm{G}(\mathrm{~s}) \mathrm{R}(\mathrm{~s})-\mathrm{H}(\mathrm{~s}) \mathrm{G}(\mathrm{~s}) \mathrm{C}(\mathrm{~s})(4.1 .18)
$$

The closed-loop transfer function, or the relationship between output and input becomes:

$$
\begin{equation*}
f(s)=\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s) H(s)} \tag{4.1.19}
\end{equation*}
$$

## TRANSFER FUNCTION OF A DC MOTOR

Now that we have reviewed the basic block diagram, what a transfer function is, and how to derive it, we will derive the transfer
function of one specific block used in the closed-loop system - the block representing the motor.

A simplified model of a DC motor with permanent magnet field can be derived by assuming armature inductance to be zero and ignoring the resonance effect. A complex model of a DC motor was presented in section 2.3. and will be further discussed in section 4.2.1. With these stipulations, the DC motor equations are:

$$
\begin{equation*}
V=I_{a} R+K_{E} \omega \tag{4.1.20}
\end{equation*}
$$

[ $\mathrm{V} ; \mathrm{A}, \Omega, \mathrm{V} / \mathrm{rad} \mathrm{s}^{-1}, \mathrm{rad} / \mathrm{s}$ ]

$$
T_{g}=K_{T} I_{a}
$$

[ $\mathrm{Nm} ; \mathrm{Nm} / \mathrm{A}, \mathrm{A}$ ]

$$
\begin{equation*}
\mathrm{T}_{\mathrm{g}}=\mathrm{J} a \tag{4.1.22}
\end{equation*}
$$

$\left[\mathrm{Nm} ; \mathrm{kg} \mathrm{m}{ }^{2}, \mathrm{rad} / \mathrm{s}^{2}\right]$

If we make a small closed-loop system, and assign each block a specific symbol, the block diagram would appear as shown in Fig. 4.1.5.


Fig. 4.1.5. Block diagram of a DC motor as the closed-loop system with each block representing a specific motor parameter.

Combining blocks to simplify the diagram, we arrive at the form shown in Fig. 4.1.6, from which the motor transfer function relating velocity $\omega$ to the input voltage $\mathbf{V}$ can be written:

$$
\begin{align*}
G_{m}(s) & =\frac{\omega(s)}{V(s)}=\frac{\frac{K_{T}}{R J s}}{1+\frac{K_{T} K_{E}}{R J s}} \\
& =\frac{1 / K_{E}}{1+s \frac{R J}{K_{T} K_{E}}} \tag{4.1.23}
\end{align*}
$$

or:

$$
\begin{equation*}
G_{m}(s)=\frac{1 / K_{E}}{1+s \tau_{m}} \tag{4.1.24}
\end{equation*}
$$

where $\tau_{m}$ is the mechanical time constant.


Fig. 4.1.6. By combining the blocks of Fig. 4.1.5, we will arrive at this simplified block diagram for a DC motor transfer function.

This transfer function contains terms that describe motor response to cyclic or variable frequency signals. We can plot the frequency response characteristic of the motor by setting s equal to $\mathrm{j} \omega$. Then:

$$
\begin{equation*}
s=j \omega=j 2 \pi f \tag{4.1.25}
\end{equation*}
$$

The transfer function is then evaluated for an entire range of frequencies and the results can be plotted. It is customary to plot the transfer function magnitude in decibels vs. frequency plotted in a logarithmic scale, and is termed the gain-frequency characteristic of the motor. Such a plot is called the Bode plot.

At a frequency where $2 \pi f$ is equal to $1 / \tau_{m}$ the slope of the curve shows a sharp change as can be seen in Fig. 4.1.7.


Fig. 4.1.7. Bode plot of a simplified motor model which ignores motor inductance.


Fig. 4.1.8. Phase angle-frequency characteristic of a DC motor.

This frequency, which can be evaluated by the ratio $K_{E} K_{T} / R \mathrm{~J}$, is called the break frequency of the motor:

$$
\begin{aligned}
f_{b}= & \frac{1}{2 \pi \tau_{m}}=\frac{K_{E} K_{T}}{2 \pi J R} \\
& {\left[\mathrm{~Hz} ; \mathrm{V} / \mathrm{rad} \mathrm{~s}^{-1}, \mathrm{Nm} / \mathrm{A}, \mathrm{~kg} \mathrm{~m}^{2}, \Omega\right] }
\end{aligned}
$$

The phase angle-frequency characteristic of the motor can also be plotted as illustrated in Fig. 4.1.8.

The phase angle is determined by the denominator of the transfer function, so that:

$$
\begin{equation*}
\varphi=-\tan ^{-1}\left(\frac{\mathrm{RJ}}{\mathrm{~K}_{\mathrm{E}} \mathrm{~K}_{\mathrm{T}}}\right) \tag{4.1.27}
\end{equation*}
$$

$\left[\mathrm{rad} ; \Omega, \mathrm{kg} \mathrm{m}^{2}, \mathrm{~V} / \mathrm{rad} \mathrm{s}^{-1}, \mathrm{Nm} / \mathrm{A}\right]$

EXAMPLE: Two motors are under consideration for an application where they are required to respond to control signals which recur at rates as high as 125 times per second. Without regard to the torque and heat dissipation requirements, which motor would be the first choice if the following data is available:

## Motor " $A$ "

$$
\begin{aligned}
\mathrm{K}_{\mathrm{T}} & =9.5 \mathrm{oz}-\mathrm{in} / \mathrm{A}=6.7 \times 10^{-2} \mathrm{Nm} / \mathrm{A} \\
\mathrm{~K}_{\mathrm{E}} & =7 \mathrm{~V} / \mathrm{krpm}=6.7 \times 10^{-2} \mathrm{~V} / \mathrm{rad} \mathrm{~s}^{-1} \\
\mathrm{R} & =0.5 \Omega \\
\mathrm{~J} & =0.68 \times 10^{-3} \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}} \\
& =4.8 \times 10^{-6} \mathrm{~kg} \mathrm{~m}^{2}
\end{aligned}
$$

Motor "B"

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{T}}=12 \mathrm{oz}-\mathrm{in} / \mathrm{A}=8.48 \times 10^{-2} \mathrm{Nm} / \mathrm{A} \\
& \mathrm{~K}_{\mathrm{E}}=8.9 \mathrm{~V} / \mathrm{krpm}=8.48 \times 10^{-2} \mathrm{~V} / \mathrm{rad} \mathrm{~s}^{-1}
\end{aligned}
$$

$$
\mathrm{R}=0.75 \Omega
$$

$$
\mathrm{J}=2.4 \times 10^{-3} \mathrm{oz}-\mathrm{in}^{-\mathrm{s}^{2}}
$$

$$
=1.695 \times 10^{-5} \mathrm{~kg} \mathrm{~m}^{2}
$$

The application requires the motor to accelerate the load with moment of inertia of $0.3 \times 10^{-3} \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}=2.12 \times 10^{-6} \mathrm{~kg} \mathrm{~m}^{2}$. Calculating the break frequency for motor " A ", we obtain:

$$
\begin{aligned}
f_{b} & =\frac{K_{E} K_{T}}{2 \pi J R} \\
& =\frac{6.7^{2} \times 10^{-4}}{2 \pi(4.8+2.12) \times 10^{-6} \times 0.5} \\
f_{b} & =206.5 \mathrm{~Hz}
\end{aligned}
$$

Then calculate break frequency for motor "B":

$$
\begin{aligned}
& f_{b}=\frac{8.48^{2} \times 10^{-4}}{2 \pi(16.95+2.12) \times 10^{-6} \times 0.75} \\
& f_{b}=80 \mathrm{~Hz}
\end{aligned}
$$

It can be seen that insofar as dynamic characteristics are concerned, motor " A " provides significant advantage in its higher break frequency which would permit easier
compensation of the servo system to achieve the desired bandwidth.

## TRANSFER FUNCTION OF

AN AMPLIFIER

An ideal DC amplifier would have a constant gain for all input frequencies. However, practical amplifiers are limited to a certain frequency range, and the corresponding transfer function in most cases is:

$$
\begin{equation*}
G_{a}(s)=\frac{V_{0}(s)}{V_{i}(s)}=\frac{A}{1+s \tau_{a}} \tag{4.1.28}
\end{equation*}
$$

where $\tau_{\mathrm{a}}$ is the time constant of the amplifier and $A$ is the $D C$ gain.

## STABILITY

We have just presented the transfer function for two blocks in the servo system, the motor and the amplifier. We shall now discuss the three possible types of system response to a step input.

Let a system be described by the transfer function:

$$
\begin{equation*}
\mathrm{G}(\mathrm{~s})=\frac{\mathrm{N}(\mathrm{~s})}{\mathrm{D}(\mathrm{~s})} \tag{4.1.29}
\end{equation*}
$$

Roots of $\mathrm{N}(\mathrm{s})=\mathbf{0}$ are zeros of the system, and roots of $D(s)=0$ are poles of the system. Poles play a major role in the system's stability, and in order to analyze stability, these poles must be determined.

If a pole is given by:

$$
\begin{equation*}
\mathrm{p}=\sigma+\mathrm{j} \omega \tag{4.1.30}
\end{equation*}
$$

then three different types of responses may be possible. These are (if all poles meet the following equations):

$$
\begin{equation*}
\text { Case 1: } \sigma<0 \text { and }|\omega|<|\sigma| \tag{4.1.31}
\end{equation*}
$$

Case 2: $\quad \sigma \leqslant 0$ and $|\omega|>|\sigma|$
Case 3: $\quad \sigma>0$

Case 1: the system is overdamped and response to a step input will be as illustrated in Fig. 4.1.9.


Fig. 4.1.9. Overdamped system response to step input.


Fig. 4.1.10. Underdamped system response.
Case 2: the system is underdamped and a step input will cause overshoot in the response, as can be seen in Fig. 4.1.10.

Case 3: when one or more poles exhibit the property of $\sigma>0$, the system will be unstable and response will increase with time.

## ROOT LOCUS METHOD

There are several analytical and graphical methods to determine system stability, all
of which depend on the fact that if $\sigma>0$ the system will be unstable. We will describe the root locus method for determining servo system stability.

Consider the closed-loop system of Fig. 4.1.11, whose transfer function is:

$$
\begin{equation*}
f(s)=\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s) H(s)} \tag{4.1.34}
\end{equation*}
$$

The entire expression is termed the closedloop transfer function; $\mathbf{G}(\mathbf{s}) \mathbf{H}(\mathbf{s})$ is called the open-loop transfer function.


Fig. 4.1.11. Block diagram used for explanation of root locus method.

The objective is to determine poles of the closed-loop transfer function since they characterize the response of the system. Therefore, the equation to be solved is:

$$
\begin{equation*}
1+G(s) H(s)=0 \tag{4.1.35}
\end{equation*}
$$

which is called the characteristic equation.
The equation can be solved numerically to determine the poles. However, we want to find the dependence of the poles on some parameter such as the system gain. This is necessary since the parameter may not be known exactly, or because we may wish to
change the parameter and will want to observe how this will affect location of poles.

A way to accomplish this is by utilizing the root locus method, which basically gives the closed-loop poles in relation to the open-loop poles and zeros, and a parameter $\mathbf{K}$, which is the parameter of interest (usually gain). The method is described by the following set of rules, each of which is illustrated by an example:

## RULE \# 1:

In order to construct the root loci, obtain the characteristic equation and rearrange it to the format:

$$
\begin{equation*}
1+K \frac{\left(s-z_{1}\right)\left(s-z_{2}\right) \ldots\left(s-z_{m}\right)}{\left(s-p_{1}\right)\left(s-p_{2}\right) \ldots\left(s-p_{n}\right)}=0 \tag{4.1.36}
\end{equation*}
$$

where K is the parameter of interest (usually gain) which is assumed to be positive. The second term in the left side of (4.1.36) is the open-loop transfer function. Then locate the open-loop poles and zeros in the s-plane.

Example: consider a system with:

$$
\begin{align*}
& G(s)=\frac{K}{s(s+1)}  \tag{4.1.37}\\
& H(s)=\frac{(s+2)}{(s+3)(s+4)} \tag{4.1.38}
\end{align*}
$$

rearranging to the desired format:

$$
\begin{equation*}
1+K \frac{(s+2)}{s(s+1)(s+3)(s+4)}=0 \tag{4.1.39}
\end{equation*}
$$

The open loop poles and zero are located in the s-plane as shown in Fig. 4.1.12.


Fig. 4.1.12. Open-loop poles and zero plot of (4.1.39).

## RULE \# 2:

Find the starting and termination points of the root loci. Since in all real systems the number of open-loop poles is greater or equal to the number of zeros ( $\mathbf{n} \geqslant \mathrm{m}$ ), the root loci start for $\mathrm{K}=\mathbf{0}$ at the open-loop poles and terminate at either an open-loop zero or at infinity. There are n branches, m of which will terminate at a zero, and ( $\mathrm{n}-\mathrm{m}$ ) branches will terminate at infinity along asymptotes.

Example: for the system of the previous example, the number of open-loop poles is $\mathrm{n}=4$, and the number of open-loop zeros is $\mathrm{m}=1$. So that one branch will terminate at the zero, and three will terminate at infinity.

## RULE \#3:

Determine the root loci on the real axis. A point on the real axis lies on a root locus
if the total number of open-loop poles and zeros on the real axis to the right of the point is odd.

Example: for the given example, portions of the root loci on the real axis are marked by the heavy line illustrated in Fig. 4.1.13.


Fig. 4.1.13. Illustration of Rule \#3 in which portions of the root loci on the real axis are marked.

Notice that all the points left of -4 up to $-\infty$ are part of the root loci. Therefore, this branch of the root loci approaches infinity along the negative axis.

## RULE \#4:

Determine the asymptotes of the root loci. ( $\mathrm{n}-\mathrm{m}$ ) branches of the root loci terminate at infinity along asymptotes. The angles of the asymptotes equal:

$$
\begin{align*}
\beta[0]= & \frac{ \pm 180(2 N+1)}{n-m}  \tag{4.1.40}\\
& {[N=0,1,2, \ldots,(n-m-1)] }
\end{align*}
$$

All the asymptotes intersect the real axis at the point $\sigma$ which is given by:

$$
\begin{equation*}
\sigma=\frac{\left(\mathrm{p}_{1}+\mathrm{p}_{2}+\ldots+\mathrm{p}_{\mathrm{n}}\right)-\left(\mathrm{z}_{1}+\mathrm{z}_{2}+\ldots \mathrm{z}_{\mathrm{m}}\right)}{\mathrm{n}-\mathrm{m}} \tag{4.1.41}
\end{equation*}
$$

Example: for the given example we have $\mathbf{n}-\mathbf{m}=\mathbf{3}$. Therefore, the angles of asymptotes are

$$
\begin{equation*}
\beta=\frac{ \pm 180(2 \mathrm{~N}+1)}{3} \tag{4.1.42}
\end{equation*}
$$

$$
[\mathrm{N}=0,1,2]
$$

Thus, the angles will be $+180^{\circ},+60^{\circ}$, and $-60^{\circ}$, and they intersect at a point given by:

$$
\begin{equation*}
\sigma=\frac{(0-1-3-4)-(-2)}{3}=-2 \tag{4.1.43}
\end{equation*}
$$

The asymptotes are shown in Fig. 4.1.14.


Fig. 4.1.14. Illustration of Rule \#4 in which the angles of the asymptotes are plotted.

## RULE \#5:

Find the breakaway and break-in points. If the root locus lies between two adjacent poles on the real axis, there is at least one breakaway point. Similarly, if the root locus lies between two adjacent zeros on
the real axis, there is at least one break-in point. However, if the root locus is between a zero and a pole on the real axis, there may exist no breakaway or break-in points.

If the characteristic equation is given by:

$$
\begin{equation*}
1+\frac{K B(s)}{A(s)}=0 \tag{4.1.44}
\end{equation*}
$$

then the location of breakaway or break-in points is given by:

$$
\begin{equation*}
A^{\prime}(s) B(s)-A(s) B^{\prime}(s)=0 \tag{4.1.45}
\end{equation*}
$$

where the prime indicates differentiation with respect to $s$. The value of K at those points equals the product of distances to all poles divided by distances to all zeros.

Example: in the given example, the charteristic equation is:

$$
\begin{equation*}
1+K \frac{.(s+2)}{s(s+1)(s+3)(s+4)}=0 \tag{4.1.39}
\end{equation*}
$$

Thus:

$$
\begin{equation*}
B(s)=s+2 \tag{4.1.46}
\end{equation*}
$$

$$
\begin{align*}
A(s) & =s(s+1)(s+2)(s+3) \\
& =s^{4}+6 s^{3}+11 s^{2}+6 s \tag{4.1.47}
\end{align*}
$$

Then, differentiating with respect to s :

$$
\begin{align*}
& B^{\prime}(s)=1  \tag{4.1.48}\\
& A^{\prime}(s)=4 s^{3}+18 s^{2}+22 s+6 \tag{4.1.49}
\end{align*}
$$

The breakaway point can be found from (4.1.45):

$$
\begin{align*}
& \left(4 s^{3}+18 s^{2}+22 s+6\right)(s+2)- \\
& -\left(s^{4}+6 s^{3}+11 s^{2}+6 s\right)=0 \\
& 3 s^{4}+20 s^{3}+47 s^{2}+44 s+12=0 \tag{4.1.50}
\end{align*}
$$

Eq. (4.1.50) will have to be solved numerically, and for this case it is found that the solution is:

$$
\begin{equation*}
\mathrm{s} \approx-0.5 \tag{4.1.51}
\end{equation*}
$$

RULE \#6:

Find the points where the root loci cross the imaginary axis. The points where the root loci intersect the imaginary axis can be found by substituting $s=j \omega$ in the characteristic equation. Equating both the real and imaginary part to zero allow the solution for $K$ and $\omega$. The value of $K$ at that point is important since it determines the value of the parameter which will cause the system to become unstable.

Example: using the characteristic equation from the above example, we set $\mathrm{s}=\mathrm{j} \omega$ and obtain:

$$
\begin{equation*}
1+K \frac{(j \omega+2)}{j \omega(j \omega+1)(j \omega+3)(j \omega+4)}=0 \tag{4.1.52}
\end{equation*}
$$

$$
\begin{equation*}
j \omega(j \omega+1)(j \omega+3)(j \omega+4)+K(j \omega+2)=0 \tag{4.1.53}
\end{equation*}
$$

This becomes:

$$
\begin{equation*}
\omega^{4}-8 j \omega^{3}-19 \omega^{2}+12 j \omega+K j \omega+2 K=0 \tag{4.1.54}
\end{equation*}
$$

Separating this equation into real and imaginary parts:

$$
\begin{align*}
& \text { Real: } \omega^{4}-19 \omega^{2}+2 K=0  \tag{4.1.55}\\
& \text { Imaginary: }-8 \omega^{3}+12 \omega+K \omega=0 \tag{4.1.56}
\end{align*}
$$

Using (4.1.56), we obtain:

$$
\begin{equation*}
K=8 \omega^{2}-12 \tag{4.1.57}
\end{equation*}
$$

This is substituted into (4.1.55):

$$
\begin{align*}
& \omega^{4}-19 \omega^{2}+16 \omega^{2}-24=0 \\
& \omega^{4}-3 \omega^{2}-24=0 \tag{4.1.58}
\end{align*}
$$

If we substitute $\mathrm{X}=\omega^{\mathbf{2}}$ the equation becomes:

$$
\begin{equation*}
x^{2}-3 x-24=0 \tag{4.1.59}
\end{equation*}
$$

which has a solution of:

$$
\begin{aligned}
& x_{1,2}=\frac{3 \pm \sqrt{9+96}}{2} \\
& x_{1}=6.62 \\
& x_{2}=-3.62
\end{aligned}
$$

Since only positive values are allowed for $X$, we obtain $X=6.62$; so that $\omega^{2}=6.62$, and the root locus crosses the imaginary
axis at $\omega=$ 2.57. The value of the gain, $\mathbf{K}$, at that point is:

$$
\begin{equation*}
K=8 \omega^{2}-12=41 \tag{4.1.60}
\end{equation*}
$$

The root loci for this example are shown in Fig. 4.1.15.


Fig. 4.1.15. Example of locating the point where root locus crosses the imaginary axis.

This concludes the presentation of the six rules for constructing of the root locus. To further illustrate construction of root loci, another problem is presented. The openloop transfer function will be $\mathbf{G}(\mathrm{s}) \mathrm{H}(\mathrm{s})$, where

$$
\begin{align*}
& G(s)=\frac{K}{s}  \tag{4.1.61}\\
& H(s)=\frac{s+2}{s+1} \tag{4.1.62}
\end{align*}
$$

Step 1: The characteristic equation is arranged to the prescribed format:

$$
\begin{equation*}
1+K \frac{s+2}{s(s+1)}=0 \tag{4.1.63}
\end{equation*}
$$

and plotted in Fig. 4.1.16.


Fig. 4.1.16. An example of locating and plotting poles and zeros. This is the plot of Eq. (4.1.63).

Step 2: The starting points are $\mathbf{0}$ and $\mathbf{- 1}$; termination points are $\mathbf{- 2}$ and since $\mathbf{n}-\mathbf{m}=\mathbf{1}$, another termination point is at infinity.

Step 3: The root loci on the real axis are between 0 and $\mathbf{- 1}$ and to the left of $\mathbf{- 2}$.

Step 4: There is one asymptote (since $\mathrm{n}-\mathrm{m}=1$ ) and the angle of that asymptote is:

$$
\begin{gather*}
\beta= \pm \frac{180(2 N+1)}{n-m}= \pm 180^{\circ}  \tag{4.1.64}\\
(N=0)
\end{gather*}
$$

Step 5: To determine the breakaway and break-in points (note that there must be a breakaway point between $\mathbf{0}$ and $\mathbf{- 1}$ and a break-in point to the left of -2 ) solve the equation:

$$
\begin{equation*}
A^{\prime} B-A B^{\prime}=0 \tag{4.1.45}
\end{equation*}
$$

where:

$$
\begin{equation*}
A(s)=s(s+1)=s^{2}+s \tag{4.1.65}
\end{equation*}
$$

$$
\begin{equation*}
B(s)=s+2 \tag{4.1.66}
\end{equation*}
$$

Then, differentiating:

$$
\begin{align*}
& A^{\prime}=2 s+1  \tag{4.1.67}\\
& B^{\prime}=1 \tag{4.1.68}
\end{align*}
$$

and substituting (4.1.65) thru (4.1.68) into (4.1.45) we obtain:

$$
\begin{align*}
& (2 s+1)(s+2)-\left(s^{2}+s\right)=0 \\
& s^{2}+4 s+2=0 \tag{4.1.69}
\end{align*}
$$

Then:

$$
\begin{aligned}
& s_{1}=-2+1.414=-0.586 \\
& s_{2}=-2-1.414=-3.414
\end{aligned}
$$

Thus, a breakaway point is at $\mathbf{s}=-0.586$ and a break-in point is at $s=-3.414$. The gain at the breakaway point equals:

$$
\begin{equation*}
K_{1}=\frac{0.586 \times 0.414}{1.414}=0.17 \tag{4.1.70}
\end{equation*}
$$

and at the break-in point:

$$
\begin{equation*}
K_{2}=\frac{3.414 \times 2.414}{1.414}=5.8 \tag{4.1.71}
\end{equation*}
$$

Step 6: To see whether the root loci cross the imaginary axis substitute $\mathrm{s}=\mathrm{j} \omega$ in the characteristic equation:

$$
\begin{equation*}
1+K \frac{\mathrm{j} \omega+2}{\mathrm{j} \omega(\mathrm{j} \omega+1)}=0 \tag{4.1.72}
\end{equation*}
$$

Then:

$$
\begin{aligned}
& \mathrm{j} \omega(\mathrm{j} \omega+1)+\mathrm{K}(\mathrm{j} \omega+2)=0 \\
& -\omega^{2}+\mathrm{j} \omega(1+\mathrm{K})+2 \mathrm{~K}=0
\end{aligned}
$$

equating real and imaginary parts:

$$
\begin{align*}
& \omega^{2}+2 K=0  \tag{4.1.73}\\
& \omega(1+K)=0 \tag{4.1.74}
\end{align*}
$$

The only point that satisfies these equations is when $\omega=0$ and $\mathrm{K}=0$, which indicates the starting point. Thus, the root loci do not cross the imaginary axis. The construction of the root loci is shown in Fig. 4.1.17.


Fig. 4.1.17. Construction of root loci for Step 6 of the example.

### 4.2. SERVO COMPONENTS

After reviewing servo theory, we will describe the common servo components by their transfer functions and block diagrams. This is necessary before we attempt to analyze the system as a whole.

### 4.2.1. DC MOTOR

A detailed model for a DC permanent magnet motor was derived in Chapter 2. The motor with the load and the tachometer was approximated as three bodies, coupled by inertialess shafts, and the torsional resonance was considered.

The model shows the relations between the following variables:

$$
\begin{aligned}
& \mathrm{V}(\mathrm{~s})=\text { motor voltage } \\
& \mathrm{I}_{\mathrm{a}}(\mathrm{~s})=\text { armature current } \\
& \theta_{\mathrm{m}}(\mathrm{~s})=\text { angular position of armature } \\
& \theta_{1}(\mathrm{~s})=\text { angular position of load } \\
& \theta_{2}(\mathrm{~s})=\text { angular position of tachometer }
\end{aligned}
$$

The relation among the variables was expressed (2.3.44) in a matrix form:

$$
\left[\begin{array}{c:c:c:c}
s L_{2}+R & s K_{E} & 0 & 0  \tag{4.2.1}\\
\hdashline K_{T} & s^{2} J_{\mathrm{J}}+\mathrm{sD} & s^{2} J_{1} & s^{2} J_{2} \\
0 & \mathrm{sD}_{1}+\mathrm{K}_{1} & -\left(s^{2} J_{1}+s D_{1}+K_{1}\right) & 0 \\
0 & \mathrm{sD}_{2}+K_{2} & 0 & -\left(s^{2} J_{2}+s \mathrm{SD}_{2}+\mathrm{K}_{2}\right)
\end{array}\right]\left[\begin{array}{c}
T_{\mathrm{a}}(\mathrm{~s}) \\
0_{\mathrm{m}}(\mathrm{~s}) \\
\theta_{1}(s) \\
0_{2}(s)
\end{array}\right]=\left[\begin{array}{c}
\mathrm{V}(s) \\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

The above model may be too general and unnecessarily complicated in many cases. When the structural resonance is of no concern, one may assume

$$
\begin{equation*}
\theta_{m}(s)=\theta_{1}(s)=\theta_{2}(s) \tag{4.2.2}
\end{equation*}
$$

and the motor equations may be simplified:

$$
\left[\begin{array}{cc}
s L_{a}+R & s K_{E}  \tag{4.2.3}\\
-K_{T} & s^{2} J+s D
\end{array}\right]\left[\begin{array}{l}
\mathrm{I}_{\mathrm{a}}(\mathrm{~s}) \\
\theta_{\mathrm{m}}(\mathrm{~s})
\end{array}\right]=\left[\begin{array}{l}
\mathrm{V}(\mathrm{~s}) \\
0
\end{array}\right]
$$

where J is the total moment of inertia:

$$
\begin{equation*}
J=J_{m}+J_{1}+J_{2} \tag{4.2.4}
\end{equation*}
$$

Since, in many cases, the motor velocity, $\omega(\mathrm{s})$, is of interest, the motor equations may be written in terms of the voltage, current, and velocity.

$$
\left[\begin{array}{cc}
\mathrm{sL}_{\mathrm{a}}+R & K_{E}  \tag{4.2.5}\\
-K_{T} & s J+D
\end{array}\right]\left[\begin{array}{l}
\mathrm{I}_{\mathrm{a}}(\mathrm{~s}) \\
\omega(\mathrm{s})
\end{array}\right]=\left[\begin{array}{l}
\mathrm{V}(\mathrm{~s}) \\
0
\end{array}\right]
$$

The transfer function between the input voltage and the velocity is then

$$
\begin{equation*}
G_{m}(s)=\frac{\omega(s)}{V(s)}=\frac{K_{T}}{(s J+D)\left(s L_{a}+R\right)+K_{E} K_{T}} \tag{4.2.6}
\end{equation*}
$$

The above models of (4.2.1), (4.2.5), and (4.2.6) assumed that the motor is driven by a voltage source. In case the amplifier is a current source, the model of (4.2.5) will be reduced to the form

$$
\begin{equation*}
(\mathrm{s} J+\mathrm{D}) \omega(\mathrm{s})=\mathrm{K}_{\mathrm{T}} \mathrm{l}_{\mathrm{a}}(\mathrm{~s}) \tag{4.2.7}
\end{equation*}
$$

and the motor transfer function is

$$
\begin{equation*}
G_{m}^{\prime}(s)=\frac{\omega(s)}{I_{a}(s)}=\frac{K_{T}}{s J+D} \tag{4.2.8}
\end{equation*}
$$

Since, in most cases, the amplifiers are voltage sources and the resonance is not significant, (4.2.6) is the most common description of a DC motor. This equation may be further simplified when some of the parameters are small and can be ignored, e.g., $\mathrm{L}_{\mathbf{a}} \approx \mathbf{0}$ for moving coils motors, or $\mathrm{D} \approx \mathbf{0}$ for some motors.

### 4.2.2. AMPLIFIER

Servo amplifiers may be divided into two main groups, according to the type of feedback they use: voltage amplifiers and current amplifiers. The two types will be studied separately, and the transfer function model for each one will be given. Further details on the amplifier design can be found later in this chapter.

## Voltage Amplifier

Ideally, an amplifier should have constant gain for all frequencies. However, this is not the case in practice, and all amplifiers have limited bandwidth. If the amplifier
transfer function has a single pole, the transfer function between the input command, $\mathrm{V}_{\text {in }}(\mathrm{s})$, and the output, $\mathrm{V}_{\text {out }}(\mathrm{s})$, is

$$
\begin{equation*}
\frac{V_{\text {out }}(s)}{V_{\text {in }}(s)}=\frac{A}{s \tau_{a}+1} \tag{4.2.9}
\end{equation*}
$$

where $A$ is the DC gain of the amplifier and $\tau_{\mathrm{a}}$ is the time constant.

If the amplifier has more than one pole, the transfer function should be constructed accordingly.

## Current Amplifier

Current amplifiers differ from voltage amplifiers by the fact that the current is fed back and, therefore, the output current is proportional to the input voltage. Consequently, the transfer function in the case of a single pole is

$$
\begin{equation*}
\frac{I_{\text {out }}(s)}{V_{\text {in }}(s)}=\frac{A_{1}}{s \tau_{\text {a }}+1} \tag{4.2.10}
\end{equation*}
$$

Note that $A_{1}$ has a unit of $[A / V]$.

### 4.2.3. AMPLIFIER-MOTOR SYSTEM

In some cases, both current and voltage feedback are used to achieve a certain performance. The general arrangement in such a system is shown in Fig. 4.2.1. The block of $\frac{A}{s \tau_{a}+1}$ represents an amplifier with a voltage feedback. We will show how a current feedback may be added and discuss its effect on the transfer function.


Fig. 4.2.1. A system of motor and amplifier with current and voltage feedbacks.

The transfer function between $\mathrm{V}_{\mathrm{in}}$ and the velocity, $\omega$, equals

Note that if $\mathbf{b}=0$, then (4.2.11) is the product of (4.2.6) and (4.2.9), indicating that the system is a series combination of a voltage amplifier and a motor. If, on the other hand, $\mathbf{b}$ becomes large, the values of the system poles are altered considerably.

When a positive current feedback is used, then (4.2.11) holds under the condition $\mathbf{b}<0$. Note that in this case large values of b will cause instability.

### 4.2.4. TACHOMETER

An ideal tachometer is a device whose output voltage, $\mathrm{V}_{\mathrm{g}}$, is proportional to the shaft angular velocity $\omega$. Thus:

$$
\begin{equation*}
V_{g}=K_{g} \omega \tag{4.2.12}
\end{equation*}
$$

In the real case, however, the voltage will have other components, which appear as ripple in the voltage given by (4.2.12). Expanding (4.2.12) to include ripple frequency and amplitude terms, we arrive at:

$$
\begin{equation*}
V_{g}=K_{g} \omega\left[1+k_{1} \cos (u \omega t)+k_{2} \cos (2 u \omega t)\right] \tag{4.2.13}
\end{equation*}
$$

where $\mathbf{u}$ is the number of commutator segments.

It is desirable to have $\mathbf{k}_{\mathbf{1}}$ and $\mathbf{k}_{\mathbf{2}}$ as small as possible to reduce the ripple factor, $\xi$, so that tachometer output voltage would be proportional to $\omega$ only. The ripple factor is:

$$
\begin{equation*}
\xi=\sqrt{k_{1}^{2}+k_{2}^{2}} \tag{4.2.14}
\end{equation*}
$$

In most cases, the tachometers are selected so that the ripple is not significant. Then (4.2.12) holds, and the tachometer transfer function becomes:

$$
\begin{equation*}
\frac{\mathrm{V}_{\mathrm{g}}(\mathrm{~s})}{\omega(\mathrm{s})}=\mathrm{K}_{\mathrm{g}} \tag{4.2.15}
\end{equation*}
$$

The block diagram is shown in Fig. 4.2.2.


Fig. 4.2.2. Block diagram of tachometer showing velocity input and voltage output.


Fig. 4.2.3. A linear potentiometer.

### 4.2.5. POTENTIOMETER

A linear potentiometer provides an output voltage, $\mathrm{V}_{\mathrm{p}}$, proportional to the angular position, $\theta$, of its shaft (Fig. 4.2.3). It operates on the principle of a voltage divider, where rotation moves the wiper
across a fixed resistor, resulting in an output voltage proportional to the angular position. The output voltage of an ideal potentiometer is given by:

$$
\begin{equation*}
V_{p}=K_{p} \theta \tag{4.2.16}
\end{equation*}
$$

In real potentiometers, the relationship of $\mathrm{V}_{\mathrm{p}}$ and $\theta$ is not truly linear, as indicated by (4.2.16), due to two main factors: first, potentiometer resistance is not uniform, which results in non-linear response; and second, in connecting the wiper, we load the circuit slightly, also affecting potentiometer linearity.

The transfer function of an ideal potentiometer is:

$$
\begin{equation*}
\frac{\mathrm{v}_{\mathrm{p}}(\mathrm{~s})}{\theta(\mathrm{s})}=\mathrm{K}_{\mathrm{p}} \tag{4.2.17}
\end{equation*}
$$

and since the shaft position is an integral of the shaft angular velocity, we may write:

$$
\begin{equation*}
\frac{\theta(s)}{\omega(s)}=\frac{1}{s} \tag{4.2.18}
\end{equation*}
$$

Combining (4.2.17) with (4.2.18) yields:

$$
\begin{equation*}
\frac{V_{p}(s)}{\omega(s)}=\frac{K_{p}}{s} \tag{4.2.19}
\end{equation*}
$$

This is represented by the block diagram in Fig. 4.2.4.


Fig. 4.2.4. Block diagram of potentiometer.

In some systems linear motion is required. In that case, this motion can be converted into rotation and a potentiometer can be used. The other alternative is to use a linear variation differential transformer, which is described in the following paragraph.

### 4.2.6. LINEAR VARIATION DIFFERENTIAL TRANSFORMER (LVDT)

The LVDT is the linear equivalent of the potentiometer, and produces a voltage $\mathrm{V}_{\mathrm{L}}$ proportional to the linear position $\mathbf{x}$ of its movable part so that:

$$
\begin{equation*}
V_{L}=K_{L} x \tag{4.2.20}
\end{equation*}
$$

Since $\mathbf{x}$ is the integral of the linear velocity $v$, we can write:

$$
\begin{equation*}
\frac{x(s)}{v(s)}=\frac{1}{s} \tag{4.2.21}
\end{equation*}
$$

We can equate linear velocity v to angular velocity $\omega$ by:

$$
\begin{equation*}
v=r \omega \tag{4.2.22}
\end{equation*}
$$

Thus, (4.2.20), (4.2.21) and (4.2.22) can be combined to:

$$
\begin{equation*}
\frac{V_{L}(s)}{\omega(s)}=\frac{K_{L} r}{s} \tag{4.2.23}
\end{equation*}
$$

Eq. (4.2.23) is represented by the block diagram in Fig. 4.2.5.

### 4.2.7.ENCODERS

## Incremental Encoder

An incremental encoder is a device which generates a pulse for a given angular increment of shaft rotation. The pulse may be square, trapezoidal, or sinusoidal. For example, assume that an unidirectional encoder generates N sinusoidal waves per revolution. The output voltage will be given by:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{e}}=\mathrm{A} \sin (\mathrm{~N} \theta) \tag{4.2.24}
\end{equation*}
$$

where $\theta$ is the angular position of the shaft in radians.

Note that the relationship of $\theta$ and $\mathrm{V}_{\mathrm{e}}$ is non-linear and, therefore, cannot be described by a transfer function. However, in the case where the angle $\theta$ is kept within some small variations, (4.2.24) can be linearized, resulting in the equivalent equations of a potentiometer.

Bidirectional encoders, on the other hand, have two output signals shifted by some phase angle, $\varphi$, thus:

$$
\begin{align*}
& \mathrm{V}_{\mathrm{e} 1}=\mathrm{A} \sin (\mathrm{~N} \theta)  \tag{4.2.25}\\
& \mathrm{V}_{\mathrm{e} 2}=\mathrm{A} \sin (\mathrm{~N} \theta+\varphi) \tag{4.2.26}
\end{align*}
$$

The relative timing of $\mathrm{V}_{\mathrm{e} 1}$ and $\mathrm{V}_{\mathrm{e} 2}$ yields direction of the rotation and rotation angle.

A common use of the incremental encoder is to input encoder signal into a digital


Fig. 4.2.5. Block diagram of a linear variation differential transformer.
counter, whose output is proportional to the angular position, $\theta$. Other applications use an encoder in a hybrid control, in which a motor is driven in a velocity control mode for some angle and then switched to position control mode which stops the motor in the next neighboring pulse position.

## Absolute Encoder

An absolute encoder is equivalent to an incremental encoder, connected to a digital counter with channels whose outputs form a number proportional to the shaft posi-
tion. Output data is periodic for every revolution, as shown in Tab. 4.2.1 and in Fig. 4.2.6.

| $\theta$ <br> [rad] | Output binary numbers | Equivalent <br> decadic <br> number |  |
| :--- | :---: | :---: | :---: |
|  | 0 | 0 | 0 |
| $\pi / 2-\pi$ | 1 | 0 | 1 |
| $\pi-3 \pi / 2$ | 0 | 1 | 2 |
| $3 \pi / 2-2 \pi$ | 1 | 1 | 3 |

Tab. 4.2.1. An absolute encoder output data.


Fig. 4.2.6. Plot of an absolute encoder output.

### 4.3. SERVO SYSTEMS

### 4.3.1. INTRODUCTION

One means of controlling the velocity or angular position of a motor is by an openloop control system as shown in Fig. 4.3.1.

In the open-loop system, the output will follow the desired function as long as all the system variables are constant. Any change in load, amplifier gain, or any other system variable will, however, cause a deviation from the desired value. In order for the motor to conform to a desired function independently of changes in these variables, a closed-loop servo system, such as the one illustrated in Fig. 4.3.2, must be constructed.

In the closed-loop system, the output variable is measured, fed back and compared to the desired input function. Any difference between the two is a deviation from the desired result; the deviation is amplified and used to correct the error. In this manner, the closed-loop system is essentially insensitive to variations in parameters, and therefore performs correctly despite changes in load condition and other system parameters. However, now the response of the system depends on the closed-loop configuration and as such it may be overdamped, underdamped, or even unstable. Special care must be given to the design of the closed loop in order to obtain the desired response.

Servo systems are divided according to the variable being controlled. The most common systems are:

1. Velocity control systems - motor velocity is to follow a given velocity profile;
2. Position control systems - motor angular position is to be controlled;
3. Torque control systems - motor torque is to be controlled;
4. Hybrid control systems - the system switches from one control mode to the other. For example, we may want to control the velocity for some time and then switch to position control mode.

Since these modes of system control are the most common, the following paragraphs describe them in detail.

### 4.3.2. VELOCITY CONTROL SYSTEMS

In a velocity control system, motor velocity is controlled to follow a signal describing the desired velocity profile. In the following analysis, simplified transfer functions are used to describe the motor and the amplifier in order to describe the method of velocity control. The block diagram for such a system is shown in Fig. 4.3.3, using the following to describe the system:


Fig. 4.3.1. A typical open-loop control system.


Fig. 4.3.2. A typical closed-loop control system.


Fig. 4.3.3. Velocity control system.

A DC gain of the amplifier
$\tau_{\mathrm{a}}$ time constant of the amplifier
$\tau_{\mathbf{m}}$ mechanical time constant of loaded motor
$\mathrm{K}_{\mathrm{E}}$ motor voltage constant
$\mathbf{K}_{\mathbf{g}}$ tachometer voltage constant

The closed-loop transfer function for the velocity control system, which is the ratio of the Laplace transforms of the output velocity $\omega_{o}$ and the desired velocity $\omega_{d}$, is:

$$
\begin{equation*}
G_{v}(s)=\frac{\omega_{o}(s)}{\omega_{d}(s)}=\frac{\frac{A}{1+s \tau_{a}} \frac{1}{1+\frac{A}{1+s \tau_{a}} \frac{K_{E}\left(1+s \tau_{m}\right)}{K_{E}\left(1+s \tau_{m}\right)} K_{g}}}{1} \tag{4.3.1}
\end{equation*}
$$

In order to determine the system response, the root locus method is used to evaluate the effect of the amplifier gain $A$ on the system response. The root locus for the velocity control system is shown in Fig. 4.3.4. Note that for the small values of $A$ the roots are negative real, which indicates that the system is overdamped. As the gain is increased, the system becomes less and less damped, and will tend to overshoot in response to a step input. Because of the simplification used on the transfer functions, the system remains stable for any value of $A$. This is not the case in reality, where the system has some more poles and, therefore, will become unstable for a certain value of $A$.


Fig. 4.3.4. Root locus for a velocity control system.

### 4.3.3. POSITION CONTROL SYSTEMS

The objective in position control systems is to control the angular position of the motor shaft, either locking the shaft at a desired
position, or rotating it at a given rate. The block diagram for such a system, with simplified transfer functions for motor and amplifier, is given in Fig. 4.3.5.

The system parameters are explained as follows:

Motor velocity is denoted by $\omega_{0}$ and the angular position is $\theta_{\mathbf{0}}$. Since $\theta_{\mathbf{o}}$ is the integral of $\omega_{0}$, this corresponds to a transfer function $1 / \mathrm{s}$ as shown in the diagram. $\mathrm{K}_{\mathrm{g}}$ represents the tachometer voltage constant $\left[\mathrm{V} / \mathrm{rad} \mathrm{s}^{-1}\right.$ ], and $\mathrm{K}_{\mathrm{p}}$ is the gain of the position sensor [ $\mathrm{V} / \mathrm{rad}$ ]. The sensor can be a potentiometer or equivalent device. Note that the combination $\mathrm{K}_{\mathrm{p}} \theta_{\mathrm{o}}+\mathrm{K}_{\mathrm{g}} \omega_{\mathrm{o}}$ is fed back and subtracted from $\theta_{\mathbf{d}}$. This is done to achieve stability, as will be shown later.

The transfer function of the system is:

$$
\begin{align*}
& \mathrm{G}_{\mathrm{p}}(\mathrm{~s})=\frac{\theta_{\mathrm{o}}(\mathrm{~s})}{\theta_{\mathrm{d}}(\mathrm{~s})} \\
&=\frac{1}{\mathrm{~s} \frac{A}{1+s \tau_{a}} \frac{1}{K_{E}\left(1+\mathrm{s} \tau_{m}\right)}}  \tag{4.3.2}\\
& 1+\frac{A}{1+s \tau_{a}} \frac{1}{K_{E}\left(1+s \tau_{m}\right)}\left(K_{g}+\frac{K_{p}}{s}\right)
\end{align*}
$$

In order to analyze the system response, we look at the roots of the second term of the characteristic equation:


Fig. 4.3.5. Position control system.

$$
\begin{equation*}
1+\frac{\frac{A}{K_{E}}}{1+s \tau_{a}} \quad \frac{K_{g} s+K_{p}}{s\left(1+s \tau_{m}\right)}=0 \tag{4.3.3}
\end{equation*}
$$

The root locus for this equation is shown in Figs. 4.3.6 and 4.3.7. Note that if no tachometer feedback is used, $\mathbf{K}_{\mathbf{g}}=\mathbf{0}$ and the characteristic equation will have three poles and no zeros. The root loci for this case are shown in Fig. 4.3.6, where it can be seen that the system becomes unstable for large values of gain A.


Fig. 4.3.6. Root locus plot of position control system with no tachometer feedback.


Fig. 4.3.7. Root locus plot of position control system with tachometer feedback. Note the improvement in stability.

The introduction of $\mathrm{K}_{\mathrm{g}}$ adds a zero to the open loop transfer function and alters the root loci as shown in Fig. 4.3.7. The new configuration is more stable and can accommodate higher gain.

Note again that the above analysis is based on simplified transfer functions of the motor and amplifier. In the real case, the transfer functions will contain more poles and the system will tend to become unstable more rapidly. However, the effect of the velocity feedback is to increase the stability of the system as shown here. Moreover,
one can control the system response by changing the value of $\mathrm{K}_{\mathrm{g}}$. When $\mathrm{K}_{\mathrm{g}}$ is small, the system may be unstable or underdamped, and as $\mathbf{K}_{\mathbf{g}}$ increases, the system becomes more damped.

### 4.3.4. TORQUE CONTROL SYSTEMS

In some cases it is desired to keep the torque of the motor constant. Since torque is proportional to the motor current, it means that a constant current must be provided to the motor. The method for achieving this is illustrated in Fig. 4.3.8.


Fig. 4.3.8. Torque control system.

The input voltage $V_{i}=-I_{d} R$ is compared with the feedback voltage $\mathbf{V}_{\mathbf{f}}=I_{\mathbf{o}} \mathbf{R}$ and input to a high gain amplifier. If the output current $I_{0}$ is different from the desired current $\mathbf{I}_{\mathbf{d}}$, the difference is amplified and used to correct the situation.

### 4.3.5. HYBRID CONTROL SYSTEMS

In some applications, the advantages of various control modes can be combined to
establish the desired performance by switching the system from one control mode to another.

One example of a hybrid system is one which alternates between velocity and position control modes. Such a system can be used for incremental motion, where the motion is performed under velocity control in obedience to a desired velocity profile, whereas stopping is done by position control mode to achieve greater accuracy.

Another example would be when forwardbackward motion is to be performed, and different velocity profiles are required for each direction. If, for instance, the forward motion is to be done at a constant velocity and the backward, or return, motion is to be in minimum time, it could be done by a hybrid control in which velocity control is used in the forward mode and position control is used for the return or "homing" mode.

A velocity/position hybrid control system is shown in Fig. 4.3.9. When the mode selector switch is open, the position feedback is disconnected, and the system operates as a velocity control servo. Closing the switch adds the circuitry required for position control. This hybrid system provides the advantages of both velocity and position controls, since it allows to follow a desired velocity profile in velocity control mode, while stopping with position accuracy. The shaft will be locked in the desired position after stopping. Achieving these advantages results in additional costs. First, a switch-


Fig. 4.3.9. Velocity/position hybrid control system.
ing circuit is required to select system modes at the right time. Secondly, the system has to be designed so that it responds to a desired manner in both modes, which makes
the design more difficult. However, the additional design effort is justified if alternate control modes are dictated by specific applications.

### 4.4. SYSTEM CHARACTERISTICS

The performance of a servo system is completely determined by its closed-loop transfer function. However, there are certain system characteristics which are especially significant, and even though they can be derived from the transfer function, it is worthwhile to describe them in detail. These characteristics are discussed below.

### 4.4.1.RESPONSE OF THE SYSTEM TO A STEP COMMAND

System response to a step input command depends on all the poles and zeros of the closed-loop transfer function. However, in most cases, one can obtain a good approximation of this response by considering only the dominant pair of poles - which may be real or complex. The system response may be any one of the following:

## Overdamped Response

The two dominant poles are either negative real or complex, with the real part negative and with the imaginary part smaller (in absolute value) than the real part. Thus, if the poles are:

$$
\begin{aligned}
& \mathbf{p}_{1}=\sigma_{1}<0 \\
& \mathbf{p}_{2}=\sigma_{2}<0
\end{aligned}
$$

or

$$
\begin{align*}
& \mathrm{p}_{1,2}=\sigma \pm \mathrm{j} \omega \\
& \sigma<\mathbf{0}  \tag{4.4.2}\\
& |\sigma|>\omega
\end{align*}
$$

then the system will be overdamped, as shown by curve (1) of Fig. 4.4.1.

## Critically Damped Response

The dominant poles are complex with equal negative real and imaginary parts. Thus:

$$
\begin{equation*}
|\sigma|=\omega \tag{4.4.3}
\end{equation*}
$$

The response in this case is fast and without an overshoot. It is shown by curve (2) of Fig. 4.4.1.

## Underdamped Response

The imaginary part of the poles is larger than the absolute value of the real part; that is:

$$
\begin{equation*}
\omega>|\sigma| \tag{4.4.4}
\end{equation*}
$$

The system response overshoots, but finally settles at the steady state value. The settling time is reciprocal of $|\sigma|$. Therefore, as $|\sigma|$ increases, the settling time becomes shorter. Curve (3) in Fig. 4.4.1 illustrates an underdamped response.

## Unstable Response

When any of the system poles has a positive real part, the response is unstable. In


Fig. 4.4.1. Response of a system to a step input command: Curve 1 -overdamped response; Curve 2critically damped response; Curve 3 - underdamped response.
practical systems instability leads to oscillations or saturation, and they are both undesired results.

The above discussion was based on the approximation of the system transfer function by its dominant pair of poles. The contribution of the remaining poles and zeros may vary, depending on the case. However, there are two common features caused by the additional poles:

1) Additional poles tend to introduce delay in the system response.
2) Complex poles which are close to the imaginary axis (e.g., poles due to structural resonance) will produce some "ringing" that rides on the response, and may have a long settling time before it decays. Fig. 4.4.2 shows a curve of the approximate response and a curve of the exact response with "ringing".


Fig. 4.4.2. Complex poles close to imaginary axis will cause "ringing" which will ride on the response.

Another question concerning the response of a system is its ability to follow input signals of different types. In analyzing a system, we observe that the system's following ability depends on the number of integrators in the open loop, which is defined as the type of the system. For this system, the open-loop transfer function has generally the form

$$
\begin{equation*}
G(s)=K \frac{\left(s-z_{1}\right)\left(s-z_{2}\right)\left(s-z_{3}\right) \ldots\left(s-z_{m}\right)}{s^{N}\left(s-p_{1}\right)\left(s-p_{2}\right)\left(s-p_{3}\right) \ldots\left(s-p_{n}\right)} \tag{4.4.5}
\end{equation*}
$$

The integer $\mathbf{N}$ indicates the number of integrators in the loop and it determines the type of the system. Thus, a system is called Type 0, Type 1, Type 2. . . if $\mathbf{N}=\mathbf{0}, \mathbf{1 , 2} \ldots$ respectively.

As mentioned above, the type of the system determines its following ability. The higher the type, the better the following. A Type 0 system will follow a step input signal with a constant steady-state error. If the input signal is a ramp, the error in following will increase with the time.

A Type 1 system, on the other hand, follows a step input signal with no steady-state error and follows a ramp input signal with a constant error. Clearly, as the type of a system is increased, its following ability is improved; a Type 2 system can follow either step or ramp input signals with no steadystate error.

### 4.4.2. SYSTEM BANDWIDTH

If the input signal to the system is sinusoidal with a low frequency, the system can respond and follow the input signal. Beyond a certain point when the input frequency becomes too high, the system cannot follow the command. The range of frequencies that the system can follow is called the bandwidth of the system.

For the system with open-loop transfer function $\mathbf{G}(\mathrm{s})$, the closed-loop transfer function $F(s)$ is given by:

$$
\begin{equation*}
F(s)=\frac{G(s)}{1+G(s)} \tag{4.4.6}
\end{equation*}
$$

Now, if we substitute $s=j \omega$ and plot the absolute value of $F(j \omega)$ in $d B$ versus the frequency $f=\omega / 2 \pi$ in logarithmic scale, we find that $|F(j \omega)|_{d B} \cong 0$ for a certain range of frequencies before dropping down, as shown in Fig. 4.4.3.

The bandwidth (or the break frequency) of a system, $f_{b}$, is defined as the frequency where $|F(j \omega)|=1 / \sqrt{2}=0.707$ or, equivalently, where:

$$
\begin{equation*}
|\mathrm{F}(\mathrm{j} \omega)|_{\mathrm{dB}}=-3 \mathrm{~dB} \tag{4.4.7}
\end{equation*}
$$



Fig. 4.4.3. Bode plot illustrating the system bandwidth.

The bandwidth of a servo system varies according to the components and the need. In general, velocity control systems using moving coil motors can reach a bandwidth of 600 Hz and more. Position control systems are more limited in general and could reach a bandwidth of about 200 Hz .

### 4.4.3. EFFECT OF TORSIONAL RESONANCE

In most servo systems, the open loop transfer function includes complex poles. As long as these poles are far enough from the imaginary axis, they do not pose a special problem. However, it becomes more and more difficult to compensate the system for complex poles as they come closer to the imaginary axis. A typical source for those poles is the effect of torsional resonance of the load-motor-tachometer combination (see section 2.3.4), and it is highly desirable to reduce the effect of those poles.

Since it is impossible to eliminate the effect of resonance completely, it is desirable to be able to measure or express quantitatively the effect of resonance, or the sensitivity of the system to sinusoidal inputs at the resonance frequency.

A simple way to describe the resonance effect is by observing the system response to sinusoidal input signals at various frequencies. The ratio of the response amplitude to the amplitude of the input signal indicates the closed-loop gain, or sensitivity of the system at various frequencies.

A typical frequency response is shown in Fig. 4.4.4, where it can be seen that the effect of resonance is represented by the peak at the frequency $f_{r}$.

It is desirable that the amplitude of the peak at $f_{r}$ is kept as low as possible to reduce


Fig. 4.4.4. Bode plot showing resonance point.
the effect of resonance. A good indication of the system sensitivity at resonance is the height of this peak, which could be described as the absolute value of system gain or in decibels.

### 4.5. SERVO AMPLIFIERS

The discussion of amplifiers in Chapter 3 centered around the speed control systems operating in the "first quadrant". We will now describe the true servo amplifiers, capable of providing positive and negative output voltages or currents, and thus being able to operate in four quadrants.

Again, we can classify the amplifiers into three categories: the linear transistor amplifiers, the SCR amplifiers, and the switching amplifiers. The following paragraphs describe them in detail.

### 4.5.1. LINEAR AMPLIFIERS

Transistor servo amplifiers are characterized by two principal output stage designs, the " H " and " T " type basic configurations, as shown in Figs. 4.5.1 and 4.5.2. The " H " or bridge output stage consists of four transistors using a single DC power supply. This output stage has the advantage of a simple, unipolar power supply and some sharing of voltage protection between the transistors. However, it is not easy to drive in linear cases, and current and voltage feedback is not easy to achieve, since the motor is "floating".

The "T" stage needs two power supplies and complementary transistors, but is easy to drive and can supply voltage and current feedback signals in a simple fashion. For this reason the " $T$ " is used most often in linear amplifiers. The biasing of the output transistors requires careful attention,


Fig. 4.5.1. " H " type output stage (one power supply required).
since a simultaneous conduction of both transistors would result in a short circuit between the two power supplies.


Fig. 4.5.2. " $T$ " type output stage (two power supplies required).
The input-output characteristics of the output stage will generally have some dead zone - an undesirable feature from linear feedback point of view. Negative feedback around the amplifier will generally reduce such nonlinearity to a negligible amount. In cases where the linear amplifier will be used in a "bang bang" mode, the dead zone
is of no primary consequence, since the amplifier will be used only in its conducting or nonconducting state.

The primary limitations on the output power handling capacity of the linear amplifier are the thermal characteristics of its output state. Since power dissipation of the output stage is the product of current and voltage across the transistors, the transistor and its associated heat sink assembly must be capable of dissipating this heat. In addition, the designer has to consider the secondary breakdown characteristics of the transistor, which will limit peak current time duration at given voltage levels, in order to ensure that such limits are not exceeded. Since amplifiers discussed in this book are designed to drive servomotors, which have a small electrical impedance, it is easy to envision situations in which an amplifier could be easily overloaded under conditions of stall or excessive torque. For this reason, current limiting circuits are generally found useful in servo amplifiers to prevent amplifier breakdown or blown fuses.

In some instances where high peak acceleration currents are needed with protection for prolonged current overload conditions, two-stage current limiters are used where the first current limit may be set at a high level with a time limit of a fraction of a second, after which a lower sustaining current limit will be brought into action. Such a circuit will not only protect the amplifier output stage, but also may prevent the motor from accidental destructive overload.

### 4.5.2. SCR AMPLIFIERS

In the last decade, the bidirectional SCR amplifier has found increasing use for servo circuits handling bandwidths up to 30 Hz . To achieve these bandwidths, a two-phase or three-phase power supply is generally used. Such systems are used to drive from one to five horsepower servo motors for the machine tool industry.

Just as in the case of the single quadrant amplifiers described in Chapter 3, close attention must be given to the form factor of the amplifier and its influence on the servo motor heating.

### 4.5.3. SWITCHING AMPLIFIERS

## Introduction

A switching amplifier is perhaps the most versatile and popular servo amplifier today. With power transistors available which can switch in the MHz region, it is now easy to design power switching amplifiers capable of handling square wave switching rates of 50 kHz . This enables the servo designer to achieve amplifier bandwidths on the order of several kHz .

Power dissipation in such an amplifier is caused by two elements: the output transistor forward voltage drop, which may be on the order of 1 to 2 V ; and the finite transition time from one polarity to the other. The latter effect is especially prominent in amplifiers with high switching rates, where the transition time may be a
significant portion of the total switching time. Therefore, while a high switching rate may seem desirable for optimum bandwidth, the switching rate may be tempered by the practical effect of output stage power dissipation.

Amplifier switching may be done in various ways. One can change either the switching frequency or the duty cycle, or both. The net result, however, has to be that the average value of the output voltage is proportional to the command. The most common method of amplifier switching is the pulse width modulation (PWM). Here the transistors are switched at a constant frequency and the resulting output voltage varies between the two extreme values. By varying the pulse width, or the duty cycle, the average value of the output voltage can be changed in accordance with the desired drive. Such output voltage is illustrated in Fig. 4.5.3.


Fig. 4.5.3. Output voltage of PWM Amplifier. The output voltage may have zero average (a), positive average (b), or negative average (c).

This modulation method is described in detail in the following section.

## PWM Amplifiers

A switching amplifier may be of an " H " or " $T$ " type, as is the case for linear amplifiers. The main advantage of the " H " type is that only one power supply is needed. The main disadvantage is that the motor voltage is floating and it is more difficult to sense its current for constructing a current feedback circuit. It appears that the advantages outweigh the disadvantages and the " H " type is the more common form, especially when high voltage is required.

A circuit diagram of an " H " type PWM amplifier is shown in Fig. 4.5.4.


Fig. 4.5.4. " H " type PWM amplifier.

The transistors of the circuit in Fig. 4.5.4 can be switched in various sequences to provide the desired voltage polarity. We will discuss the two common modes: the bipolar and the unipolar drive methods.

The bipolar drive is characterized by the fact that the transistors are turned on in pairs. When we want a positive current we turn on both $\mathbf{Q}_{\mathbf{1}}$ and $\mathbf{Q}_{\mathbf{4}}$. Whereas, for negative current, we turn on $\mathrm{Q}_{2}$ and $\mathrm{Q}_{3}$. By alternatively switching between the two pairs we can control the current. The combination of switching on one pair of transistors at a time provides a motor voltage that varies between +Vs and -Vs , with an average value which is dependent on the duty cycle.

The bipolar drive method has a limitation, however. When a pair of transistors is turned off, it takes a finite time for the transistors to get to the off state. Therefore, we must allow for some time delay between the turning off of one pair and the turning on of the other pair. This time delay, which approximates $5 \mu \mathrm{~s}$, limits the switching frequency to approximately 20 kHz .

In order to eliminate the need for the time delay, a different switching scheme was devised. This is the unipolar drive method. Here, as long as the desired current is positive, $\mathbf{Q}_{\boldsymbol{1}}$ is turned on and $\mathbf{Q}_{4}$ is periodically switched on and off to regulate the current.

The two drive methods are described in detail in the following discussion.

## Bipolar Drive

In bipolar drive, the switching period, $\mathbf{T}$, is divided into three phases: phase a, phase b and the delay phase. These phases are shown in Fig. 4.5.5.


Fig. 4.5.5. The phases of the switching period.
The phases are defined as follows:

| phase $a:$ | $\mathrm{O}_{1}, \mathrm{o}_{4}$ | On |
| :--- | :--- | :--- |
|  | $\mathrm{O}_{2}, \mathrm{o}_{3}$ | Off |
| phase $\mathrm{b}:$ | $\mathrm{o}_{2}, \mathrm{o}_{3}$ | On |
|  | $\mathrm{o}_{1}, \mathrm{o}_{4}$ | Off |

Delay phase: $\mathrm{O}_{1}, \mathrm{O}_{2}, \mathrm{O}_{3}, \mathrm{O}_{4}$ Off
Accordingly, the output voltage during phases $a$ and $b$ is:

$$
\begin{equation*}
V_{m}=V_{s} \text { during phase a } \tag{4.5.1.}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{m}=-V_{s} \text { during phase } b \tag{4.5.2}
\end{equation*}
$$

During the delay phase the voltage is determined by the circuit equation which requires continuity of the current. The current will continue its flow through the motor and the freewheeling diodes.

In order to analyze the motor voltage during the delay phase, recall the circuit equation for the DC motor:

$$
\begin{equation*}
V_{m}=r l+L \frac{d l}{d t}+K_{E} \omega \tag{4.5.3}
\end{equation*}
$$

Now assume that the current I is positive. When all four transistors are turned off the motor current will flow through the diodes $D_{2}$ and $D_{3}$. If we ignore the forward drop voltage of the diodes, we may conclude that:

$$
V_{m}=-V_{s} \text { for } I>0 \text { and delay phase (4.5.4) }
$$

Similarly, if the motor current is negative, it will flow through $D_{1}$ and $D_{4}$ resulting:

$$
V_{m}=+V_{s} \text { for } I<0 \text { and delay phase (4.5.5) }
$$

Finally, if the motor current is zero during the delay phase, no current will flow through the diodes and the motor voltage becomes:

$$
V_{m}=K_{E} \omega \text { for } I=0 \text { and delay phase (4.5.6) }
$$

Typical examples of voltage and currents of the bipolar amplifier are shown in Figs. 4.5.6 and 4.5.7. The condition where the current equals zero for a finite time, as illustrated in Fig. 4.5.7, is known as current discontinuity.


Fig. 4.5.6. Continuous current mode.


Fig.4.5.7. Current and voltage during discontinuous mode.

The current discontinuities are not common and occur only when the current is small. The effect of the current discontinuities is to cause nonlinearities in the amplifier transfer function. However, these nonlinearities can be minimized, as will be shown later, by the use of current feedback. In the following discussion we will limit ourselves to the continuous current case. The interested reader may find a detailed analysis of the discontinuous case in [4-7].

In order to analyze systems which use PWM amplifiers, we need to develop a mathematical model for the amplifier and the motor. We will divide the model into two elements, as illustrated in Fig. 4.5.8.


Fig. 4.5.8. Two element model of the PWM amplifier.

The input to the model, $\mathbf{V}_{\mathrm{in}}$, is the command signal; the output of the first element is $T_{a}$, the duration of phase $a$, and the overall output is $\mathrm{V}_{\mathrm{m}}$, the motor voltage.

Since $\mathrm{V}_{\mathrm{m}}$ is a switching voltage, we can describe it by a Fourier series as a DC term and harmonics. However, since the switching frequency is much beyond the bandwidth of the motor (and preferably beyond all resonances), the contribution of those harmonics is zero. Therefore, they can be ignored, and we may consider the average value of $\mathrm{V}_{\mathrm{m}}$ as the output of the model.

In order to analyze the first element of the model, consider the circuit that performs this function. Such a circuit is shown in Fig. 4.5.9.


Fig. 4.5.9. Phase Generating Circuit.

The circuit consists of a comparator with a deadband and an input element that sums $\mathrm{V}_{\text {in }}$ with a triangular waveform $\mathbf{S}(\mathrm{t})$. When the combined input is above a threshold, phase $a$ is chosen. When the combined input is below a negative threshold, phase $b$ is chosen and finally, when the input is within the deadband, the delay phase is selected. Accordingly, the times $\mathrm{T}_{\mathrm{a}}$ and $\mathrm{T}_{\mathrm{b}}$ are given by

$$
\begin{align*}
& T_{a}=\frac{T}{2}-T_{0}+K V_{i n}  \tag{4.5.7}\\
& T_{b}=\frac{T}{2}-T_{0}-K V_{i n} \tag{4.5.8}
\end{align*}
$$

Note that the constant K has a unit of time over voltage. K is given by

$$
\begin{equation*}
K=\frac{T}{\Delta \mathbf{S}} \tag{4.5.9}
\end{equation*}
$$

where $\Delta \mathbf{S}$ is the peak-to-peak magnitude of $\mathbf{S}(\mathrm{t})$. In order to complete the model, define the time $\mathrm{t}_{1}$ as the time interval during which the output voltage equals $\mathrm{V}_{\mathrm{s}}$. Accordingly,

$$
t_{1}= \begin{cases}T_{a} & \text { for } I>0  \tag{4.5.10}\\ T_{a}+2 T_{0} & \text { for } I<0\end{cases}
$$

or

$$
\begin{equation*}
t_{1}=T_{a}+T_{o}-T_{0} \operatorname{Sgn}(I) \tag{4.5.11}
\end{equation*}
$$

The average value of the output voltage equals

$$
\begin{align*}
V_{m a} & =\frac{V_{s} t_{1}}{T}-\frac{V_{s}\left(T-t_{1}\right)}{T} \\
& =\frac{2 V_{s} t_{1}}{T}-V_{s} \tag{4.5.12}
\end{align*}
$$

The complete model of the amplifier is now illustrated in Fig. 4.5.10.

The model of Fig. 4.5.10 is a complete one and therefore it is suitable for detailed analysis or simulation of the system.


Fig. 4.5.10. Mathematical model of PWM bipolar amplifier.

However, for a linearized analysis we may ignore constant inputs. If we further ignore $\mathrm{T}_{\mathbf{0}}$, we may simplify the model

$$
\begin{equation*}
V_{m}=\frac{2 K V_{s}}{T} V_{i n} \tag{4.5.13}
\end{equation*}
$$

This simplified model is illustrated in Fig. 4.5.11.


Fig. 4.5.11. Simplified model of PWM bipolar amplifier.

Next we will investigate the dependence of the current variations on the motor and the amplifier parameters. For simplicity, assume that the motor velocity is constant and that the amplifier is in a steady state. Furthermore, assume that the motor velocity, $\omega$, does not vary during one switching cycle because the switching frequency is too high. Also assume that the variations in the term $I_{r}$ are not significant and can be ignored.

Next define $I_{a}$ and $V_{a}$ as:

$$
\begin{equation*}
I_{a}=\frac{1}{T} \int_{0}^{T} I(t) d t \tag{4.5.14}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{a}=r I_{a}+K_{E} \omega \tag{4.5.15}
\end{equation*}
$$

Now recall the circuit equation for the motor (4.5.3), and approximate the term $\mathbf{r l}$ by $\mathbf{r l}_{\mathrm{a}}$. This simplifies the equation to:

$$
\begin{equation*}
\mathrm{L} \frac{\mathrm{dl}}{\mathrm{dt}}=\mathrm{V}_{\mathrm{m}}-\mathrm{V}_{\mathrm{a}} \tag{4.5.16}
\end{equation*}
$$

This simplified equation has a triangular waveform solution, instead of the exponential solution of (4.5.3).

The peak-to-peak variation of the motor current during $\mathrm{t}_{\mathbf{1}}$ is:

$$
\begin{equation*}
\mathrm{I}=\frac{V_{s}-V_{a}}{L} t_{1} \tag{4.5.17}
\end{equation*}
$$

However, the total current variation in one period is zero.

$$
\begin{equation*}
\frac{V_{s}-V_{a}}{L} t_{1}-\frac{V_{s}+V_{a}}{L}\left(T-t_{1}\right)=0 \tag{4.5.18}
\end{equation*}
$$

Equation 4.5.18 can be manipulated to form:

$$
\begin{equation*}
V_{a}=V_{s}\left(\frac{2 t_{1}}{T}-1\right) \tag{4.5.19}
\end{equation*}
$$

When this is substituted in 4.5 .17 it results in:

$$
\begin{equation*}
\Delta I=\frac{2 V_{s} T}{L} \cdot \frac{t_{1}}{T}\left(1-\frac{t_{1}}{T}\right) \tag{4.5.20}
\end{equation*}
$$

The quantity $\Delta I$ depends on $t_{1}$. It reaches its maximum when $\mathrm{t}_{\mathbf{1}}=\mathrm{T} / \mathbf{2}$. That maximum value equals

$$
\begin{equation*}
\Delta I_{\max }=\frac{V_{s} T}{2 L} \tag{4.5.21}
\end{equation*}
$$

Next, we repeat the analysis for the unipolar drive amplifier.

## Unipolar Drive

The switching amplifier selects phase a when the input command is positive. Conversly, when $\mathrm{V}_{\mathrm{in}}<0$, phase b is selected. The two phases are further divided into the on and off intervals which are defined as the conditions:

$$
\begin{array}{cccc}
\text { phase } a \text { - on } & Q_{1} & Q_{4} & \text { on } \\
\left(T_{a}\right) & Q_{2} & Q_{3} & \text { off }
\end{array}
$$

$$
\begin{array}{ccccc}
\text { phase a - off } Q_{1} & & \text { on } \\
\left(T_{a o}\right) & Q_{2} & Q_{3} & Q_{4} & \text { off }
\end{array}
$$

similarly,

$$
\begin{array}{rlll}
\text { phase } b-\text { on } & Q_{2} & Q_{3} & \text { on } \\
\left(T_{b}\right) & Q_{1} & Q_{4} & \text { off }
\end{array}
$$

$$
\begin{array}{cccccc}
\text { phase } b-\text { off } Q_{3} & & \text { on } \\
\left(T_{b o}\right) & Q_{1} & Q_{2} & Q_{4} & \text { off }
\end{array}
$$

Since the operation is symmetric for the two phases, it is sufficient to analyze one only. We will study the case where $\mathrm{V}_{\text {in }}>0$, and phase a is selected.

The motor voltage during the on interval is given by:

$$
\begin{equation*}
V_{m}=V_{s} \text { for phase } a-\text { on } \tag{4.5.22}
\end{equation*}
$$

During the off interval, the three possibilities exist. If the current is positive, it will continue its flow through $\mathbf{Q}_{\mathbf{1}}$ and $\mathrm{O}_{3}$ resulting in zero voltage.

$$
\begin{equation*}
V_{m}=0 \text { for phase } a-\text { off and } I>0 \tag{4.5.23}
\end{equation*}
$$

If the current $\mathbf{I}$ is negative, which may occur after changes in $\mathrm{V}_{\mathrm{in}}$, the current will flow through $\mathrm{O}_{1}$ and $\mathrm{O}_{4}$. This results in:

$$
\begin{equation*}
V_{m}=V_{s} \text { for phase a - off and } 1<0 \tag{4.5.24}
\end{equation*}
$$

Notice that this implies that the motor voltage remains constant at $\mathrm{V}_{\mathrm{s}}$ until the negative current increases to zero.

Finally, if the motor current is zero, it will remain so during the off interval. This results in a motor voltage of:

$$
\begin{align*}
V_{m} & =K_{E} \omega \text { for phase } a-\text { off and } I \\
& =0 \tag{4.5.25}
\end{align*}
$$

Examples of the voltage and current waveforms are shown in Fig. 4.5.12 and Fig. 4.5.13.

Next we will derive the mathematical model of this amplifier. The duration of the on interval is proportional to the command signal

$$
\begin{equation*}
T_{a}=K V_{i n} \tag{4.5.26}
\end{equation*}
$$

where K is given by Eq. (4.5.9).


Fig. 4.5.12. Voltage and current waveforms for unipolar PWM amplifier.


Fig. 4.5.13. Voltage and current waveforms for unipolar PWM amplifier in the discontinuous current mode.

In deriving the model we consider the common case where $\mathbf{l}>0$ during phase a. In this case the voltage is given by:

$$
V_{m}=\left\{\begin{array}{l}
V_{s} \text { for phase a - on }  \tag{4.5.27}\\
0 \text { for phase a off }
\end{array}\right.
$$

and the average voltage is:

$$
\begin{equation*}
V_{m a}=\frac{V_{s} T_{a}}{T} \tag{4.5.28}
\end{equation*}
$$

The resulting model for the amplifier is therefore:

$$
\begin{equation*}
V_{m}=\frac{K V_{s}}{T} V_{i n} \tag{4.5.29}
\end{equation*}
$$

Next we will derive the magnitude of the current variations. Consider the simplified motor equation (4.5.16) in which the motor voltage is given by Eq. (4.5.27). The current variation is given by:

$$
\begin{equation*}
\Delta I=\frac{V_{s}-V_{a}}{L} T_{a} \tag{4.5.30}
\end{equation*}
$$

However, since the total current variation during one period is zero, we obtain:

$$
\begin{equation*}
\frac{V_{s}-V_{a}}{L} T_{a}-\frac{V_{a}}{L}\left(T-T_{a}\right)=0 \tag{4.5.31}
\end{equation*}
$$

or

$$
\begin{equation*}
V_{a}=\frac{V_{s} T_{a}}{T} \tag{4.5.32}
\end{equation*}
$$

Now substitute (4.5.32) in (4.5.30) to obtain:

$$
\begin{equation*}
\Delta I=\frac{V_{s} T}{L}\left(1-\frac{T_{a}}{T}\right) \frac{T_{a}}{T} \tag{4.5.33}
\end{equation*}
$$

The current variation is maximized when $T_{a}=T / 2$. The resulting current variation is:

$$
\begin{equation*}
\Delta I_{\max }=\frac{V_{s} T}{4 L} \tag{4.5.34}
\end{equation*}
$$

## Self Oscillating Amplifiers

In the previous discussion it was assumed that the triangular waveforms signal, $\mathbf{S}(\mathbf{t})$, is generated externally and that its frequency is constant. Therefore, the switching frequency of the PWM amplifier is constant. Although this is the case in most systems, there are still amplifiers which are built to be self oscillating.

Self oscillating amplifiers generate the signal $\mathbf{S}(\mathrm{t})$ internally. An example of such
an amplifier is shown in Fig. 4.5.14. In order to be specific, the circuit and the following discussion are limited to bipolar PWM amplifiers.

The amplifier includes the following components:

## Comparator -

High gain amplifier with positive feedback. This results in a hysteresis, as shown in Fig. 4.5.15. The output of the comparator is either $\mathrm{V}_{\text {max }}$ or $-\mathrm{V}_{\text {max }}$.

The comparator is followed by a dead-zone generator. This element is necessary in bipolar amplifiers to guarantee that one set of transistors is turned completely off before the other set is turned on.


Fig. 4.3.14. Self oscillating PWM amplifier. Note that the amplifier may use current feedback or delayed voltage feedback.


Fig. 4.5.15. Hysteresis effect in comparator.
The dead-zone may be generated in several ways. An example of such a circuit is shown in Fig. 4.5.16.


Fig. 4.5.16. Dead zone generator.
The relationships between the signals in the dead-zone generation circuit is shown in Fig. 4.5.17.

The signal $\mathbf{V}_{2}$ is later applied to the driver. According to the described operation of bipolar PWM amplifiers, the switching period has 3 phases: phase $a$, phase $b$ and the delay phase.

The feedback can be either the motor current or the delayed voltage. In the case of current feedback, there is a need for a differential amplifier to sense the motor current. The voltage sensing can be


Fig. 4.5.17. Relationships between variables in the dead-zone generations circuit.
simplified as it is sufficient to measure the voltage on one side of the motor.

In the case of motor voltage feedback the voltage is applied to a low-pass filter and the filter output is the desired waveform S(t).

It is clear that in a self-oscillating amplifier the switching frequency is a function of the parameters of the motor and the amplifier. Moreover, the switching frequency depends on the motor current and velocity. As a consequence, the switching frequency will vary with the motor velocity. This feature is undesirable as it may cause the switching frequency to coincide with resonance modes of the motor.

Next we will consider the mathematical model of the amplifier with feedback.

## Amplifier Transfer Function

In the previous discussion we derived a simplified model for the PWM amplifier. It was indicated that the delay phase introduces nonlinearities to the system and that those nonlinearities are minimized with feedback. Now we will discuss the effect of the feedback on the amplifiers transfer function.

Let the amplifiers open-loop nonlinear relationship between input and output voltages be $\mathbf{N}(\mathrm{e})$. If we use voltage feedback we obtain a system as shown in Fig. 4.5.18.


Fig. 4.5.18. PWM amplifier with voltage feedback.

The overall transfer function is given by:

$$
\begin{equation*}
\frac{V_{m}}{V_{i n}}=\frac{N(e)}{1+F(s) N(e)} \tag{4.5.35}
\end{equation*}
$$

If the open loop gain is sufficiently high, within the system bandwidth,

$$
\begin{equation*}
N(e) F(s) \gg 1 \tag{4.5.36}
\end{equation*}
$$

we can ignore the term 1 in the denominator of Eq. (4.5.35), leading to the simplified expression:

$$
\begin{equation*}
\frac{V_{m}}{V_{i n}}=\frac{1}{F(s)} \tag{4.5.37}
\end{equation*}
$$

Since $F(s)$ is a low-pass filter, the net result of Eq. (4.5.37) is to provide phase lead in the loop.

Next we consider the effect of current feedback on the transfer function. The mathematical model in this case is more complex, as shown in Fig. 4.5.19.


Fig. 4.5.19. Block diagram of a switching amplifier and a motor.

The system components may be represented by their transfer functions, as shown in Fig. 4.5.19. The comparator and the driver are nonlinear elements and their output depends on the input signal e. It is assumed here that $\mathrm{N}(\mathrm{e})$ is a describing function which represents their behavior.

The motor impedance is considered to be a resistance $R$ and an inductance $L_{\mathbf{t}}$. The inductance $L_{t}$ includes the internal and the external parts.

The element $\mathrm{K}_{\mathrm{E}}$ represents the back emf voltage in the motor and $a$ is the current feedback. The transfer function of the system is given by:
$N(e) \cdot \frac{1}{s L_{t}+R} \cdot \frac{K_{T}}{s J}$
or

$$
\begin{equation*}
\frac{\omega(s)}{V_{\text {in }}(s)}=\frac{N(e) K_{T}}{\left(s L_{t}+R\right) s J+N(e) a s J+K_{E} K_{T}} \tag{4.5.39}
\end{equation*}
$$

In most cases, the amplifier gain is very high and $\mathbf{N}(\mathrm{e})$ is very large. Under those conditions, the transfer function may be approximated by:

$$
\begin{equation*}
\operatorname{limit}_{N(e)^{+\infty}} \frac{\omega}{V_{i n}}(\mathrm{~s})=\frac{\mathrm{K}_{\mathrm{T}}}{a \mathrm{sJ}} \tag{4.5.40}
\end{equation*}
$$

The simplified transfer function is represented by a block diagram in Fig. 4.5.20. The interpretation of this simplified model is that due to high forward gain in the switching amplifier, its response is determined by the feedback. In this case, the amplifier becomes a current amplifier, and for such a source, the transfer function of the motor is simplified considerably (see 4.2.8).


Fig. 4.5.20. Block diagram of simplified transfer function model for switching amplifier and motor.

It can be seen how one can alter this transfer function by changing the current feedback. If a dynamic element with a transfer function $a(\mathrm{~s})$ is used, the transfer function becomes:

$$
\begin{equation*}
\frac{\omega(\mathrm{s})}{\mathrm{V}_{\mathrm{in}}(\mathrm{~s})}=\frac{1}{a(\mathrm{~s})} \frac{\mathrm{K}_{\mathrm{T}}}{\mathrm{sJ}} \tag{4.5.41}
\end{equation*}
$$

Thus, all the poles of $a(s)$ become zeros of the transfer function and vice versa.

The model of (4.5.41) may be used to analyze the behavior of the amplifier and the motor as parts of a larger system. It is useful in analyzing system stability, bandwidth, and other general characteristics. However, it cannot describe the unique characteristics of a switching amplifier, and for those we have a special discussion below.

## Power Dissipation in the Motor

A switching amplifier considerably reduces the dissipation in the amplifier by turning the transistors completely "on" or completely "off". However, the amplifier switching may adversely affect the power dissipation in the motor, due to the current ripples. The additional power losses in the motor and their dependence on the system parameters are described below.

The power dissipation in the motor can be divided into two major elements: the dissipation in the winding and the dissipation in the core.

$$
\begin{equation*}
P_{\text {dissipation }}=P_{w}+P_{c} \tag{5.4.42}
\end{equation*}
$$

We start by considering the effect of switching on the dissipation in the winding. Note that the dissipation in the winding is given by

$$
\begin{equation*}
P_{w}=I^{2}(t) R \tag{5.4.43}
\end{equation*}
$$

where $\mathbf{R}$ is the armature resistance. When switching frequencies are much higher than 10 KHz , one should consider the ac resistance of the armature which is greater than R. However, in this discussion it is assumed that the dc resistance is a correct representation. In the case of PWM amplifiers the current is given by

$$
\begin{equation*}
I_{p}=I+S(t) \tag{5.4.44}
\end{equation*}
$$

where $\mathbf{S}(\mathbf{t})$ is a triangular current with peak-to-peak variations of $\Delta I$ amperes.

Assume that the current $\mathbf{I}$ is constant over one switching period. The average power dissipation in the motor winding is

$$
\begin{align*}
& \frac{1}{T_{0}} \int_{0}^{T} I^{2} R \mathrm{Rdt}=\frac{1}{T_{0}} \int^{T}[I+S(t)]{ }^{2} R d t= \\
& R I^{2}+\frac{1}{T_{0}} \int_{0}^{T} R I S(t) d t+\frac{1}{T_{0}} \int^{T} R^{2}(t) d t= \\
& R I^{2}+\frac{R \Delta I^{2}}{12} \tag{5.4.45}
\end{align*}
$$

Note that the power dissipation has two parts: the first term, $I^{2} R$, is the dissipation in the motor regardless of the switching. The second, $\frac{\mathrm{R} \Delta 1^{2}}{12}$, is the additional dissipation due to the switching. Note that this quantity is fairly small. We may therefore conclude that the switching does not significantly affect the power dissipation in the winding.

Core losses, on the other hand, are significantly affected by the amplifier switching. The main components of the core losses are the eddy current and the hysteresis losses. The hysteresis losses are equal

$$
\begin{equation*}
P_{h}=K_{1} f_{s} B_{\Delta}{ }^{1.6} \tag{5.4.46}
\end{equation*}
$$

where $K_{1}$ is a constant, $f_{s}$ is the switching frequency and $B_{\Delta}$ is the incremental flux density corresponding to a change of current of $\Delta \mathrm{l}$. The eddy current losses on the other hand, are given by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{e}}=\frac{\mathrm{K}_{2} \mathrm{f}_{\mathrm{s}}{ }^{2} \mathrm{~B}_{\Delta}{ }^{2}}{\rho} \tag{5.4.47}
\end{equation*}
$$

where $\mathrm{K}_{\mathbf{2}}$ is a different constant and $\rho$ is the resistivity of the material.

It can be seen that both the hysteresis and the eddy current losses increase with the switching frequency and with the magnitude of the current ripple.

The hysteresis losses are found in ferromagnetic materials whereas the eddy current losses can affect any electricallyconductive material. The primary areas in dc motors where significant core losses may occur are the stator laminations and magnetic material with low resistivity, such as Alnico and Rare Earth materials. Ceramic magnets, on the other hand, due to their high resistivity, are totally unaffected by the current ripple.

## The Selection of the Switching Frequency

The switching frequency, $f_{s}$, is probably the most important parameter in the design of PWM amplifiers. The following discussion provides guidelines for selecting $f_{s}$.

The switching rate must be high enough so that the servo system does not respond to it. Thus, if the motion requirements are such that a bandwidth $f_{B W}$ is needed, one should select

$$
\begin{equation*}
\mathrm{f}_{\mathrm{s}}>10 \mathrm{f}_{\mathrm{BW}} \tag{5.4.48}
\end{equation*}
$$

The switching frequency must also be above all the significant resonance frequencies of the loop.

$$
\begin{equation*}
f_{s}>f_{\text {resonance }} \tag{5.4.49}
\end{equation*}
$$

On the other hand, the switching period should be much longer than the transistor switching delay time $\mathrm{T}_{\mathrm{d}}$. Otherwise, the amplifier may have significant nonlinearities. Thus,

$$
\begin{equation*}
\mathrm{f}_{\mathrm{s}}<\frac{10}{\mathrm{~T}_{\mathrm{d}}} \tag{5.4.50}
\end{equation*}
$$

Having established upper and lower limits for $f_{s}$, we can proceed to select $f_{s}$ which will minimize the losses.

The power dissipation in the amplifier transistors is directly proportional to $\mathrm{f}_{\mathrm{s}}$. Therefore, in order to minimize those losses, we need to select $f_{s}$ as small as possible.

The switching frequency affects the core losses in two ways: first, it affects the hysteresis losses and the eddy current losses through the term $f_{s}$ in Eqs. (5.4.46) and (5.4.47). The switching frequency also affects the current variations, $\Delta \mathbf{I}$, reciprocally, and this has an opposite effect on the losses. The total effect of $f_{s}$ on the core losses is dependent on the particular motor. Thus, there are reasons to increase $f_{s}$, and others to decrease it. The final selection is a compromise between those contradicting conditions.

### 4.6. DIGITAL CONTROL AND PHASELOCKED SYSTEMS

### 4.6.1. INTRODUCTION

A digital control system is a system in which at least one of the elements in the control loop is a digital component. Such digital components may be either a microprocessor or a digital circuit. They can also be optical encoders, magnetic position sensors and others.

The use of digital components as controllers or as sensors improves the system performance by allowing more flexibility in the design. It also makes the system less susceptible to drift due to parameter changes. On the other hand, the addition of digital components to the control loop poses a challenge due to the added difficulty in design and analysis.

The main problem with digital control systems is that when applying the control signals at discrete times, one creates the effect of time delay in the control loop. This has an adverse effect on the system stability.

A second problem with digital control systems is that they cannot be analyzed by the methods of Laplace transform and
frequency-domain. Instead, they require different tools for analysis as discussed below.

### 4.6.2. DIGITAL CONTROL SYSTEMS

A typical digital control system is shown in Figure 4.6.1. In general, the system has analog elements, such as the motor and the amplifier, and digital elements, such as the controller. The system also has a third type of element - that needed to convert the signal from analog to digital form and vice-versa.

The motor and the amplifier are represented by the transfer functions $M(s)$ and A(s) respectively. Although the amplifier may be a switching element, as long as the switching frequency is much higher than the system bandwidth, the continuous representation is valid.

The digital controller processes the signal in a digital form and outputs a digital signal at discrete times. In order to interface this output signal with the analog element, there must be an analog-digital converter (ADC) and a hold circuit to hold the output constant between samplings. Such an element is the zero-order-hold $(\mathrm{ZOH})$ circuit which is shown in Fig. 4.6.1.


Fig. 4.6.1. Block diagram representation of a digital control system.

On the input end of the digital controller we need to perform the opposite conversion. Here an analog signal has to be converted to a digital form by a digitalanalog converter (DAC). Quite often the conversion from analog to digital form can be done by the position sensor. For example, an optical encoder quantizes the position and outputs the signal in a digital form which can be directly applied to a digital controller with no additional conversion. The two samplers at the input and the output of the digital controller are internal to the controller.

To better explain the operation of the digital control loop, consider the mathematical relationship between the variables in the loop:

Start with the error signal, e, which is the difference between the input and the feedback. e is a continuous function of the time, $\mathbf{e}(\mathrm{t})$. The error signal, $\mathbf{e}$, is applied to a DAC and the resulting output is $f(t)$. There are two differences between $e(t)$ and $f(t)$. First, $f(t)$ is quantized. It can assume a finite number of values according to the number of bits by which it is represented. For example, if $f(t)$ is represented by 8 bits, it can have 256 discrete values. On the other hand, $\mathbf{e}(\mathbf{t})$ is a continuous signal and can have an infinite number of values.

The second difference between $e(t)$ and $f(t)$ is in their form. $e(t)$ is represented by one signal which can very continuously over some range, whereas $f(t)$ is represented
by several digital signals according to some code. Although this difference is very significant for the circuit, it makes no difference mathematically. Therefore, except for the quantization process, which becomes significant under some conditions, the ADC has no effect on the signal and therefore, can be mathematically ignored.

The digital controller inputs the signal $\mathrm{f}(\mathrm{t})$. It samples it at a sampling period of $T$, to form the sequence $f(0), f(T), f(2 T) \ldots$ This sequence is processed by the controller which then generates a new sequence $g(0), g(T), g(2 T)$. . The $g(k T)$ sequence is the output of the digital controller.

The sequence $g(k T)$ is applied to a ZOH circuit which generates the function $h(t)$ as follows:

$$
\begin{equation*}
h(t)=g(k T) \text { for } k T \leq t<(k+1) T \tag{4.6.1}
\end{equation*}
$$

The signal $\mathbf{h}(\mathbf{t})$ is then applied to a DAC. This element only changes the form of representation of the signal. From the digital form of $h(t)$, it is converted to an analog form $\mathrm{V}_{1}(\mathrm{t})$. Mathematically, the transfer function of the DAC equals 1.

Finally, the voltage $\mathrm{V}_{\mathbf{1}}$ is applied to the amplifier and the amplified output is applied, in turn, to the motor. The amplifier and the motor are linear, continuous elements and their behavior was described in detail previously.

The digital controller may be a microprocessor or a dedicated digital circuit which was built for the particular application. The advantages of a microprocessor are its flexibility and its low cost availability. The main disadvantage is the computation time. Complex operations involving multiplication and division may be performed much faster by a dedicated circuit.

The system shown in Fig. 4.6.1 is the simplest one. It often involves additional elements such as internal loops or feedforward branches, as shown in Fig. 4.6.2. However, the basic principles of both systems are the same.

### 4.6.3. SYSTEM ANALYSIS

The first step in the analysis of digital control systems is to classify all the loop components as analog or digital elements. Analog elements include the motor, amplifier, tachometer and potentiometer. The
digital elements include a digital controller, a shaft encoder with digital output and other digital circuits in the loop.

After considering the elements, we can focus on the way in which the signal is converted. The conversion from analog to digital may have an effect in analyzing special stability conditions at low velocities, however, in most cases, this can be ignored. The conversion from digital to analog, however, is the key to analyzing the system.

To understand this critical point recall Fig. 4.6.1 and note that the output of the digital controller is a sequence of points $\mathbf{g}(\mathbf{k} \mathbf{T})$. Such a sequence of points cannot drive an analog device. An analog device needs an input function $\mathbf{h}(\mathbf{t})$ that has a non-zero integral. As a result, all digital control systems have some way of converting the sequence $\mathbf{g}(\mathrm{kT})$ to a function $h(t)$ with non-zero integrals.

The most common way for generating $\mathbf{h}(\mathbf{t})$ is by a ZOH circuit, as given by Eq.


Fig. 4.6.2. Multi-loop digital control system.
(4.6.1). However, there are other forms. One common way, which results from methods of linearization, is to multiply the sequence $\mathbf{g}(\mathbf{k T})$ by a sequence of impulses, thereby generating $\mathbf{h}(\mathrm{t})$. Since these are the two most common methods for generating $\mathbf{h ( t )}$, we will focus on them.

The next step is to describe the system by its block diagram form. We group all the analog components together to form $H(s)$. Similarly, we group all the digital elements into one component. Now we find that the system may be represented in the form of Fig. 4.6.3 or Fig. 4.6.4, according to the process of generating $\mathbf{h}(\mathrm{t})$.

The next question to consider concerns the sampling times. If the sampling period is constant, the system is said to be a sampled-data system and it can be analyzed by the $\mathbf{Z}$ transform method.

Often the sampling time depends on the motor position. For example, when the position information is updated by an optical encoder, the sampling times depend on the motor velocity and don't necessarily occur at constant intervals. In this case one has to be cautious in stating the
conditions that will lead to constant sampling frequency. An example for such conditions is given in the following discussion on instability at low velocities.

If the assumption of constant sampling rate cannot be justified, the $\mathbf{Z}$ transform method is not valid and one has to resort to simulation for analysis.

Next we present a simple, straightforward method for stability analysis of systems which can be represented in the form of Figs 4.6.3 or 4.6.4.

## Systems with ZOH

When a digital control system is in the form of Fig. 4.6.3, it can be analyzed using the $\mathbf{Z}$ transform method. The method is similar to the Laplace transform method except that it is designed for digital systems.

We start by defining the $\mathbf{Z}$ transform. A sequence $f(k)$ is transformed to the function $F(z)$ by the equation

$$
\begin{equation*}
F(z)=Z\{f(k)\}=\sum_{k=0}^{\infty} f(k) z^{-k} \tag{4.6.2}
\end{equation*}
$$



Fig. 4.6.3. Digital control system with ZOH.


Fig. 4.6.4. Digital control system with impulse multiplier.
For example, if the $f(k)$ sequence is

$$
\begin{equation*}
f(k):\{1,-1,1,0,0 \ldots\} \tag{4.6.3}
\end{equation*}
$$

the $F(z)$ is

$$
\begin{equation*}
F(z)=1-z^{-1}+z^{-2} \tag{4.6.4}
\end{equation*}
$$

A detailed description of the $\mathbf{Z}$ transform method can be found in numerous textbooks such as [4-16] and [4-17].

One of the most important properties of the $\mathbf{Z}$ transform is the shifting property. It states that if $\mathbf{F}(\mathbf{z})$ is the $\mathbf{Z}$ transform of the sequence $f(k)$

$$
\begin{equation*}
Z\{f(k)\}=F(z) \tag{4.6.5}
\end{equation*}
$$

then the $\mathbf{Z}$ transform of the shifted sequence $f(k-m)$ is

$$
\begin{equation*}
Z\{f(k-m)\}=z^{-m} F(z) \tag{4.6.6}
\end{equation*}
$$

Now we are ready to determine the transfer function of a digital controller. Consider the system shown in Fig. 4.6.3. Suppose the controller samples the input function $f(t)$ every $100 \mu \mathrm{~s}$. During the following $100 \mu \mathrm{~s}$ interval, the controller calculates the value of the next output, $\mathrm{g}(\mathrm{kT})$ as a function of the input and previous values of the output. For example, let the relationship between input and output be

$$
\begin{align*}
g(k T)= & f[k-1) T]-0.5 f[(k-2) T] \\
& +0.2 g[(k-1) T] \tag{4.6.7}
\end{align*}
$$

We can rewrite (4.6.7) as

$$
\begin{equation*}
g(k)-0.2 g(k-1)=f(k-1)-0.5 f(k-2) \tag{4.6.8}
\end{equation*}
$$

Next, we apply the $\mathbf{Z}$ transform to Eq (4.6.8). Denote the $Z$ transforms of $f(k)$ and $g(k)$ by $F(z)$ and $G(z)$ respectively. The transform of (4.6.8) then becomes

$$
\begin{equation*}
G(z)-0.2 z^{-1} G(z)=z^{-1} F(z)-0.5 z^{-2} F(z) \tag{4.6.9}
\end{equation*}
$$

The transfer function $\mathrm{D}(\mathrm{z})$ can be written as

$$
\begin{equation*}
D(z)=\frac{G(z)}{F(z)}=\frac{z^{-1}-0.5 z^{-2}}{1-0.2 z^{-1}}=\frac{z-0.5}{z^{2}-0.2 z} \tag{4.6.10}
\end{equation*}
$$

The transfer function of controllers with other different equations is determined in a similar manner. Note that the output at time $\mathbf{k T}, \mathbf{g}(\mathbf{k} \mathbf{T})$ depends on inputs at previous sampling times $(k-1) T$, etc. This is due to the computation time of the digital controller.

Next we will find the digital transfer function of the elements shown in Fig. 4.6.5.


Fig. 4.6.5. Elements of the digital control loop with ZOH .

If we denote the $\mathbf{Z}$ transforms of the input sequence $g(k)$ and the output sequence $y(k)$ by $G(z)$ and $Y(z)$ respectively, the transfer function of this subsystem is

$$
\begin{equation*}
E(z)=\frac{Y(z)}{G(z)} \tag{4.6.11}
\end{equation*}
$$

The function $\mathrm{E}(\mathrm{z})$ can be found from $H(s)$ by the following equation

$$
\begin{align*}
E(z) & =Z\{H(s) \text { and } Z O H\} \\
& =\left(1-z^{-1}\right) Z\left\{\frac{H(s)}{s}\right\} \tag{4.6.12}
\end{align*}
$$

Equation (4.6.12) indicates that in order to find $\mathrm{E}(\mathrm{z})$ we need to divide $\mathrm{H}(\mathrm{s})$ by s , and then find the Laplace-Z transform pair of $\frac{\mathrm{H}(\mathrm{s})}{\mathrm{s}}$. This function can be found in standard tables, such as Table 4.6.1. As an example, consider a system where

$$
\begin{equation*}
H(s)=\frac{1000}{s+1000} \tag{4.6.13}
\end{equation*}
$$

Then

$$
\begin{equation*}
\frac{H(s)}{s}=\frac{1000}{s(s+1000)} \tag{4.6.14}
\end{equation*}
$$

From the table we find that

$$
\begin{equation*}
Z\left\{\frac{H(s)}{s}\right\}=\frac{\left(1-e^{-1000 T}\right) z^{-1}}{\left(1-z^{-1}\right)\left(1-z^{-1} e^{-1000 T}\right)} \tag{4.6.15}
\end{equation*}
$$

and the transfer function $E(z)$, as given by (4.6.12) is

$$
\begin{equation*}
E(z)=\frac{\left(1-e^{-1000 T}\right) z^{-1}}{1-z^{-1} e^{-1000 T}} \tag{4.6.16}
\end{equation*}
$$

| Laplace <br> Trans- <br> form $F(s)$ | Time <br> function <br> $f(t)$ | $z-$ transform <br> $F(z)$ |
| :---: | :---: | :---: |
| 1 | $\delta(t)$ | 1 |
| $e^{-k T s}$ | $\delta(t-k T)$ | $z^{-k}$ |
| $\frac{1}{s}$ | $u_{s}(t)$ | $\frac{z}{z-1}$ |
| $\frac{1}{s^{2}}$ | $t$ | $\frac{T z}{(z-1)^{2}}$ |
| $\frac{2}{s^{3}}$ | $t^{2}$ | $\frac{T^{2} z(z+1)}{(z-1)^{3}}$ |
| $\frac{1}{s+a}$ | $e^{-a t}$ | $\frac{z}{z-e^{-a T}}$ |
| $\frac{1}{(s+a)^{2}}$ | $t e^{-a t}$ | $\frac{T z e^{-a T}}{\left(z-e^{-a t}\right)^{2}}$ |
| $\frac{a}{s(s+a)}$ | $1-e^{-a t}$ | $\frac{z\left(1-e^{-a T}\right)}{(z-1)\left(z-e^{-a T}\right)}$ |
| $\frac{a}{s^{2}(s+a)}$ | $t-\frac{1}{a}\left(1-e^{-a t}\right)$ | $\frac{z\left[A z-a T e^{-a T}+1-e^{-a T}\right]}{a(z-1)^{2}\left(z-e^{-a T}\right)}$ |
| $A=\left(a T-1+e^{-a T}\right)$ |  |  |

Table 4.6.1. Laplace Z-transform Paris
Clearly, the transfer function $E(z)$ depends on the sampling time $\mathbf{T}$. For a sampling time of $\mathrm{T}=\mathbf{1 0 0} \mu \mathrm{s}$, it becomes

$$
\begin{align*}
E(z) & =\frac{\left(1-e^{-0.1}\right) z^{-1}}{1-z^{-1} e^{-0.1}}=\frac{0.095 z^{-1}}{1-0.905 z^{-1}} \\
& =\frac{0.095}{z-0.905} \tag{4.6.17}
\end{align*}
$$

Once the transfer functions of the digital and the analog parts are known, we can find the overall transfer function and can
analyze the system stability. The overall system transfer function, which relates the input to the output, is

$$
\begin{equation*}
Q(z)=\frac{D(z) E(z)}{1+D(z) E(z)} \tag{4.6.18}
\end{equation*}
$$

The system characteristic equation which determines the loop poles is

$$
\begin{equation*}
1+D(z) E(z)=0 \tag{4.6.19}
\end{equation*}
$$

All we have to do is solve (4.6.19) and determine its roots. If each root is within the unit circle, the system is stable. Otherwise the system is unstable. For the given example we have

$$
\begin{equation*}
1+\frac{z-0.5}{z^{2}-0.2 z \quad 0.095}=0 \tag{4.6.20}
\end{equation*}
$$

This can be written as

$$
\begin{equation*}
z^{3}-1.105 z^{2}+0.276 z-0.0475=0 \tag{4.6.21}
\end{equation*}
$$

Eq (4.6.21) has three roots

$$
\begin{aligned}
& Z_{1}=0.85 \\
& z_{2,3}=0.125 \pm j 0.76
\end{aligned}
$$

Since the magnitude of each one of the roots is less than one, the system is stable.

## Systems With Impulse Multiplier

Systems with impulse multipliers can be represented by the block diagram of Fig. 4.6.4. The stability analysis of those
systems is similar to the one described before. We start with the transfer function of the digital controller, $\mathrm{D}(\mathrm{z})$ using the same procedure. The procedure differs when we find $E(z)$, the transfer function of the rest of the loop. The elements included in the remaining section are shown in Fig. 4.6.6.


Fig. 4.6.6. Elements of the digital control loop with impulse multiplier.

The transfer function $E(z)$ is defined as in Eq. (4.6.11). In this case it is found by

$$
\begin{equation*}
E(z)=Z\{H(s)\} \tag{4.6.22}
\end{equation*}
$$

where $E(z)$ and $H(s)$ are a Laplace $Z$ transform pair, as given in Table 4.6.1.

For example, if

$$
\begin{equation*}
H(s)=\frac{1000}{s+1000} \tag{4.6.23}
\end{equation*}
$$

then

$$
\begin{equation*}
E(z)=\frac{1000}{1-z^{-1} e^{-1000 T}} \tag{4.6.24}
\end{equation*}
$$

If we further assume that the sampling period equals $\mathbf{1 0 0} \mu \mathrm{s}, \mathrm{E}(\mathrm{z})$ becomes

$$
\begin{equation*}
E(z)=\frac{1000 z}{z-0.905} \tag{4.6.25}
\end{equation*}
$$

Once $D(z)$ and $E(z)$ are found, the system transfer function can be determined by Eq. (4.6.18). The loop characteristic equation can be written and its roots are found. The system is stable if each one of its roots is within the complex unit circle.

The classification of systems into the ZOH type or the impulse multiplier type is not always simple. As a rule of thumb, if the output of the controller is held constant during a sampling period, we say that a system has a ZOH. In this case the magnitude of the output may change only at sampling times.

An alternative method to output the command of a controller is to pulse-width modulate it; the output may be either high or low, but the duration of the pulse may be controlled by the processor. This case fits the impulse multiplier description. An example of such a case is given in the next section.

## Modified Analysis Method

The stability analysis method given before is limited to determining whether, for a given set of parameters, the system is stable. This method can be modified to allow more flexibility in the analysis.

Consider the transformation

$$
\begin{equation*}
z=\frac{w+1}{w-1} \tag{4.6.26}
\end{equation*}
$$

This transforms the inside of the unit circle in the $\mathbf{z}$ plane to the left half of the $\mathbf{w}$
plane. As a result we may analyze the transformed equation of $w$ by the root locus method and other analysis tools of continuous systems. As an example consider a system as in Fig. 4.6.4, with

$$
\begin{equation*}
H(s)=\frac{10 K}{s+2} \tag{4.6.27}
\end{equation*}
$$

and let the input equal 1. The digital controller is an integrator. It samples the input every $100 \mu \mathrm{~s}$ and adds the input to the previous output. Assuming that the addition time is much shorter than $100 \mu \mathrm{~s}$, we may write the difference equation for the controller as

$$
\begin{equation*}
g(k T)=f(k T)+g[(k-1) T] \tag{4.6.28}
\end{equation*}
$$

Now suppose that the controller modulates its output as follows: Upon calculating the value of the output, $\mathbf{g}(\mathbf{k} \mathbf{T})$, the controller outputs a signal of magnitude of 5 v and a duration of $\mathrm{g}(\mathrm{kT}) \mu \mathrm{s}$. Thus, if $\mathrm{g}(\mathrm{kT})=\mathbf{2}$, the pulse width is $2 \mu \mathrm{~s}$. The controller output is then reduced to zero until the next sampling time.

In this case we can approximate the controller output by an impulse, but we need to determine its weight. Note that the control signal in this case is

$$
h(t)=\left\{\begin{array}{l}
5 \text { for } k T<t<k T+10^{-3} g(k T)  \tag{4.6.29}\\
0 \text { for } k T+10^{-3} g(k T) \leq t \leq(k+1) T
\end{array}\right.
$$

The integral of $h(t)$ over the interval [ $k T,(k+1) T]$ is

$$
\begin{equation*}
\int_{k T}^{(k+1) T} h(t) d t=5 \cdot 10^{-3} g(k T) \tag{4.6.30}
\end{equation*}
$$

Then, this is equivalent to multiplying $\mathbf{g}(\mathbf{k T})$ by an impulse of magnitude of $5 \cdot 10^{-3}$. This factor is represented as gain in the loop. The resulting block diagram of the loop is given in Fig. 4.6.7.

The digital controller transfer function $D(z)$ can be found from (4.6.28). By applying the $\mathbf{Z}$ transform we obtain

$$
\begin{equation*}
\mathrm{G}(\mathrm{z})=\mathrm{F}(\mathrm{z})+\mathrm{z}^{-1} \mathrm{G}(\mathrm{z}) \tag{4.6.31}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{D}(\mathrm{z})=\frac{\mathrm{G}(\mathrm{z})}{\mathrm{F}(\mathrm{z})}=\frac{1}{1-\mathrm{z}-1} \tag{4.6.32}
\end{equation*}
$$

The transfer function of the analog part is found from 4.6.27 and Table 4.6.1.

$$
\begin{equation*}
E(z)=\frac{10 K}{1-z^{-1} e^{-2 T}} \tag{4.6.33}
\end{equation*}
$$

Given a sampling time of $100 \mu \mathrm{~s}$, we obtain

$$
\begin{equation*}
E(z)=\frac{10 K}{1-0.82 z^{-1}} \tag{4.6.34}
\end{equation*}
$$

Finally, to analyze the loop stability, we write the characteristic equation

$$
\begin{equation*}
1+0.005 \frac{1}{1-z^{-1}} \frac{10 K}{1-0.82 z^{-1}}=0 \tag{4.6.35}
\end{equation*}
$$

or

$$
\begin{equation*}
(1+0.05 K) z^{2}-1.82 z+0.82=0 \tag{4.6.36}
\end{equation*}
$$

We want to find the range of values of K for which the system is stable. Applying the transformation (4.6.26) to (4.6.36) we obtain

$$
\begin{equation*}
0.05 K w^{2}+(0.36+0.1 \mathrm{~K}) \mathrm{w}+3.24+0.05 \mathrm{~K}=0 \tag{4.6.37}
\end{equation*}
$$

Now we can apply the Routh-Hurwitz stability criterion to Eq. (4.6.37). Since


Fig. 4.6.7. Block diagram of system in example.
(4.6.37) is a second order equation, the condition for stability is that all the coefficients must have the same sign. This condition is met for all the positive values of $\mathbf{K}, \mathbf{0}<\boldsymbol{K}<^{\infty}$. We may therefore conclude that the system is stable for all the positive values of $K$.

## Low Speed Stability Analysis

In digital control systems where the position signal is quantized by a position sensor, such as optical encoders, the sampling interval of the digital controller equals the time interval between position pulses. As the motor velocity decreases, the encoder frequency decreases, resulting in longer time intervals between pulses. The longer sampling periods usually have adverse effects on the system stability. In most cases, if the sampling period increases above a given value, the system becomes unstable. In the following discussion we describe, by example, how to analyze the instability at low speeds.

Consider the system shown in Fig. 4.6.8. The digital controller generates its output according to the equation

$$
\begin{equation*}
g(k T)=g[(k-1) T]+2 f[(k-1) T] \tag{4.6.38}
\end{equation*}
$$

This corresponds to a digital transfer function of

$$
\begin{equation*}
D(z)=\frac{G(z)}{F(z)}=\frac{2}{z-1} \tag{4.6.39}
\end{equation*}
$$

The output is multiplied by an impulse function and later is applied to the analog element

$$
\begin{equation*}
H(s)=\frac{K}{s+10} \tag{4.6.40}
\end{equation*}
$$

The sampling period $\mathbf{T}$ may vary with the velocity. The objective is to determine how the parameters $\mathbf{T}$ and K affect the system stability.

A necessary assumption to be made is that the motor velocity is constant during $\delta(\mathbf{t})$


Fig. 4.6.8. Block diagram of system in example.
a sampling interval. This guarantees that the sampling frequency is constant, and allows the use of the $\mathbf{Z}$ transform method.

The digital transfer function of the impulse multiplier and the analog element can be found from Table 4.6.1.

$$
\begin{equation*}
E(z)=\frac{Y(z)}{G(z)}=\frac{K}{1-z^{-1} e^{-10 T}} \tag{4.6.41}
\end{equation*}
$$

Then the loop equation is

$$
\begin{equation*}
1+\frac{2}{z-1} \frac{K z}{z-e^{-10 T}}=0 \tag{4.6.42}
\end{equation*}
$$

or

$$
\begin{equation*}
z^{2}+z\left(-1-e^{-10 T}+2 K\right)+e^{-10 T}=0 \tag{4.6.42}
\end{equation*}
$$

Now apply the transformation (4.6.26) to Equation (4.6.42). The result is

$$
\begin{align*}
& 2 K w^{2}+\left(2-2 e^{-10 T}\right) w+\left(2-2 K+2 e^{-10 T}\right) \\
& =0 \tag{4.6.43}
\end{align*}
$$

For stability we require all the coefficients of (4.6.43) to have the same sign. For positive values of $K$, the first two coefficients are positive. Therefore, it is necessary and sufficient, for stability, to require

$$
\begin{equation*}
2-2 \mathrm{~K}+2 \mathrm{e}^{-10 \mathrm{~T}}>0 \tag{4.6.44}
\end{equation*}
$$

or

$$
\begin{equation*}
K<1+e^{-10 T} \tag{4.6.45}
\end{equation*}
$$

Equation 4.6.45 shows how the stability depends on K and T . At high sampling rates, as $\mathbf{T}$ approaches zero, the maximum allowed gain is 2 . For low speeds, with $\mathbf{T}$ approaching infinity, the maximum allowed gain is 1 .

In this simple example, the system is stable at all speeds. In most practical cases, the equations are more complex and the system becomes unstable at low velocities.

### 4.6.4 SPECIAL FEATURES

The implementation of control functions by digital controllers is done by programming the processor or by the use of look-up tables. The flexibility of these design methods enables the implementation of various control functions.

Digital compensations are designed by programming the processor to perform a difference equation. The coefficients of the difference equations determine the poles and zeros of the transfer function, as discussed previously.

Digital controllers can generate nonlinear functions such as squares, or velocitydependent gains. It also allows the addition of constraints to the control function. For example, it can control the position of the system while limiting the acceleration rate or the velocity to given values.

The design flexibility also allows different design approaches. Two such approaches are the feedforward and the adaptive control.

## Feedforward

Consider the system shown in Fig. 4.6.9.

Under normal conditions, the system will have a steady state error due to the finite gain of the loop. This characteristic is undesirable in position control systems where the position has to follow the input closely. In those cases one can add a feedforward function that generates a signal equal to the desired steady state error, allowing the position to equal the input.

The main advantage of the feedforward method is that the feedforward branch is not a part of the closed loop. Therefore, it does not affect the system stability, although it improves the performance.

## Adaptive Control

In order to understand this feature, we start with the fundamental question: Why do we need to close the control loop? We want the system to perform in spite of changes in the system parameters and the outside inputs, such as load. If all the parameters were fixed and known, open loop control could do just as well.

As the parameter changes become broader, there is a need for increased loop gains, which results in stability problems. Therefore, if we can narrow the range of parameter variations, we need less gain to obtain the same performance. The range of parameters uncertainty can be narrowed if we note that most parameters change slowly. This is due to temperature change


Fig. 4.6.9. Digital control system with feedforward.
or mechanical wear. If we can measure and determine the slowly changing variables, we can reduce the range of the parameter uncertainty. The measurement of parameter values, and compensation for the changes is part of the adaptive control feature.

System parameters can change in different ways, and the parameter estimation has to be designed accordingly. For example, if the only parameter that varies is the inertial load or the friction, this parameter can be estimated by determining the change in motor velocity due to known current. However, if several parameters may change simultaneously, the parameter estimation process requires a more sophisticated procedure.

### 4.6.5. PHASE LOCKED SYSTEMS

As the needs for precise velocity control systems increase, and with the availability of suitable integrated circuits, phase-locked servo loops are becoming increasingly popular. They exhibit excellent speed regulation and inherent insensitivity to parameter changes.

The basic phase-locked loop and its components are shown in Fig. 4.6.10. When the system is "phase-locked", the frequencies of the command and feedback signals become identical, and if the frequency of the command signal is constant, it produces a constant motor velocity.

As long as the system is "phase-locked", the motor velocity follows the frequency
of the command independently of motor and amplifier parameter variations. This guarantees a motor speed regulation which is as good as the stability of the command frequency.

Note that although phase-locked servo systems (PLS) are commonly used to control velocity, they really are position control systems in which the position control results in velocity control.

The early PLS used analog devices for position sensing and for phase detection, therefore their model and analysis were based on analog signal methods. Most of the literature on PLS, therefore, describes them by continuous models.

In recent years, with the availability of digital circuits for phase detection, the use of digital elements in the loop has increased and consequently most PLS have become digital. However, since the encoder frequency is usually very high as compared with the loop bandwidth, the digital phase detector may be approximated by an analog device, resulting in an analog model. Therefore, most of the results of analog PLS apply equally to digital PLS.

The main differences between the two loop types are in the model of the phase detector and in the behavior at low velocities. For a smooth presentation, we start with the description of the analog PLS, and later will discuss the outstanding features of the digital PLS. We start by describing the system components and their operations.


### 4.6.6. SYSTEM COMPONENTS

The phase-locked servo loop includes the following components:

1. Phase comparator
2. Low-pass filter
3. Amplifier
4. Motor
5. Encoder (optical tachometer)

The operation of these components is discussed below.

## Phase Comparator

The purpose of the phase comparator is to detect differences between the phases of the command signal and the feedback signal, and produce an error signal to indicate them.

Some comparators are designed to detect frequency differences as well as phase differences between the command and feedback signals, and produce an output
voltage which is a function of the phase error and the frequency error. Such detectors are called phase-frequency comparators.

In the following analysis, we derive the mathematical equations for the most simple phase comparator, the multiplier. For the simplicity of derivation, it is assumed that both the command signal and the feedback signal are sinusoidal. This is only a simplifying assumption and the results are valid for square-wave signals as well. Let the command signal and the feedback signal be $S_{1}(t)$ and $S_{2}(t)$, respectively, and denote the multiplier gain by $\mathrm{K}_{\mathrm{m}}$. The multiplier output is

$$
\begin{equation*}
V_{c}=K_{m} S_{1} S_{2} \tag{4.6.46}
\end{equation*}
$$

Since the command signal is sinusoidal, it can be written as

$$
\begin{equation*}
\mathrm{S}_{1}=-\mathrm{V}_{\mathrm{s}} \cos \theta_{\mathrm{i}} \tag{4.6.47}
\end{equation*}
$$

Let $\theta_{\mathrm{m}}$ be the angular position of the motor shaft and define the electric angle of the encoder, $\theta_{0}$, as

$$
\begin{equation*}
\theta_{0}=n \theta_{m} \tag{4.6.48}
\end{equation*}
$$

where $\mathbf{n}$ is the number of pulses generated by the encoder each revolution. The feedback signal, $\mathbf{S}_{2}$, may be written as

$$
\begin{equation*}
\mathrm{S}_{2}=\mathrm{V}_{\mathrm{o}} \sin \theta_{0} \tag{4.6.49}
\end{equation*}
$$

The comparator output voltage, $\mathbf{V}_{\mathbf{c}}$, becomes

$$
\begin{equation*}
\mathrm{V}_{\mathrm{c}}=-\mathrm{K}_{\mathrm{m}} \mathrm{~V}_{\mathrm{s}} \mathrm{~V}_{\mathrm{o}} \cos \theta_{\mathrm{i}} \sin \theta_{\mathrm{o}} \tag{4.6.50}
\end{equation*}
$$

This can be written as
$\mathrm{V}_{\mathrm{c}}=\frac{\mathrm{K}_{\mathrm{m}} \mathrm{V}_{\mathrm{s}} \mathrm{V}_{\mathrm{o}}}{2}\left[\sin \left(\theta_{\mathrm{i}}-\theta_{0}\right)-\sin \left(\theta_{\mathrm{i}}+\theta_{\mathrm{o}}\right)\right]$

The voltage component proportional to $\sin \left(\theta_{\mathbf{i}}+\theta_{\mathbf{o}}\right)$ is sinusoidal with high frequency and will be blocked by the lowpass filter. Therefore, we may ignore it and consider only the voltage component proportional to $\boldsymbol{\operatorname { s i n }}\left(\theta_{\mathbf{i}}-\theta_{\mathbf{o}}\right)$. Also define the comparator gain, $\mathrm{K}_{\mathrm{p}}$, as

$$
\begin{equation*}
K_{p}=\frac{K_{m} V_{s} V_{o}}{2} \tag{4.6.52}
\end{equation*}
$$

Then the effective comparator output voltage is

$$
\begin{equation*}
V_{c}=K_{p} \sin \left(\theta_{i}-\theta_{0}\right) \tag{4.6.53}
\end{equation*}
$$

Define the phase error, $\theta_{e}$, as

$$
\begin{equation*}
\theta_{\mathrm{e}}=\theta_{\mathbf{i}}-\theta_{\mathbf{o}} \tag{4.6.54}
\end{equation*}
$$

and note that when $\theta_{\mathrm{e}}$ is small, (4.6.53) may be approximated by

$$
\begin{equation*}
V_{c}=K_{p}\left(\theta_{i}-\theta_{o}\right)=K_{p} \theta_{e} \tag{4.6.55}
\end{equation*}
$$

Equation 4.6 .55 indicates that for small phase errors, the comparator output is proportional to the phase difference. This is illustrated in Fig. 4.6.11.


Fig. 4.6.11. Linearized model for phase comparator.
The model (4.6.55) was developed for a multiplier, but it is valid for all the phase comparators, as long as the phase error, $\theta_{e}$, is small. In cases where phase-frequency comparators are used the output voltage may be approximated as the sum of two components, the phase error voltage and the frequency error voltage:

$$
\begin{equation*}
v_{c}=K_{p} \theta_{e}+K_{v} \frac{d \theta_{e}}{d t} \tag{4.6.56}
\end{equation*}
$$

When Laplace transformation is used, this becomes

$$
\begin{equation*}
V_{c}(s)=\left[s K_{v}+K_{p}\right] \theta_{e}(s) \tag{4.6.57}
\end{equation*}
$$

A block diagram model for the phasefrequency comparator is illustrated in Fig. 4.6.12.


Fig. 4.6.12. Linearized model for phase-frequency comparator. Note that $K_{p}$ gives the phase difference, whereas $K_{v}$ indicates frequency differences.

## Low-Pass Filter

The low-pass filter is needed to block the high frequency components of the comparator output. A single-pole filter may be sufficient for attenuation, but the final selection is made on the basis of loop design. The transfer function of the filter is denoted by $\mathrm{F}(\mathrm{s})$.

## Amplifier

The amplifier is needed to raise the power level of the error signal in order to drive the motor. The amplifier transfer function, $\mathrm{H}(\mathrm{s})$, may be selected to provide additional compensation if needed.

## Motor

The transfer function of the motor was discussed in detail in section 4.2. In the following analysis, it is denoted by G(s).

## Encoder

The encoder, or optical tachometer, is a nonlinear device which generates $\mathbf{n}$ sinusoi-
dal pulses for each revolution of the motor shaft. Thus the encoder output, $\mathrm{S}_{2}(\mathrm{t})$, is

$$
\begin{equation*}
\mathbf{S}_{2}(\mathrm{t})=\mathrm{V}_{\mathrm{o}} \sin n \theta_{\mathrm{m}}=\mathrm{V}_{\mathrm{o}} \sin \theta_{\mathrm{o}} \tag{4.6.58}
\end{equation*}
$$

However, since the comparator model is linearized, the encoder may be represented as a block with constant gain of $\mathbf{n}$ (Fig. 4.6.13).


Fig. 4.6.13. Block diagram model for the encoder, relating the mechanical shaft position $\theta_{\mathrm{m}}$ to the encoder electrical angle $\theta_{0}$.

Now we are ready to construct a detailed block diagram for the phase-locked servo system. This is shown in Fig. 4.6.14.

### 4.6.7. SYSTEM DESIGN AND STABILITY

When we examine the open-loop transfer function of the system in Fig. 4.6.14, we find it to be

$$
\begin{equation*}
L(s)=n\left(s K_{v}+K_{p}\right) F(s) G(s) H(s) \frac{1}{s} \tag{4.6.59}
\end{equation*}
$$

It can be seen that the term $1 / \mathrm{s}$ introduces $90^{\circ}$ phase shift to the loop. Additional lag is produced by motor and the filter. If we couple this with the fact that the open-loop gain is very high ( n is between 500 and 2000), it is apparent that the system will have stability problems, unless carefully designed.


Fig. 4.16.14. Block diagram of phase-locked servo system.

The design is made more difficult by the fact that the filter $F(s)$ is needed, and any lead network on the comparator will cancel the filtering effect. Consequently, all efforts are directed at determining compensations which will reduce the lag and loop phase shift at frequencies about the break frequency. This can be achieved by lead-lag, or similar type compensations.

Next, we want to examine the effect of a lead-lag compensation network on the loop stability.

Consider a phase-locked loop where the phase detector has a gain $\mathrm{K}_{\mathrm{p}}$. The low-pass filter has a single pole at -1000 rad/s, and a transfer function

$$
\begin{equation*}
F(s)=\frac{1000}{s+1000} \tag{4.6.60}
\end{equation*}
$$

The amplifier is assumed to have a constant gain

$$
\begin{equation*}
H(s)=5 \tag{4.6.61}
\end{equation*}
$$

The motor transfer function is

$$
\begin{equation*}
G(s)=\frac{1000}{s+100} \tag{4.6.62}
\end{equation*}
$$

and the encoder line density is

$$
\begin{equation*}
n=500 \tag{4.6.63}
\end{equation*}
$$

The corresponding open-loop transfer function is

$$
\begin{equation*}
L(s)=\frac{2.5 \times 10^{9} K_{p}}{s(s+100)(s+1000)} \tag{4.6.64}
\end{equation*}
$$

and the corresponding root-locus diagram is shown in Fig. 4.6.15.


Fig. 4.6.15. Root locus diagram for the system of (4.6.64).

In order to determine the limit on the comparator gain, solve the equation

$$
\begin{equation*}
1+L(j \omega)=0 \tag{4.6.65}
\end{equation*}
$$

or

$$
1+\frac{2.5 \times 10^{9} K_{p}}{j \omega(\mathrm{j} \omega+100)(\mathrm{j} \omega+1000)}=0
$$

$$
\begin{equation*}
\mathrm{j} \omega(\mathrm{j} \omega+100)(\mathrm{j} \omega+1000)+2.5 \times 10^{9} \mathrm{~K}_{\mathrm{p}}=0 \tag{4.6.66}
\end{equation*}
$$

Separating the real and imaginary parts of (4.6.67) and solving for $\omega^{2}$ and $K_{p}$ yields

$$
\begin{align*}
& \omega^{2}=10^{5} \\
& K_{p}=0.044 \tag{4.6.68}
\end{align*}
$$

Thus, the comparator gain has to be limited below $0.044 \mathrm{~V} / \mathrm{rad}$ to guarantee stability. Next, suppose that we add a lead-lag compensation to the amplifier by introducing a zero at $\mathbf{- 1 0 0} \mathrm{rad} / \mathrm{s}$ and a pole at $-5000 \mathrm{rad} / \mathrm{s}$. The modified amplifier transfer function becomes

$$
\begin{equation*}
H_{m}(s)=\frac{250(s+100)}{(s+5000)} \tag{4.6.69}
\end{equation*}
$$

Note that at low frequencies, the amplifier gain remains unchanged.

The modified open-loop transfer function becomes

$$
\begin{equation*}
L_{m}(s)=\frac{1.25 \times 10^{11} K_{p}}{s(s+1000)(s+5000)} \tag{4.6.70}
\end{equation*}
$$

The maximum gain for the modified system can be determined by the same procedure as before. When this is done, we find that the maximum comparator gain and the corresponding frequency are

$$
\begin{align*}
& \omega^{2}=5 \times 10^{6} \\
& K_{p}=0.24 \mathrm{~V} / \mathrm{rad} \tag{4.6.71}
\end{align*}
$$

Thus, by introducing the compensation, one can increase the gain by a factor of 5.5

### 4.6.8. SPECIAL CHARACHERISTICS

In the previous analysis the model of the phase-locked servo system was linearized and the loop was treated as a regular position control system. While this analysis is valid, there are some special characteristics that result from the nonlinear nature of phase-locked loops, and these are described below.

## Velocity Acquisition and Locking Range

In order for the system to remain locked, it has to satisfy the following conditions:
a) The system must be stable.
b) The frequency of the effective error signal, $\mathrm{V}_{\mathbf{c}}$, should be within the bandwidth of the system.

As long as the two conditions are satisfied, the system can correct phase errors and remain locked. On the other hand, if the frequency of the error signal is higher than the system bandwidth, the loop cannot respond fast enough and the system will "lose lock".

The frequency of the error signal is proportional to the difference between the motor velocity and the desired velocity. Large differences between the two may occur during start-up condition when the motor starts from rest, or due to large speed disturbances.

To illustrate this point consider a stable phase-locked servo system where the encoder line density is $\mathbf{n}=1000$. Let the frequency of the input signal be $f_{i}=20 \mathrm{kHz}$, and suppose that the system bandwidth is 500 Hz . The system will remain locked as long as the frequency of the feedback signal is between 19500 and 20500 Hz . With the given encoder, this corresponds to a motor velocity in the range of $19.5-20.5 \mathrm{r} / \mathrm{s}$, or 1170-1230 rpm.

In other words, as long as the motor velocity is within the range of $1200 \pm 30 \mathrm{rpm}$, locking is guaranteed. Any change in velocity beyond those limits will result in loss of lock.

In order to guarantee velocity acquisition and prevent loss of lock, one can use a phase-frequency comparator, where the velocity error signal may be used to accelerate the motor until lock is achieved. This enables the system to acquire the desired velocity; however, the acquisition range is limited, as explained by the following example.

A motor with an encoder of line density $\mathrm{n}=1000$ is required to rotate at a velocity of 2400 rpm in CW direction. A stable
phase-locked system is used where a positive error signal accelerates the motor in the CW direction. The frequency of the input signal is $f_{i}=40000 \mathrm{~Hz}$, and the feedback frequency, $f_{0}$, is proportional to the velocity. Furthermore, note that the velocity error of the frequency comparator is proportional to $\left(f_{i}-f_{0}\right)$. Thus,

$$
\begin{equation*}
V_{c}=K_{c}\left(f_{i}-f_{o}\right) \tag{4.6.72}
\end{equation*}
$$

Next, consider various initial velocities, and the system response.
a) Initial velocity above $\mathbf{2 4 0 0} \mathbf{r p m}$ in CW direction. Here the error signal is negative and the motor will decelerate until the required velocity is achieved.
b) Initial velocity below 2400 rpm in either direction. Here the error signal is positive and the motor is accelerated in CW direction to the required velocity.
c) Initial velocity above 2400 rpm in CCW direction. Now the error signal is negative, which accelerates the motor further in CCW direction until the amplifier is saturated. Under this condition, velocity acquisition is not possible.

In conclusion, phase-freqeuncy comparator will accelerate the motor to a desired velcoity, $\omega_{d}$, if the initial velcoity, $\omega_{m}$, is

$$
\begin{equation*}
\omega_{m}>-\omega_{d} \tag{4.6.73}
\end{equation*}
$$

Otherwise, the motor will run away in the opposite direction.

There are several ways to solve this problem. The most simple one is to use a unidirectional amplifier if possible. When the motor is required to run in both directions, it is advisable to use an optical encoder with two output channels in quadrature. The phase relationship between the two output channels indicates the direction of rotation.

Further discussion on the modeling and design of phase-frequency detectors can be found in [4-11] and [4-12].

## Speed Variations

Phase-locked systems are used primarily for velocity control because of the high accuracy and regulation they can provide. It is therefore important to investigate the sources of the velocity errors and the methods for reducing them. The most common sources of velocity disturbances are listed below.

1. Variations in input frequency - As the frequency of the command signal varies with time or temperature, the system will follow the command frequency and produce a velocity error. Clearly, the speed regulation of the system cannot be better than that of the command signal and a stable oscillator is needed for good regulation.
2. Load variations - Variation in load torque, $\mathbf{T}_{\mathbf{L}}$, will produce speed distur-
bances. These variations will be minimized by increasing the loop gain and bandwidth. An alternative method for reducing velocity disturbances is to add a balanced inertial load to the system which attenuates the effect of the load changes.
3. Motor torque disturbances - The torque produced in the motor during rotation is not constant, but it varies with the angular position. The torque ripple is equivalent to load disturbance and results in velocity variations. In order to minimize those, one can select a motor with low torque ripple, in addition to increasing the loop gain or using an additional inertial load.
4. Encoder-generated errors - In the previous analysis it was assumed that the encoder is an ideal device with an output

$$
\begin{equation*}
\mathrm{S}_{2}=\mathrm{V}_{\mathrm{o}} \sin \theta_{\mathrm{o}} \tag{4.6.74}
\end{equation*}
$$

In reality, the encoder has some error and its output may be written as

$$
\begin{equation*}
\mathrm{S}_{2 \mathrm{a}}=\mathrm{V}_{\mathrm{o}} \sin \theta_{\mathrm{a}} \tag{4.6.75}
\end{equation*}
$$

where $\theta_{a^{\prime}}$, the actual electrical angle, is given by

$$
\begin{equation*}
\theta_{\mathrm{a}}=\theta_{\mathrm{o}}+\theta_{L}+\theta_{\mathrm{h}} \tag{4.6.76}
\end{equation*}
$$

${ }^{\theta} L$ represents the low-frequency phase errors which result from mounting errors. This can be approximated by

$$
\begin{equation*}
\theta_{L}=K \sin \theta_{m} \tag{4.6.77}
\end{equation*}
$$

$\theta_{\mathrm{h}}$, on the other hand, results from errors in encoder mask, and therefore will have high-frequency elements.

When the actual feedback signal, $\mathbf{S}_{\mathbf{2 a}}$, is fed back to the phase comparator, the output voltage becomes

$$
\begin{align*}
\mathbf{V}_{\mathbf{c}} & =\mathrm{K}_{\mathrm{p}}\left[\theta_{\mathrm{i}}-\theta_{\mathbf{a}}\right] \\
& =\mathrm{K}_{\mathrm{p}}\left[\theta_{\mathbf{i}}-\theta_{\mathbf{o}}-\theta_{L}-\theta_{\mathbf{h}}\right] \tag{4.6.78}
\end{align*}
$$

$\theta_{h}$ will be blocked by the filter and hence may be ignored. $\theta_{L}$, which may be within the system bandwidth, will pass through the filter as an error, and will try to correct the system phase accordingly. Thus, the phase-locked loop will attempt to correct errors
which are produced in the encoder, and when the frequency of those errors is low, they result in speed disturbances.
5. Low speed instability - PLS with digital phase detectors often experience velocity disturbances at low velocities. These disturbances get worse as the velcoity decreases, and eventually the whole loop becomes unstable.

The reason for this phenomenon is that the position sampling rate in those systems depends on the motor velocity. As the sampling period increases at lower speeds, the system tends to become less stable and eventually it becomes unstable. This subject was previously discussed in this section 4.6.3. Further discussion can be found in [4-14].

### 4.7. HOW TO MAKE SYSTEMS WORK

Some system requirements may be contradictory to the basic requirement of stability, and compensation must be added. There is no simple rule for compensation, but some worthwhile ideas will be discussed.

The designer of a servo system usually desires high gain in the open-loop transfer function, as this reduces the sensitivity of the system to disturbances and parameter variations, and widens the system bandwidth. For example, a position control system with high open-loop gain will appear as a "stiff" system that can follow a position command despite disturbing torques.

On the other hand, as gain is increased, the system tends to become unstable. In fact, all practical servo systems become unstable if their open-loop gain is increased beyond a certain limit. This indicates that the plurality of the number of poles over the number of zeros of the characteristic equation is at least three. Then, as gain is increased, at least one of the poles moves to the right half-plane, as follows from the asymptotes rule of the root loci.

Thus, the requirements for high gain and wide bandwidth are contradictory to the basic requirement for stability. To overcome this difficulty, compensation networks are used to change the open-loop transfer function so that higher gain can be achieved under stable conditions.

There is no simple selection rule for compensation networks. Instead, a network has to be tailored for a given case after careful analysis. There are, however, some general ideas which are worth discussing here.

In general, servo system stability is enhanced if the phase shift, or the lag, in the open-loop transfer function is reduced. Thus, phase shift should be reduced whenever possible. The first major source of phase shift is the motor; it is possible to reduce system phase shift by selecting a motor with a short mechanical time constant. This possibility should be considered, however, only if gain or bandwidth requirements are extremely high.

The other major source of lag is the amplifier. It is easy to stabilize an amplifier by using a large capacitor in the feedback, but such an amplifier will cause stability problems when it is used in a closed loop. Therefore, as a general rule, capacitive feedback in the amplifier should be avoided as much as possible. Actually, an ideal amplifier should have a constant gain and no phase shift over a wide bandwidth. ElectroCraft's Servo Control Amplifiers (SCA) can maintain a constant gain up to a frequency of 20 kHz , so for the purpose of analysis they can be considered as constant gain devices.

Thus, in general, by reducing the phase shift, a system can stand higher gain without disturbing its stability. In cases where further reduction in phase shift is needed,
a lead-lag compensation network can be used with the transfer function:

$$
\begin{equation*}
H(s)=K \frac{s-a}{s-b} \tag{4.7.1}
\end{equation*}
$$

where both $\mathbf{a}$ and $\mathbf{b}$ are positive and:

$$
\begin{equation*}
\mathbf{b}>\mathbf{a} \tag{4.7.2}
\end{equation*}
$$

Such a network has a zero at the frequency $\omega=\mathrm{a}$ and a pole at $\omega=\mathrm{b}$. This combination results in some phase lead, whose amplitude depends on the parameters a and b. Such a compensation network can be constructed using either active or passive components. The two possible circuits are shown in Figs. 4.7.1 and 4.7.2. The two transfer functions are of the lead-lag type, and the circuit components are selected to obtain the right values of $\mathbf{a}$ and $\mathbf{b}$.


Fig. 4.7.1. Compensation network using an active component (amplifier).


Fig. 4.7.2. Compensation network using passive components.

In some cases, the desired velocity profile is to have a constant acceleration and
deceleration. While this is extremely difficult to achieve with linear compensation networks, it can be obtained easily using some nonlinear techniques. The simplest device to be used here is a current limiter. Thus, amplifier output current is limited to a certain adjustable value. During acceleration and deceleration the amplifier current saturates, resulting in constant velocity change. Because of the simplicity and usefulness of the current limiter, all ElectroCraft SCA amplifiers are equipped with this feature, which can be used or disconnected according to individual application requirements.

In some applications, it is desired to follow the input command with zero error so that the system output is less sensitive to parameter variations. This requires the use of an additional integrator in the loop, which increases the DC gain to infinity. However, an additional integrator adds to the phase shift and, hence, causes stability problems. To design an integrator without the phase shift, a lag-lead compensation network can be used. The transfer function of the network is:

$$
\begin{equation*}
H(s)=K \frac{s-a}{s} \tag{4.7.3}
\end{equation*}
$$

Thus, the network has a pole at $\mathbf{s}=\mathbf{0}$ and a zero at the frequency $s=a$. The compensation network causes a phase shift which is noticed at the low frequencies. However, beyond the frequency of $s=3 a$ the effect of the network is minor, and can be ignored. A possible network is shown in Fig. 4.7.3, having a transfer function of:


Fig. 4.7.3. Example of a compensation network.

$$
\begin{equation*}
H(s)=\frac{V_{0}(s)}{V_{i}(s)}=-\frac{1+s R C_{i}}{s R C_{f}} \tag{4.7.4}
\end{equation*}
$$

When a system has to operate in several control modes, it is inefficient to design one compensation network for all the modes. Instead, it will be more desirable to design the best compensation for each mode and switch between them at the appropriate time.

For example, a system that has to run at a specified velocity and then stop exactly at a certain position is a two control mode system. It operates on either velocity or position control; the system, as shown in Fig. 4.7.4, is suggested for variable compensation.


Fig. 4.7.4. Hybrid control system.
The position sensor, $\mathbf{P}$, may be a potentiometer or a digital encoder, and the compensation network may use analog circuits, digital circuits, or both. The mode selector
switch will be in the desired position according to whether the velocity or the position are to be controlled. For example, suppose that the desired velocity profile is trapezoidal, as shown in Fig. 4.7.5, and it is required that the motor will stop after rotating through a given angle $\theta$.

During this time interval between 0 and $\mathrm{t}_{2}$, the system will be in velocity control mode, and between $t_{2}$ and $t_{3}$ it operates in position control mode.


Fig. 4.7.5. A trapezoidal velocity profile.


Fig. 4.7.6. System with unstable response at higher values of gain.

A system which is difficult to compensate is one where the open-loop transfer function has a pair of complex poles which are close to the imaginary axis. A common
source for such a pair is the torsional resonance between the motor and the load or in the load. The difficulty can be seen easily if we observe the root locus curves for this system in Fig. 4.7.6. Assuming that the system has at least two more open-loop poles (from motor and amplifier), the root loci branches will tend to move into the right half-plane, causing underdamped response when the gain is low and unstable response for a higher gain.

Such a response is not acceptable, and the system has to be compensated to divert the root locus branches which originate at the complex poles and move them to the left, away from the imaginary axis. Since open-
loop zeros have the tendency to attract root locus branches, a good approach will be to use some zeros in the compensation network and locate them in the appropriate place for best results. As the detailed design of the compensation network depends on the location of all poles and zeros of the open-loop transfer function, an analysis should be done to locate them. This can be done by measuring exactly the frequency response of the system by recording amplitude of gain and phase shift.

On the basis of this information, the structure of the compensation network is determined, and a realizing circuit can be synthesized.

### 4.8. OPTIMIZATION

### 4.8.1. INTRODUCTION

In recent years the automation industries have pressed for increased performance (and at the same time higher reliability) in automatically controlled processes and equipment. As a result, in many instances mechanical systems are being replaced by all-electronic servo controls driving a highly responsive electromechanical transducer. The field of incremental motion control by this method is relatively new; and, therefore, many potential pitfalls await the system designer. This discussion is intended to provide insights into the selection of velocity profiles and coupling ratios so that these system considerations may aid in realizing an optimum system design.

The incremental motion system to be analyzed consists of a load with moment of inertia $J_{L}$ which is to be accelerated, rotated through an angle $\theta$, decelerated and brought to rest, all in time $\mathrm{t}_{\mathbf{c}}$. A retarding load torque, $T_{L}$, will be considered constant. The driving torque will be provided by a DC servomotor having a torque constant $\mathrm{K}_{\mathrm{T}}$, an armature resistance $\mathbf{R}$, and a moment of inertia $\mathbf{J}_{\mathbf{m}}$.

### 4.8.2. VELOCITY PROFILE OPTIMIZATION

The velocity profile (i.e., the motor and load angular velocity as a function of time) could be optimized around minimum peak speed or minimum peak current or any of several parameters. In general, however, the
limiting factor in the performance of the type of incremental motion control system we are analyzing is motor armature heat dissipation. Therefore, it is the parameter we will minimize.

The basic equations of this incremental motion are:

$$
\begin{equation*}
\mathrm{J} \frac{\mathrm{~d} \omega}{\mathrm{dt}}+\mathrm{T}_{\mathrm{L}}=\mathrm{K}_{\mathrm{T}} \mathrm{I}_{\mathrm{a}} \tag{4.8.1}
\end{equation*}
$$

where $\mathbf{J}=\mathbf{J}_{\mathrm{m}}+\mathrm{J}_{\mathrm{L}}$, and

$$
\begin{equation*}
\int_{0}^{t_{c}} \omega(t) d t=\theta \tag{4.8.2}
\end{equation*}
$$

The resulting energy dissipation in the motor armature per step, $\mathrm{W}_{\mathbf{c}}$, is given by:

$$
\begin{equation*}
W_{c}=R \int_{0}^{t_{c}} I_{a}^{2}(t) d t \tag{4.8.3}
\end{equation*}
$$

The problem now reduces to the determination of the optimum velocity profile $\omega(\mathrm{t})$, which minimizes $W_{c}$ and still satisfies (4.8.1) and (4.8.2). We shall discuss three types of velocity profiles (parabolic, triangular, and trapezoidal) and determine energy dissipation for each.

## Parabolic Velocity Profile

The solution to this problem results in two conclusions:

1. The constant opposing torque $T_{L}$ adds a fixed energy dissipation:
$\frac{R}{K_{T}^{2}} T_{L}^{2} t_{c}$
which is independent of velocity profile.
2. The optimal velocity profile is parabolic and is given by the equation:

$$
\begin{equation*}
\omega_{o}(t)=6 \theta \frac{t_{c}-t}{t_{c}^{3}} t \tag{4.8.4}
\end{equation*}
$$

A graph of this profile is shown in Fig. 4.8.1. The resulting energy dissipation per step becomes:

$$
\begin{equation*}
W_{c o}=\frac{R}{K_{T}^{2}}\left[12 \frac{J^{2} \theta^{2}}{t_{c}^{3}}+T_{L}^{2} t_{c}\right] \tag{4.8.5}
\end{equation*}
$$



Fig. 4.8.1. Optimized velocity profile.

## Triangular Velocity Profile

As a result of limitations in practical control system complexity, velocity profiles other then optimum are frequently used. In the triangular velocity profile the load is accelerated at a fixed rate and then decelerated


Fig. 4.8.2. Triangular velocity profile.
at the same rate, as shown in Fig. 4.8.2. The energy dissipation per step is:

$$
\begin{equation*}
W_{c}=\frac{R}{K_{T}^{2}}\left[16 \frac{J^{2} \theta^{2}}{t_{c}^{3}}+T_{L}^{2} t_{c}\right] \tag{4.8.6}
\end{equation*}
$$



Fig. 4.8.3. Trapezoidal velocity profile.

## Trapezoidal Velocity Profile

The trapezoidal profile is divided into three parts: acceleration, run, and deceleration as shown in Fig. 4.8.3. Frequently there is a feeling among engineers that the triangular velocity profile is more efficient than the trapezoidal, as the latter's run time seems wasted. The results show, however, that for a trapezoidal velocity profile in which all three parts are of equal time interval, the energy dissipation per step is:

$$
\begin{equation*}
W_{c}=\frac{R}{K_{T}^{2}}\left[13.5 \frac{J^{2} \theta^{2}}{t_{c}^{3}}+T_{L}^{2} t_{c}\right] \tag{4.8.7}
\end{equation*}
$$

Again, if we assume $T_{L}=0$, the energy dissipation in the motor armature is $12.5 \%$ higher than in the case of a parabolic velocity profile, but $20.83 \%$ less than in the case of a triangular velocity profile. For convenience at a later point, let us now define $\eta$ as the velocity profile efficiency when $\mathrm{T}_{\mathrm{L}}=0$ :

$$
\begin{equation*}
\eta=\frac{W_{c o}}{W_{c}} @ T_{L}=0 \tag{4.8.8}
\end{equation*}
$$

Then the energy dissipation per step equals:

$$
\begin{equation*}
\mathrm{W}_{\mathrm{c}}=\frac{\mathrm{R}}{\mathrm{~K}_{\mathrm{T}}^{2}}\left[\frac{12}{\eta} \frac{\mathrm{~J}^{2} \theta^{2}}{\mathrm{t}_{\mathrm{c}}^{3}}+\mathrm{T}_{\mathrm{L}}^{2} \mathrm{t}_{\mathrm{c}}\right] \tag{4.8.9}
\end{equation*}
$$

Thus, $\eta=1$ for the parabolic velocity profile; $\eta=0.75$ for the triangular velocity profile; and $\eta=0.889$ for the trapezoidal velocity profile.

### 4.8.3. COUPLING RATIO OPTIMIZATION

A second important system parameter affecting motor armature power dissipation is the coupling ratio between the motor and the load. The term coupling ratio is loosely defined as the angular movement of the motor compared to the movement of the load. For a system in which the load is rotated this would be more commonly called the gear ratio. For a system in which
the load is linearly translated, the coupling ratio is the motor rotation for one unit of linear motion (i.e., it is expressed in units of $[\mathrm{rad} / \mathrm{m}]$ or $[\mathrm{rad} / \mathrm{in}])$. Three methods of coupling will be considered; (1) the gear transmission, (2) the belt and pulley, and (3) the lead screw. The objective will be to determine the coupling ratio in which the motor armature energy dissipation is minimized. For the sake of simplicity it will be assumed that total load moment of inertia is fixed and does not change with coupling ratio and that all mechanical losses are contained in the $\mathrm{T}_{\mathrm{L}}$ term.


Fig. 4.8.4. Gear coupled system.

## Gear Ratio Selection

In a coupling system such as the one shown in Fig. 4.8.4 using a gear ratio of N:1, the following relationships reflect all parameters to the motor shaft:

$$
\begin{align*}
\theta & =N \theta_{L}  \tag{4.8.10}\\
J & =J_{m}+\frac{1}{N^{2}} J_{L}  \tag{4.8.11}\\
T_{L}^{\prime} & =\frac{1}{N} T_{L} \tag{4.8.12}
\end{align*}
$$

The equation (4.8.9) for energy dissipation, derived from (4.8.5) and (4.8.8) therefore becomes:

$$
\begin{align*}
\mathrm{W}_{\mathrm{c}} & =\frac{\mathrm{R}}{\mathrm{~K}_{\mathrm{T}}^{2}}\left[\frac{12}{\eta} \frac{\left(\mathrm{~J}_{\mathrm{m}}+\frac{1}{\mathrm{~N}^{2}} \mathrm{~J}_{\mathrm{L}}\right)^{2} \mathrm{~N}^{2} \theta_{\mathrm{L}}^{2}}{\mathrm{t}_{\mathrm{c}}^{3}}+\right. \\
& \left.+\frac{\mathrm{T}_{\mathrm{L}}^{2}}{\mathrm{~N}^{2}} \mathrm{t}_{\mathrm{c}}\right] \tag{4.8.13}
\end{align*}
$$

For convenience now define:

$$
\begin{equation*}
\gamma=\frac{\eta}{12}\left[\frac{\mathrm{~T}_{\mathrm{L}} \mathrm{t}_{\mathrm{c}}^{2}}{\theta_{\mathrm{L}} \mathrm{~J}_{\mathrm{L}}}\right]^{2} \tag{4.8.14}
\end{equation*}
$$

Substituting into (4.8.13) and rearranging gives:

$$
\begin{equation*}
\mathrm{W}_{\mathrm{c}}=\frac{\mathrm{R}}{\mathrm{~K}_{\mathrm{T}}^{2}} \frac{12}{\eta} \frac{\theta_{\mathrm{L}}^{2} \mathrm{~J}_{\mathrm{L}}^{2}}{\mathrm{t}_{\mathrm{c}}^{3}}\left[\mathrm{~N}^{2}\left(\frac{\mathrm{~J}_{\mathrm{m}}}{\mathrm{~J}_{\mathrm{L}}}+\frac{1}{\mathrm{~N}^{2}}\right)^{2}+\frac{\gamma}{\mathrm{N}^{2}}\right] \tag{4.8.15}
\end{equation*}
$$

Performing the differentiation of the part of (4.8.15) in brackets, i.e.

$$
\frac{d}{d\left(N^{2}\right)}[\cdots \cdots]=0
$$

we obtain the gear ratio $N_{0}$ which minimizes $W_{c}$ :

$$
\begin{equation*}
N_{o}^{2}=\frac{J_{L}}{J_{m}} \sqrt{1+\gamma} \tag{4.8.16}
\end{equation*}
$$

If there were no load opposing torque, $\mathrm{T}_{\mathrm{L}}=0$, then $\gamma=0$ and (4.8.16) becomes:

$$
\begin{equation*}
N_{o}=\sqrt{\frac{J_{L}}{J_{m}}} \tag{4.8.17}
\end{equation*}
$$

This result is known as an inertia match since the motor moment of inertia equals the reflected load moment of inertia, $J_{m}=\frac{J_{L}}{N^{2}}$ In cases where for practical reasons a nonoptimum gear ratio must be used it would be of interest to examine the resultant increase in armature energy dissipation. Therefore, let

$$
\begin{align*}
\rho=\frac{W_{c}(N)}{W_{c}\left(N_{o}\right)} & =\frac{N^{2}\left(J_{m}+\frac{1}{N^{2}} J_{L}\right)^{2}}{N_{o}^{2}\left(J_{m}+\frac{1}{N_{o}^{2}} J_{L}\right)^{2}}= \\
& =\frac{1}{4}\left(\frac{N}{N_{o}}+\frac{N_{o}}{N}\right)^{2} \tag{4.8.18}
\end{align*}
$$

The energy ratio $\rho$ increases as shown in Fig. 4.8 .5 if the gear ratio changes from optimum value. It can be seen that a small deviation from optimum is not critical, however, as the deviation increases the penalty becomes increasingly severe. If we deviate by $\pm 10 \%$ the energy increases by less than $1 \%$, but if we deviate by a factor of 2 the energy increases by $56 \%$.


Fig. 4.8.5. Energy dissipation increase due to nonoptimum gear ratio.

## Belt-Pulley Drive

The belt-pulley drive is shown in Fig. 4.8.6, and consists of a motor turning a pulley which pulls a belt to which the load is attached; thus converting rotary motion into translation. The coupling ratio in this case is the reciprocal of the radius $r$ of the pulley on the motor. The load mass, $m$, is to be moved a distance $\mathbf{x}$ in one step, i.e. in time $t_{c}$. The constant opposing force is $F$. Thus:

$$
\begin{align*}
& \theta=\frac{x}{r}  \tag{4.8.19}\\
& J=J_{m}+m r^{2}  \tag{4.8.20}\\
& T_{L}=F r \tag{4.8.21}
\end{align*}
$$

According to the concepts previously developed [using (4.8.9)], the energy dissipated in the motor armature per step is:


Fig. 4.8.6. Belt-pulley drive system.

$$
\begin{align*}
W_{c}= & \frac{R}{K_{T}^{2}}\left[\frac{12}{\eta} \frac{\left(\frac{x}{r}\right)^{2}}{t_{c}^{3}}\left(J_{m}+m r^{2}\right)^{2}+\right. \\
& \left.+(F r)^{2} t_{c}\right] \tag{4.8.22}
\end{align*}
$$

Now define the coupling ratio

$$
\begin{equation*}
G=\frac{1}{r} \tag{4.8.23}
\end{equation*}
$$

and:

$$
\begin{equation*}
\beta=\frac{\eta}{12}\left[\frac{\mathrm{Ft}_{\mathrm{c}}^{2}}{\mathrm{mx}}\right]^{2} \tag{4.8.24}
\end{equation*}
$$

Then the energy equation (4.8.22) becomes:

$$
\begin{equation*}
W_{c}=\frac{R}{K_{T}^{2}} \frac{12}{\eta} \frac{m^{2} x^{2}}{t_{c}^{3}}\left[G^{2}\left(\frac{J_{m}}{m}+\frac{1}{G^{2}}\right)^{2}+\frac{\beta}{G^{2}}\right] \tag{4.8.25}
\end{equation*}
$$

Note that (4.8.25) is similar to (4.8.15) where $\mathbf{G}$ is equivalent to $\mathbf{N}, \mathrm{m}$ is equivalent to $J_{L}$ and $\beta$ instead of $\gamma$. Clearly, the
optimum coupling ratio, according to (4.8.16), will be:

$$
\begin{equation*}
\mathrm{G}_{\mathrm{o}}^{2}=\frac{\mathrm{m}}{J_{\mathrm{m}}} \sqrt{1+\beta} \tag{4.8.26}
\end{equation*}
$$

The optimum pulley radius is then:

$$
\begin{equation*}
r_{0}=\left(\frac{J_{m}}{m \sqrt{1+\beta}}\right)^{\frac{1}{2}} \tag{4.8.27}
\end{equation*}
$$

If the constant opposing force $\mathbf{F}=\mathbf{0}$, then $\beta=0$, and we again have an exact inertia match:

$$
\begin{equation*}
r_{0}=\sqrt{\frac{J_{m}}{m}} \tag{4.8.28}
\end{equation*}
$$

## Lead Screw Drive

The lead screw drive is shown in Fig. 4.8.7, and consists of a motor turning the lead screw which moves a mass $m$, much in the same way as the belt-pulley drive. Assuming the motion problem variables $\mathbf{x}, \theta, \mathbf{T}_{\mathbf{L}}$, $F$, and $m$ remain the same as in the beltpulley system, the problem becomes to determine the optimum pitch, $\mathrm{P}_{\mathrm{o}}$, of the lead screw. Pitch here is taken as number of revolutions per unit length and therefore:

$$
\begin{equation*}
P=\frac{G}{2 \pi} \tag{4.8.29}
\end{equation*}
$$

Since $\mathbf{G}$ is the motor rotation in radians per unit length, the optimum pitch is therefore:

$$
\begin{equation*}
P_{o}=\frac{1}{2 \pi}\left(\frac{m \sqrt{1+\beta}}{J_{m}}\right)^{\frac{1}{2}} \tag{4.8.30}
\end{equation*}
$$

First we must calculate $\gamma$ from (4.8.14):

$$
\gamma=0.260
$$

Now we can calculate the optimum gear ratio from (4.8.16):

$$
\begin{aligned}
& N_{o}^{2}=179.6 \\
& N_{0}=13.4
\end{aligned}
$$

Example 2: The second example involves a mass to be moved linearly by means of a belt-pulley drive. The following load and performance parameters apply:

$$
\begin{aligned}
\mathrm{m} & =0.01 \mathrm{~kg} \\
\mathrm{~F} & =0.003 \mathrm{~N} \\
\mathrm{x} & =0.02 \mathrm{~m} \\
\mathrm{t}_{\mathrm{c}} & =0.01 \mathrm{~s}
\end{aligned}
$$

From (4.8.24) we have:

$$
\beta=1.667 \times 10^{-7}
$$

By observing (4.8.27) it can be seen that the factor $\beta$ is insignificant and that the inertia match formula

$$
\begin{equation*}
r_{o}=\sqrt{\frac{J_{m}}{m}} \tag{4.8.28}
\end{equation*}
$$

can be used. The optimum pulley radius is thus

$$
r_{o}=18.8 \mathrm{~mm}=0.74 \mathrm{in}
$$

Example 3: In the final example, a mass is to be driven by a lead screw in the same
manner as in Example 2. The problem is to find the optimum lead screw pitch. The following load and performance parameters apply:

$$
\begin{aligned}
\mathrm{m} & =1 \mathrm{~kg} \\
\mathrm{~F} & =0.02 \mathrm{~N} \\
\mathrm{x} & =0.02 \mathrm{~m} \\
\mathrm{t}_{\mathrm{c}} & =0.01 \mathrm{~s}
\end{aligned}
$$

Let us also consider the moment of inertia of the lead screw to be

$$
\begin{aligned}
J_{L S} & =0.4 \times 10^{-3} \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \\
& =28.25 \mathrm{~g} \mathrm{~cm}^{2}
\end{aligned}
$$

This moment of inertia is to be added to the motor moment of inertia, so that:

$$
\mathrm{J}_{\mathrm{m}}^{\prime}=\mathrm{J}_{\mathrm{m}}+\mathrm{J}_{\mathrm{LS}}=63.55 \mathrm{~g} \mathrm{~cm}^{2}
$$

We can now calculate the opposing force factor $\beta$ from (4.8.24) :

$$
\beta=7.4 \times 10^{-10}
$$

This factor can be ignored. The optimum pitch is thus from (4.8.30):

$$
\begin{aligned}
P_{o} & =\frac{1}{2 \pi}\left(\frac{\mathrm{~m}}{J_{m}^{\prime}}\right)^{\frac{1}{2}} \\
& =63.134 \text { turns } / \mathrm{m} \cong 1.6 \text { turns } / \mathrm{in}
\end{aligned}
$$

### 4.8.4. CAPSTAN OPTIMIZATION

In the following let us briefly examine some of the practical aspects of a typical servo problem in computer peripheral equipment.

## Introduction

Fast-moving technological changes in incremental tape transports of the past decade have made $200-250 \mathrm{in} / \mathrm{s}(5-6.4 \mathrm{~m} / \mathrm{s}$ ) machines commercially available. Single capstan drive systems have emerged as virtually sole survivors in the drive methods contest. As the quest for increasingly higher tape speeds and repetition rates continues, tape transport servo and capstan designers as well as users must face the increasing demands on motors, feedback elements, capstan design and power supplies.

Thus, in the decade we can expect demands on servomotors with higher $\mathrm{K}_{\mathbf{T}} / \mathrm{R}$ ratio, lower moment of inertia, and lower thermal resistances (and, lest we forget, increasing downward price pressure). We believe that capstan motor power dissipation problems and torsional resonance problems are the major technical hurdles we have to face in the future.

This discussion covers one aspect of this problem: how to pick the right motor and design the optimum capstan radius to minimize power dissipation. It also discusses what added power dissipation the application would need under non-optimum conditions.

## Optimum Capstan Selection

If we define a tape drive system with a given linear velocity profile $\mathbf{v}(\mathrm{t})$, the linear acceleration of the tape is:

$$
\begin{equation*}
a=\frac{d v}{d t} \tag{4.8.31}
\end{equation*}
$$

A direct-coupled-to-the-motor capstan of radius $r$ drives the tape. The motor angular velocity $\omega$ is therefore:

$$
\begin{equation*}
\omega=\frac{v}{r} \tag{4.8.32}
\end{equation*}
$$

Differentiating (4.8.32) gives the angular acceleration:

$$
\begin{equation*}
a=\frac{\mathrm{d} \omega}{\mathrm{dt}}=\frac{1}{\mathrm{r}} \frac{\mathrm{dv}}{\mathrm{dt}} \tag{4.8.33}
\end{equation*}
$$

Combining (4.8.31) and (4.8.33) gives:

$$
\begin{equation*}
a=\frac{\mathrm{a}}{\mathrm{r}} \tag{4.8.34}
\end{equation*}
$$

The moment of inertia of the capstan, $J_{c}$, depends on the radius $r$ and on the capstan design. Suppose that the capstan structure is approximated by a solid disc and a cylinder (Fig. 4.8.8, $A$ and $B$ ).

If the dominant portion of capstan moment of inertia would be represented by a disc of length $w_{1}$, radius $r$, density $\rho$ and mass $m_{1}$, then:

$$
\begin{equation*}
J_{c}=\frac{1}{2} m_{1} r^{2}=\frac{1}{2} \rho \pi w_{1} r^{4} \tag{4.8.35}
\end{equation*}
$$



Fig. 4.8.8. Typical capstan construction can take the form of $(A)$ a disc, and $(B)$ a cylinder. (C) indicates the hub and its small mass. Hence, it contributes a negligible amount to the capstan moment of inertia.

On the other hand, if the dominant portion of $\mathrm{J}_{\mathbf{c}}$ lies in the outside cylinder of thickness $d$, length $w_{2}$ and mass $m_{2}$, then:

$$
\begin{equation*}
J_{c}=m_{2} r^{2}=2 \pi \rho w_{2} d r^{3} \tag{4.8.36}
\end{equation*}
$$

From these extreme conditions, we can conclude that a capstan made from elements $A$ and $B$ (Fig. 4.8.8) provides a moment of inertia which is proportional to the power of $r$ between 3 and 4 . We can usually ignore the inertia contribution of the hub C . We can therefore define the inertia equation as:

$$
\begin{equation*}
\mathrm{J}_{\mathrm{c}}=\mathrm{kr} \mathrm{r}^{\mathrm{p}} \tag{4.8.37}
\end{equation*}
$$

where the exponent p :

$$
\begin{equation*}
3 \leqslant p \leqslant 4 \tag{4.8.38}
\end{equation*}
$$

A subsequent example shows how to arrive at a value for $p$ in a specific case. The total system moment of inertia is the sum of motor moment of inertia $\mathrm{J}_{\mathrm{m}}$ and the capstan moment of inertia $\mathbf{J}_{\mathbf{c}}$.

$$
\begin{equation*}
J=J_{m}+J_{c} \tag{4.8.39}
\end{equation*}
$$

Combining (4.8.37) with (4.8.39) gives:

$$
\begin{equation*}
J=J_{m}+k r^{p} \tag{4.8.40}
\end{equation*}
$$

The torque $\mathbf{T}$ needed to run the motor at acceleration $a$ is:

$$
\begin{equation*}
\mathrm{T}=\mathrm{J} a \tag{4.8.41}
\end{equation*}
$$

If we combine (4.8.34), (4.8.40) and (4.8.41), we get:

$$
\begin{equation*}
T=\frac{a}{r}\left(J_{m}+k r^{p}\right) \tag{4.8.42}
\end{equation*}
$$

From the chosen motor specification we can define the armature current during acceleration as:

$$
\begin{equation*}
I_{a}=\frac{T}{K_{T}} \tag{4.8.43}
\end{equation*}
$$

Hence the instantaneous power dissipation in the motor (neglecting friction - a realistic assumption in most cases of incremental motion) is:

$$
\begin{equation*}
P(t)=R I_{a}^{2} \tag{4.8.44}
\end{equation*}
$$

or, using (4.8.43):

$$
\begin{equation*}
\mathrm{P}(\mathrm{t})=\mathrm{R} \frac{\mathrm{~T}^{2}}{\mathrm{~K}_{\mathrm{T}}^{2}} \tag{4.8.45}
\end{equation*}
$$

Clearly, minimizing $\mathrm{P}(\mathrm{t})$ minimizes T .
Performing the differentiation

$$
\frac{d T}{d r}=0
$$

of (4.8.42) gives:

$$
-a J_{m} r^{-2}+(p-1) a k r^{p-2}=0
$$

or:

$$
\begin{equation*}
J_{m}=(p-1) k r^{p} \tag{4.8.46}
\end{equation*}
$$

Substituting (4.8.37) into (4.8.46) gives the relationship:

$$
\begin{equation*}
J_{m}=(p-1) J_{c} \tag{4.8.47}
\end{equation*}
$$

Equation (4.8.47) shows how the motor moment of inertia relates to the capstan's. For the given range of $p$, minimum power dissipation requires a capstan moment of inertia between $1 / 2$ and $1 / 3$ of the motor moment of inertia. This requirement differs from the usual rule-of-thumb concept in analog servo design that calls for motor moment of inertia equal to load moment of inertia for optimum performance conditions.

## Non-optimum Choices

Frequently, additional constraints or available equipment prevent the selection of optimal capstans. Since in this case we must choose a non-optimal capstan, how does power dissipation increase?

Recalling that the capstan moment of inertia is $J_{c}=k r^{p}$ (4.8.37), and if we call the optimal capstan radius $r_{0}$ and the realizable (non-optimum) radius $r$, and denote the power dissipation corresponding to $r_{o}$ and $r$ by $P_{o}$ and $P$, respectively, then from (4.8.45)
and (4.8.42) we obtain the power dissipation.

$$
\begin{equation*}
P=\frac{R a^{2}}{K_{T}^{2} r^{2}}\left(J_{m}+k r^{p}\right)^{2} \tag{4.8.48}
\end{equation*}
$$

and the minimum power dissipation:

$$
\begin{equation*}
P_{o}=\frac{R a^{2}}{K_{T}^{2} r_{o}^{2}}\left(J_{m}+k r_{o}^{p}\right)^{2} \tag{4.8.49}
\end{equation*}
$$

A good indicator of the "efficiency" is $\eta$, the ratio of $\mathbf{P}$ and $\mathrm{P}_{\mathbf{o}}$ :

$$
\begin{equation*}
\eta=\frac{P}{P_{o}}=\left[\frac{r_{o}\left(J_{m}+k r^{p}\right)}{r\left(J_{m}+k r_{o}^{p}\right)}\right]^{2} \tag{4.8.50}
\end{equation*}
$$

Now recall the $r_{0}$ satisfied (4.8.46), therefore:

$$
\begin{equation*}
k r_{o}^{p}=\frac{J_{m}}{p-1} \tag{4.8.51}
\end{equation*}
$$

If we denote

$$
\begin{equation*}
\gamma=\frac{r}{r_{0}} \tag{4.8.52}
\end{equation*}
$$

and substitute into (4.8.50) yields:

$$
\begin{equation*}
\eta=\left[\frac{r_{0}\left(J_{m}+k \gamma^{p} r_{o}^{p}\right)}{\gamma r_{o}\left(J_{m}+k r_{o}^{p}\right)}\right]^{2} \tag{4.8.53}
\end{equation*}
$$

This can be further simplified by substituting (4.8.51):

$$
\begin{equation*}
\eta=\left[\frac{p-1+\gamma^{p}}{\gamma p}\right]^{2} \tag{4.8.54}
\end{equation*}
$$

Equation (4.8.54) indicates the increase in power dissipation when a non-optimal radius is used. Note that for $p=3$ and $p=4$ it becomes:

$$
\begin{align*}
& \eta=\left[\frac{2+\gamma^{3}}{3 \gamma}\right]^{2} @ p=3  \tag{4.8.55}\\
& \eta=\left[\frac{3+\gamma^{4}}{4 \gamma}\right]^{2} \quad @ p=4
\end{align*}
$$

Since for most cases $3 \leqslant p \leqslant 4$, the power dissipation increase is within the limits given by (4.8.55) and presented in Fig. 4.8.9. Note that a small deviation from the optimal radius does not result in a large power dissipation increase. However, as the deviation increases, the power dissipation increases considerably. For example, if the capstan radius is half the size of the optimal, power dissipation increases between $100 \%$ and $134 \%$.


Fig. 4.8.9. Motor power dissipation increase as function of capstan radius.

State-of-the-art systems not uncommonly work the servo system close to the dissipation limits of the motor. Therefore, you should know the penalty for non-optimum capstan choice.

### 4.8.5. OPTIMUM MOTOR SELECTION FOR INCREMENTAL APPLICATION

So far, we learned how to optimize capstan design for a given motor. Now we can start over again and assume we have not chosen a specific motor. Let's select the best motor for a general capstan shape.

If we combine (4.8.44) and (4.8.45), we define power dissipation for any motor as:

$$
\begin{equation*}
P=R I_{a}^{2}=R \frac{T^{2}}{K_{T}^{2}} \tag{4.8.56}
\end{equation*}
$$

Inserting (4.8.34) and (4.8.41) into (4.8.56) produces:

$$
\begin{equation*}
P=\frac{R \mathrm{~J}^{2}}{\mathrm{~K}_{\mathrm{T}}^{2}}\left(\frac{\mathrm{a}}{\mathrm{r}}\right)^{2} \tag{4.8.57}
\end{equation*}
$$

Substitution from (4.8.39) and (4.8.47) gives:

$$
\begin{equation*}
P=\frac{R}{K_{T}^{2}}\left[J_{m} \frac{p}{p-1} \frac{a}{r}\right]^{2} \tag{4.8.58}
\end{equation*}
$$

Rearranging (4.8.46) gives us the optimal radius:

$$
\begin{equation*}
r_{0}=\left[\frac{J_{m}}{(p-1) k}\right]^{\frac{1}{p}} \tag{4.8.59}
\end{equation*}
$$

and substituting into (4.8.58) gives:

$$
\begin{equation*}
P=\frac{R}{K_{T}^{2}}\left[J_{m} \frac{a p}{p-1}\left(\frac{[p-1] k}{J_{m}}\right)^{\frac{1}{p}}\right]^{2} \tag{4.8:60}
\end{equation*}
$$

If we isolate motor dependent parameters in (4.8.60), we see that:

$$
\begin{equation*}
P=\frac{R J_{m}^{(2-2 / p)}}{K_{T}^{2}}\left[\frac{a p}{p-1}([p-1] k)^{\frac{1}{p}}\right]^{2} \tag{4.8.61}
\end{equation*}
$$

and the term to be minimized is:

$$
\begin{equation*}
\frac{R J_{m}^{\beta}}{K_{T}^{2}} \tag{4.8.62}
\end{equation*}
$$

where

$$
\beta=2-2 / p
$$

For the given range of $p$, it becomes:

$$
\begin{equation*}
\frac{4}{3} \leqslant \beta \leqslant \frac{3}{2} \tag{4.8.63}
\end{equation*}
$$

Thus, we now observe that for optimum power dissipation we must minimize the following term:
$\frac{R J_{m}(1.33 \text { to } 1.5)}{K_{T}{ }^{2}}$

In some cases, we may find that the design objective is to minimize the motor temperature rise, $\Theta_{r}$. We find that $\Theta_{r}$ is related to the power dissipation by:

$$
\begin{equation*}
\Theta_{r}=P R_{t h} \tag{4.8.65}
\end{equation*}
$$

where $\mathbf{R}_{\mathbf{t h}}$ is the thermal resistance of the motor. Then the motor dependent parameters to be minimized are:

$$
\begin{equation*}
\frac{R J_{\mathrm{m}}^{\beta} R_{\mathrm{th}}}{\mathrm{~K}_{\mathrm{T}}^{2}} \tag{4.8.66}
\end{equation*}
$$

Thus, we note that (4.8.66) matches the formula of (4.8.62) with the addition of $R_{\text {th }}$.

We can establish the value of $\beta$ from the design philosophy of the capstan design, which gives a value of $\mathbf{p}$ (see subsequent example). To select the optimum motor, we can now take specification data for various motors and plug it into (4.8.64). The lowest value indicates the optimum motor from the power dissipation point of view. Of course, we must consider other reasons for selecting a motor, such as compatibility with power supply voltage and current limits, resonant frequencies, tachometer accessories, physical size, available shaft dimensions and any other parameters specific to the user's need.

We have now acquired an insight into factors which affect power dissipation in an incremental motion system. In the following example we compare two motors of
rather diverse performance characteristics to show the drastic differences in power dissipation.

## EXAMPLE

When designing a capstan drive, we can use either of two available typical capstans with the following parameters:

$$
\begin{aligned}
& r_{1}=0.75 \text { in, } J_{c 1}=0.3 \times 10^{-3} \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}} \\
& r_{2}=1.0 \text { in, } J_{c 2}=0.85 \times 10^{-3} \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
\end{aligned}
$$

We can therefore assume that we can design the optimum capstan of a radius that ranges between 0.55 in to 1.0 in . Which of the following two available motors should we select?

| Motor | $K_{T}[o z-i n / A]$ | $R[\Omega]$ | $J_{m}\left[o z-i n-s^{2}\right]$ |
| :---: | :---: | :---: | :---: |
| $M-1030$ | 5.8 | 0.85 | $0.45 \times 10^{-3}$ |
| $M-1438$ | 9.5 | 0.55 | $0.65 \times 10^{-3}$ |

To determine the equation of capstan moment of inertia, we want to arrive at the form:

$$
\begin{equation*}
J_{c}=k r^{p} \tag{4.8.37}
\end{equation*}
$$

Then:

$$
\begin{equation*}
\mathrm{J}_{\mathrm{c} 1}=\mathrm{kr}_{1}^{\mathrm{p}} \text { and } \mathrm{J}_{\mathrm{c} 2}=\mathrm{kr}_{2}^{\mathrm{p}} \tag{4.8.67}
\end{equation*}
$$

Dividing the two equations yields:

$$
\begin{equation*}
\frac{J_{c 2}}{J_{c 1}}=\left(\frac{r_{2}}{r_{1}}\right)^{p} \tag{4.8.68}
\end{equation*}
$$

Now take the logarithm of the two sides:

$$
\begin{equation*}
p=\frac{\log \left(\frac{J_{c 2}}{J_{c 1}}\right)}{\log \left(\frac{r_{2}}{r_{1}}\right)} \tag{4.8.69}
\end{equation*}
$$

Here:

$$
p=\frac{\log \left(\frac{8.5}{3}\right)}{\log \left(\frac{1.0}{0.75}\right)}=3.6
$$

To determine $\mathbf{k}$ :

$$
\mathrm{k}=\frac{\mathrm{J}_{\mathrm{c} 2}}{\mathrm{r}_{2}^{\mathrm{p}}}=\frac{0.85 \times 10^{-3}}{1^{3.6}}=0.85 \times 10^{-3}
$$

Since $p=3.6$, then:

$$
\beta=2-\frac{2}{p}=2-\frac{2}{3.6}=1.45
$$

To select the motor, evaluate (4.8.64) the term:

$$
\frac{R J_{m}^{1.45}}{K_{T}^{2}}
$$

for the two motors.

In evaluating the previous term, use any unit system, as long as you use it for all the motors. The ratio of power dissipation of the two motors will then be:

$$
\begin{aligned}
\frac{(P)_{M-1030}}{(P)_{M-1438}}= & \frac{\frac{0.85 \times 4.5^{1.45}}{5.8^{2}}}{\frac{0.55 \times 6.5^{1.45}}{9.6^{2}}} \\
& =\frac{0.223}{0.072} \cong 3.1
\end{aligned}
$$

The second motor, M-1438, will dissipate power at a much lower rate than M-1030, by a factor of 3.1 !

If M-1438 motor fulfills other requirements, such as price, size, power supply needs, we will select it as the capstan driver.

Since the optimal capstan moment of inertia $\mathrm{J}_{\mathrm{c}}$ (4.8.47) for M -1438 motor is:

$$
\begin{aligned}
J_{c}=\frac{J_{m}}{p-1} & =\frac{0.65 \times 10^{-3}}{2.6} \\
& =0.25 \times 10^{-3} \mathrm{oz}-\mathrm{in}^{-\mathrm{s}^{2}}
\end{aligned}
$$

The capstan radius should be (4.8.37):

$$
\begin{aligned}
& 0.85 \times 10^{-3} r^{3.6}=0.25 \times 10^{-3} \\
& r^{3.6}=\frac{0.25}{0.85}=0.294
\end{aligned}
$$

## $3.6 \log r=\log 0.294$

From this,

$$
r_{o}=0.71 \mathrm{in}
$$

The optimum capstan radius is 0.71 in and the optimum capstan moment of inertia is $0.25 \times 10^{\mathbf{- 3}} \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}$. These values come close to those of one of the sample capstans listed earlier. If we use the 0.75 in radius capstan rather than the 0.71 in optimum, then

$$
\gamma=\frac{r}{r_{0}}=\frac{0.75}{0.71}=1.056
$$

and the power dissipation increase due to non-optimal radius approximates $1 \%$ - a small penalty.

### 4.9. PERMANENT MAGNET MOTORS FOR SERVO APPLICATIONS

In this section, we will discuss a few reasons why permanent magnet motors are useful for servo applications.

The easily controlable speed feature of the DC motor, coupled with its extremely linear torque-speed control characteristic makes the PM motor an ideal servo component. An expression relating motor speed to applied voltage is:

$$
\begin{equation*}
n=\frac{V}{K_{E}} \tag{4.9.1}
\end{equation*}
$$

This relationship allows motor speed to be controlled by proper adjustment of the applied terminal voltage. If the terms of the above expression are rearranged we obtain:

$$
\begin{equation*}
V=K_{E} n \tag{4.9.2}
\end{equation*}
$$

In this form it is easy to see that if driven at the shaft and used in a generator mode, a PM device will provide an output voltage signal that is proportional to the shaft speed. This feature makes the PM generator an ideal and compatible feedback device for use in combination with a PM motor in a DC servo system. The basic elements of a simple speed control system are shown in Fig. 4.9.1. The armature current is regulated by the transistor Q which in turn is controlled to be more or less conductive by the generator connected between its base and the wiper arm of a potentiometer. Each
setting of the potentiometer corresponds to a given shaft speed. If a given setting is supplied by the potentiometer, the circuit will then function automatically, holding the motor shaft at the desired speed, independent of variation in the load torque.


Fig. 4.9.1. Elementary speed control system.
In order to exploit the adaptability to servo systems, PM machine design has assumed some rather unique forms not normally seen on general purpose devices. In the speed control scheme shown in Fig. 4.9.1, it is necessary for the motor shaft and the generator shaft to be rigidly coupled together. To accomplish this, many servo systems are built with both motor and generator on one shaft. Such a device is called a motor-tachometer-generator (tachometer means speed counter) or simply a motor-tach.

While motor or generator action is reversible, depending on whether the shaft is driven or the armature excited, the electrical characteristics of the devices are often quite different. The motor is designed to
convert electrical power to a mechanical form. Because of this power handling role it must have large wire size and is usually characterized by low armature resistance. On the other hand, the generator is not required to convert significant amounts of power, but rather to provide a voltage signal. The generator can function well using relatively fine wire sizes. These different demands of motor and generator function have resulted in a unique motortach design patented and marketed by Electro-Craft under the trade name "Motomatic". The concept of the Motomatic design is that the fine wire for generator armature winding could easily occupy the interstices of the regular large motor armature winding, and thus occupy the same armature core without actually losing space for the motor winding. The widespread acceptance of the Motomatic components has proved the design concept to be correct. Motomatic armatures are shown in Figs. 4.9.2 and 4.9.3. The motor and generator windings may be wound simultaneously or consecutively. In either case, the windings occupy the same slots and are electrically isolated from one another. Electrical power flows in through a set of brushes at the motor end, develops torque and mechanical power to a load, and at the same time


Fig. 4.9.2. Motomatic design concept - combining motor and generator on one armature core.
the voltage proportional to shaft speed is provided off the set of brushes at the opposite (generator) end. The concept allows very significant economies in manufacturing costs.

The Motomatic style (integral motor-tach), of construction provides acceptable characteristics for most applications. There are two reasons, however, why it might be desirable in some instances to place the motor and generator windings on separate cores (Fig. 4.9.3) but still on the same shaft. One is that in the case of an SCR type controller the pulsating DC current in the motor produces a large voltage ripple in the generator voltage because of the transformer action of the common iron core.


Fig. 4.9.3. Motor-tach design for magnetic isolation of motor and generator windings.

The other - and entirely different - reason is that if either a very wide controlled speed range is desired or a high degree of instantaneous or long-term speed stability is required at the very low speed, then these objectives might be better achieved with a higher voltage gradient tachometer on a separate core.

In general applications where the motor operates for long periods at very low speeds and at the same time extremely close speed
control (better than $0.2 \%$ of the set speed) is required and any drift in speed over a long time cannot be tolerated, then it may be necessary to use a silver commutator instead of the conventional copper one in the
generator. Under the above conditions, the film on a copper commutator tends to change, introducing a drift in generator voltage - hence in motor speed. This is not the case when silver is used as a commutator surface.

### 4.10. SYSTEMS AND CONTROLS TROUBLESHOOTING

Prior to undertaking the troubleshooting of a system, a basic comprehension of each part of the system is essential, as well as a knowledge of operating and calibration controls. Cognizance of a normally operating system, such as voltage and current waveforms at key points, will facilitate isolation of the defective components.

To aid troubleshooting, we can divide failure modes into three classes:

1) Total
2) Partial
3) Intermittent

In the case of a total failure, start troubleshooting at the incoming power lines. If power is present and no fuses or circuit breakers are defective, check the system power supply. Then follow normal power flow through the system toward the load until the defective stage is located. Substitution of a function generator signal for normal command will facilitate signal tracing.

When troubleshooting a partially defective system, a more unique approach is required. By substituting a signal midstream in the system, say at the driver amplifier, it is possible to localize the fault to a before or after stage. Removing the load from the amplifier, opening the velocity or position feedback loop, the loading of an amplifier
with a resistive load will allow conventional amplifier signal tracing.

The intermittent fault can best be isolated by long-term monitoring of key points with an events' recorder or similar instrument. Knowing the rate of the intermittent fault may lead to causes. Also the application of heat, vibration, cold, moisture, stress or other variables to areas of the system may cause the intermittent fault to appear.

When the failure is isolated, the symptom which causes the failure should be noted and questioned as to why the symptom occurred. Refer to the theoretical failure symptom chart, and as an example take an inoperative motor, noting that the symptom was a blown fuse or tripped circuit breaker. Before replacing the fuse or resetting the breaker, question why the fuse blew. Consider the following:

1) Did the fuse blow because of age or fatigue, or was it overloaded?
2) Did the motor fail, causing the fuse to blow or was it a case of overload?
3) Did the amplifier go into an oscillatory mode, producing high ripple currents which blew the fuse?
4) Did the motor make any unusual sounds or emit smoke prior to failure of the system?

| Nature of Failure | Symptom Noted |
| :---: | :---: |
| Motor will not run | 1. Motor or amplifier overheated <br> 2. Fuses blown <br> 3. Fuses or pilot light off |
| Uncontrollable operation at full speed | 1. Immediately when power is applied <br> 2. When load is applied |
| Poor load regulation | 1. At high speed only <br> 2. At low speed only <br> 3. Through entire speed range <br> 4. In one direction only |
| Erratic performance | 1. At high speed <br> 2. At low speed <br> 3. At all speeds <br> 4. When motor is hot <br> 5. In both directions of operation <br> 6. In one direction only <br> 7. Under extreme load conditions |
| Speed variation with time | 1. Increases or decreases <br> 2. Starts or stops |

## Speed variation with temperature <br> 1. Increases or decreases <br> Stops or starts

Repairing and replacing components must be done with the same techniques and care that the original part was installed. This is to say all heat sink mounted semiconductors must be installed with the proper insulators, hardware and thermal mounting components as originally used. Low-power devices should be installed carefully, noting the amount of heat used for soldering, lead strain, and cleaning. Coated printed circuit boards should be thoroughly cleaned, then recoated.

When testing a unit which has been repaired, it is good practice to run the supply voltage up from zero with a variable transformer. As line voltage is slowly applied any serious defects which remain may be noticed soon enough to prevent further damage by monitoring the input power on a wattmeter. The repaired unit should be tested to its original specifications. Calibration to original specifications will disclose any additional shortcomings, or verify that the proper repair has been made.

## Chapter 5

## Applications

### 5.1. INTRODUCTION

The following section has two aims: one is to provide information and guidance to the engineer who plans to design his own servo system; and the second is to illustrate the considerations which relate to the application of servo systems, so that the engineer who plans to buy a complete system can have a better understanding of the problems involved.

It is not possible to overstress the importance of complete cooperation between customer and supplier when designing a special motor or system. In many cases both the customer's engineers and the servo manufacturer's engineers may find themselves facing problems which are foreign to their previous experiences. For example, if the application is a knitting machine, the servo engineer may be confronted with problems involving stitch formation and thread handling; while the knitting machine designer may be facing problems associated with supplying electronic interfacing with the servo system. This kind of situation cannot be avoided where a new technology is applied in a traditional industry; however, the difficulty need not be an obstruction. Electro-Craft's experience as a principal manufacturer of servomotors and com-
ponents for more than ten years has shown that when a high level of cooperation exists the chances of an application project being mutually successful are high.

### 5.2. SYSTEM CLASSIFICATION AND SPECIFICATION

More and more industries remote from the technology of electronics and servo systems are making use of the many advantages that DC servo systems can offer. In many cases, difficulty occurs in communication between a design engineer in such an industry and the servo systems designer. This section is intended to show how a potential user can examine his application, pick out the important performance criteria and describe them in terms meaningful to the servo designer.

### 5.2.1. CLASSIFICATION

## Controlled Variable

An important step in examination of a system is identification of the controlled variable. By definition, a servo system monitors the condition of an output variable, comparing it to an input command and making adjustments to maintain an equality.

The most common output quantities used as controlled variables are:

1. Position
2. Speed
3. Torque (or force)

It is possible for a system to have more than one controlled variable, or for a system to switch from one variable to another during operation. Identification of the controlled variable is not always as straight forward as it might appear. For example, consider a drive system used to synchronize an audio tape recorder with a movie camera. At first, it would appear the purpose of the drive is to keep the speed of the tape recorder exactly equal to the speed of the camera. Closer study will show that the correct approach is to control the relative position of the two drives to ensure that the appropriate sounds occur while the correct picture is being projected when the film is played back. If this is done, the speeds will automatically be equalized.

The following are examples of controlled variables in typical applications:

| Position | Velocity | Torque |
| :--- | :--- | :--- |
| line printer | metering pump | reel drive <br> (constant tension) |
| matrix printer | matrix printer |  |
| teleprinter | crystal grower | muscle exercizer |
| machine tool <br> $(x-y)$ | blueprint <br> machine | weighing machine |

## Operating Mode

A servo system operates in one of two modes: incremental or continuous.

The incremental mode of operation is defined as the mode where the input command causes the controlled variable to change its value in discrete steps, with a brief static condition occurring between each step.

The continuous mode is one where the input command signal causes the controlled variable to remain constant, or to change in a linear (rather than step) mode.

## Examples

| Incremental | Continuous |
| :--- | :--- |
| line printer | audio tape recorder |
| photo typesetter | rewinding drive |
| digital tape transport | digital tape transport |
| disc head drive | pumps |
|  | energy chopper |
|  | laboratory stirrer |

The digital tape transport is an example of a dual mode system. During normal reading the capstan drive motor moves the tape in discrete variable length steps, but during rewinding - or when scanning through a length of tape - the speed is held constant. The system, therefore, can operate in both incremental and continuous modes.

## Load Classification

Loads can be classified under two headings. The first describes the kind of the motion, either rotational or linear. The second describes the means of coupling the load to the motor (capstan, lead screw, belt, gears, etc.).

### 5.2.2. SPECIFICATION

When the above classifications are established for a particular application, the next is the specification of performance. The way in which this is done will vary with the system classification.

## Incremental Mode Systems

The most common incremental mode systems are position systems.

Their function is usually to move a load from one position to another, by steps. The steps may be of varying length, or may occur at different repetition rates. A velocity profile must be drawn to describe the motion accurately (Fig. 5.2.1).


Fig. 5.2.1. Velocity profile diagram.

The velocity profile diagram represents the motion of the load during one increment. Where the peak velocity $\boldsymbol{v}_{\text {max }}$ is not fixed by other requirements, it should be selected to make $\mathrm{t}_{\mathrm{a}}=\mathrm{t}_{\mathrm{r}}=\mathrm{t}_{\mathrm{d}}$, which minimizes motor heating. If the system is an extremely high performance drive, a time interval of approximately 1 or 2 ms should be subtracted from the motion time and added to the dwell time to allow the system to settle after each step.

The units used for speed and displacement description will depend on the load classification.

If there are several different types of steps which the system is required to execute, a velocity profile must be drawn for both the average and the worst cases. An explanation of how the step varies is also required.

Next, motor velocity profiles should be drawn. This will use the units [rad/s] for speed and [rad] for angular displacement. These represent the motion which the motor has to execute to drive the load through the previously established profiles. To draw these profiles, one simply applies the various gear and coupling ratios used to the original load velocity profiles. A section covering selection of optimum coupling is presented earlier in this book (section 4.8.).

NOTE: Where the coupling type or ratio is not fixed at the time of writing the specification, it is good policy to leave them open and supply the servo designer with load velocity profile diagrams. This gives the
designer maximum flexibility in selecting a motor, and prevents the possibility of the coupling ratio forcing the designer to use a motor of higher cost than necessary.

## Load Description

All loads will possess the following properties: inertia, friction and viscous friction. Values for each of these should be given. Also, if the load is coupled to the motor by a method other than direct mounting on the motor shaft, each component of the coupling system will possess these three properties, and their values must be given. All of these values will be used to calculate the moment of inertia, friction torque and viscous damping factor as reflected to the motor shaft by the coupling system.

## Accuracy

Accuracy is defined as the deviation of the controlled variable from the desired level. Accuracy specifications divide into two categories: short-term and long-term.

Short-term accuracies are normally defined by the system design, and include such factors as gain, line densities on encoders, bandwidth and motor mechanical features.

Long-term accuracies are normally controlled by the system component. design, temperature coefficients of tachometers and operational amplifiers, etc. To meet long-term accuracy specifications, the system designer employs component selection and compensation techniques. Values of
required accuracies are needed to determine the type of feedback transducer and other system components.

## Continuous Mode Systems

The function of a continuous mode servo system is to maintain the value of the controlled variable constant against externally and internally introduced disturbances, and to execute changes in the value of the controlled variable when so commanded. Generally, the execution of intentional changes in the value of the controlled variable in a continuous mode system will either be made at a relatively low response rate, or the speed of response will not be a critical parameter. In either case, the torque required to accelerate or decelerate the load will not be a significant factor.

However, to ensure that an exception is not overlooked, it is wise to specify worst case and typical acceleration and deceleration rates; and, where applicable, repetition rates.

For a typical continuous mode servo system, the following data will be required.

## Load Description

Inertia, friction and viscous friction data for the load and coupling mechanism will all be required exactly as for the incremental system. Special attention is required to evaluate all impulse type loadings which may be generated in gears and bearings.

## Disturbances

Any impulse loading, whether generated internally or externally, should be included in this category. The important factor is the maximum impulse loading that the system will see. This may be one force or the sum of several forces. The customer needs to specify the peak value of the force, the duration and the average repetition rate. If the impulses are periodic, a graph similar to Fig. 5.2.2. showing a typical cycle, would be helpful.


Fig. 5.2.2. Periodic load disturbance.
Although all the above information may be essential for complex high accuracy systems, the majority of systems will only require a statement describing the maximum value of impulse torque.

## Error Size and Recovery Time

The maximum error size is the maximum value of instantaneous speed (or position) deviation due to the occurrence of a load or system disturbance. It should be expressed as a percentage of set speed (or position).

The recovery time describes the time the system takes to recover to the set speed or position after the disturbance ceases. A
useful way to express this is the time the system should take to correct $63.2 \%$ of the error after the impulse force is removed. This is illustrated in Fig. 5.2.3.


Fig. 5.2.3. System recovery.

Drift

Drift is a long term error due to changes in the temperature and other conditions of the system components. It should be expressed as the maximum percentage variation from the set level per unit of change in the temperature or alternative condition.

## General Specification for Incremental and Continuous Servo Systems

In general, for both incremental and continuous mode systems, the following information should be provided:

1. Operating ambient temperature range
2. Operating ambient humidity range
3. Environment, i.e. laboratory, industrial, etc.
4. Available power supplies
5. Physical size and weight limitations
6. Life requirements

In addition, as much information as possible should be given regarding the operation and use of the final machine. Although the relevance of such information may not be apparent at first, it serves to make the servo designer as familiar as possible with the application. A complete understanding of the customer's machine is an invaluable advantage.

## Cost

In general, the advantage of using a DC servo system is that higher performance, better reliability and improved marketability of the final product can be obtained at a lower cost than by upgrading more traditional devices. A typical relationship between cost and performance is shown in Fig. 5.2.4.

Performance can be speed, accuracy, reliability or any other part of a specification which presents a significant design problem.


Fig. 5.2.4. Relationship between cost and performance of servo systems.

The exact points where the alternative methods become uneconomical will vary, depending on the application and what the user defines as performance. It is true that in any application where the aim of the designer is to increase the marketability of a product by increasing its capabilities beyond present standards, then utilization of a DC servo system should always be investigated.

### 5.3. MOTOR SELECTION CRITERIA

### 5.3.1. INTRODUCTION

There is no known formal procedure for the optimum selection of a PM servomotor. One reason is that optimum is difficult to define. Here, optimum will be defined as the most economical choice which satisfies specifications. No matter what choice is made, it is important to understand how it affects the selection of adjacent system components, and to determine if the selected motor is adequate.

The following is a discussion of PM servomotor selection factors and analytical methods for analysis of the selected motor.

### 5.3.2. WHEN TO SPECIFY

The motor, drive train, and power supply should be given simultaneous consideration as early as possible in the system design process. It is a mistake to delay motor selection. Severe compromises may be required if the choice is limited by the remaining available space, or if the drive train and amplifier are fixed without having considered their effect on motor selection. For example, the moments of inertia of solid cylinders of homogenous materials (shafts, gears, pulleys) of equal length vary as the fourth power of diameter. Therefore, early consideration should be given to the materials and the geometry of these elements, because the motor and amplifier size will depend on it.

### 5.3.3. WHAT TO CONSIDER

A thorough description of the application should be given. This information is required to select the most suitable motor type with the proper ratings and design features. The information which is required to make this decision is given below. Later, there appears a detailed discussion of how the information is used.

## 1. Load Analysis

A thorough analysis of the machine loading is required. The analysis will not only help in the selection of the motor, but may also suggest design options which could lead to a more economical design approach.

Friction loads should be described from actual measurements, if possible. A plot of load torque vs. speed over the entire operating range is useful to evaluate viscous loading. Inherent design imbalances (i.e., eccentrics) should be described completely. The cyclical actuation of brakes, cams, clutches, or random disturbances on the system should be described as accurately as possible.

Inertial loads should be described. It is instructive to separate load moment of inertia into its component parts to identify the most significant sources.

A velocity profile should be described for each operational mode. The object of examining each operational mode is to determine what constitutes "worst case" operation.

## 2. Environmental Conditions

The following should be specified:

Temperature range
Altitude
Humidity range
Presence of airborne contaminants, chemical fumes, etc.

Shock and vibration

## 3. Mechanical

The selection of all interfacing mechanical components should be given consideration to assure compatibility with the motor:

Heat sinking can have a dramatic effect not only on the motor, but also on the driven load. Good design practice will allow for proper heat conduction away from temperature sensitive machine members, as well as away from the motor.

A coupling method should be selected to prevent damage to the motor due to axial or radial forces. Conversely, motor shaft end play, radial play, and concentricity should be considered to assure proper mating to the load.

Motor mounting position should be specified.

Shaft radial and axial loads should be described, including both magnitude and direction.

Resonance problems can be avoided by closely coupling all elements of the drive train. Direct coupling is normally more economical, quieter and less prone to resonance problems.

## 4. Life

An indication of the desired life should be given. There is no analytical method for calculating motor life. But we do know that motor life is a complex function of: environment (temperature, altitude, humidity, air quality, shock, vibration, etc.), design (commutation design, bearing design, etc.), application variables (shaft loads, velocity profile, current form factor, etc.). The only conclusive method of determining motor life for a specific application is to perform life tests on an actual machine under real operating conditions. It is always good policy to perform life tests, providing that the test conditions are realistic. Attempts to simulate machine loads or alter the velocity profile for accelerated life tests can yield misleading results.

## 5. Efficiency

The PM motor is the most efficient type of electrical motor. However, in applications where the motor is operated about stall, or in an incremental mode, output/ input power ratios are not primary considerations. In applications in which the motor is operated in the torque mode, output speed is zero; therefore, efficiency is zero by definition. For incrementers, a large portion of the output power is con-
sumed changing the system energy during acceleration and deceleration.

## 6. Motor Type

PM servo motors are available in five basic configurations: torque, conventional iron core, surface wound, "printed" armature and moving coil. An attempt to make a relative comparison of them is given in Tab. 5.3.1. For certain, contradictions to the table exist because of the variation of performance available from each type. There perhaps should be an additional comparison made which indicates the design flexibility of each type: armature winding, commutation system, magnetic circuit, etc. Some types can be modified more easily and extensively than others to cause significant changes in all of the performance factors listed in the table.

For example, the low speed performance of an iron core motor can be altered significantly by: adding skewed pole shoes, skewing the lamination stack, increasing the number of commutator bars and/or changing the lamination design, etc. In fact, iron core type motors may be the most flexible in terms of design features and the most versatile in terms of being capable of operating satisfactorily in a variety of modes. Each of the other types could be considered as having evolved from it, with features designed to satisfy the more special demands of a specific class of applications.

### 5.3.4. SELECTION ANALYSIS

The initial selection of a motor for a specific application is usually based on an educated guess, considering the selection factors discussed in the preceding paragraphs. The type is determined after weighing the relative importance of the characteristics of each type. The size is estimated after making a quick calculation of the required output. The choice can be analyzed by considering the following:

## 1. Thermal Effects

The ultimate winding temperature is a primary consideration for every motor application. The calculation of armature temperature is derived below. The derivation is based on the assumption that temperature rise is proportional to the product of power dissipation and armature-to-ambient thermal resistance.

Input power

$$
\begin{equation*}
P_{i}=V I_{a} \quad[W ; V, A] \tag{5.3.1}
\end{equation*}
$$

Motor voltage

$$
\begin{equation*}
V=R_{h} l_{a}+0.7395 K_{T} n \tag{5.3.2}
\end{equation*}
$$

$$
[V ; \Omega, A, o z-\mathrm{in} / \mathrm{A}, \mathrm{krpm}]
$$

$\mathbf{R}_{\mathbf{h}}$, the motor terminal resistance at the operating temperature is given by

$$
\begin{equation*}
\mathbf{R}_{\mathrm{h}}=\mathrm{R}\left(1+\psi \Theta_{\mathrm{r}}\right) \tag{5.3.3}
\end{equation*}
$$

where $\Theta_{r}$ is the temperature rise.

| Motor Type <br> Characteristic |  | Torque | Iron core | Surface wound | Printed | Moving coil |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moment of inertia |  | large | medium | small | small | very small |
| Electrical time constant |  | long | average | short | very short | very short |
| Internal damping losses |  | medium | medium | small | large | small |
| High speed performance |  | poor | good | very good | poor | average |
| Low speed performance |  | very good | average | good | very good | good |
| Armature thermal time constant |  | long | long | average | short | very short |
| Torque/inertia ratio |  | low | medium | high | high | very high |
| Relative cost |  | medium | low | high | medium | high |
| Construction (typical) | Brush springs type | leaf | coil | coil | coil | coil |
|  | Brush orientation | radial | radial | radial | axial | radial |
|  | Commutator radius | large | medium | medium | large | medium |
|  | Magnets | Alnico | Alnico/ceramic | Alnico/ceramic | ferrite/Alnico | Alnico |

Tab. 5.3.1. Motor type comparison.

Combining (5.3.2) and (5.3.3) with (5.3.1) gives:

$$
\begin{equation*}
P_{i}=R I_{a}^{2}+R I_{a}^{2} \psi \Theta_{r}+0.7395 I_{a} K_{T} n \tag{5.3.4}
\end{equation*}
$$

The output power $\mathbf{P}_{\mathbf{o}}$ is given by

$$
\begin{equation*}
P_{o}=0.7395 T_{0} n \tag{5.3.5}
\end{equation*}
$$

[W; oz-in, krpm]
where $\mathbf{T}_{\mathbf{0}}$ is the motor output torque. The power dissipation equals

$$
\begin{align*}
& P_{L}=P_{i}-P_{o}=R I_{a}^{2}+R I_{a}^{2} \psi \Theta_{r}+ \\
& \quad+0.7395 n\left(K_{T} I_{a}-T_{o}\right) \tag{5.3.6}
\end{align*}
$$

The temperature rise $\Theta_{r}$ is then given by
$\Theta_{r}=P_{L} R_{t h}=\frac{\left[R I_{a}^{2}+0.7395 n\left(K_{T} I_{a}-T_{o}\right)\right] R_{t h}}{1-R I_{a}^{2} \psi R_{t h}}$

The term ( $\mathrm{K}_{\mathrm{T}} \mathbf{I}_{\mathrm{a}}-\mathrm{T}_{\mathbf{o}}$ ) may be recognized as the difference between internally generated torque and output torque. This difference is due to internal friction losses (brush, bearing, viscous, etc.). For many applications this term can be ignored without significantly affecting accuracy. The term $I_{a}$ in (5.3.7) is the RMS value of armature current, calculated from the given velocity profile and other system parameters.

It is not always obvious what constitutes "worst case" operating conditions. If the
machine is cycled in a variety of operational modes, temperature rise should be calculated for each mode to establish what constitutes worst case.

If the motor selected on the first try is limited because of temperature rise, the following options may be available: forcedair cooling, increased heat sinking, relaxed performance specifications, reduced loading (moment of inertia, friction, viscosity, etc.) Each of these options has drawbacks. Forced-air cooling may be provided by static type blowers which direct air axially or radially over the armature; a fan type which directs air over the external housing; or an impeller attached either internally or externally to the housing.

High pressure static blowers are in some instances more costly than the motor. They require added space, and they produce noise. Forced-air cooling is generally limited to cases where an air supply is already available. Directing a small amount of air over the housing by a fan is reasonably effective for constant power output applications; but, since the most motors armature-to-ambient thermal time constants are relatively long, there will not be good short-term overload protection. Impellers are not generally attached to servomotors because they introduce additional inertial and viscous loading - which defeats their purpose. Proper attention should always be given to the natural heat sinking available by mounting the motor to a good thermal conductor. Low inertia motors are especially susceptible to armature failures which are caused by short-term high loads. If an overload is
imposed for more than several thermal time constants (armature-to-housing), trouble can be expected. If it is anticipated that overloading will occur, the best method for protecting the armature is to provide forced-air cooling.

## 2. Acceleration Effects

Motor demagnetization can occur if the required acceleration current exceeds the motor pulse current rating. Caution should be observed when the calculated value of acceleration current approaches the rated limit. Acceleration current is:

$$
\begin{equation*}
I_{\max }=\frac{T_{g \max }}{K_{T \min }} \tag{5.3.8}
\end{equation*}
$$

where $\mathrm{K}_{\mathrm{T} \text { min }}$ is the minimum value of $\mathrm{K}_{\mathrm{T}}$, specified at $25{ }^{\circ} \mathrm{C}$ and derated by $0.2 \% /{ }^{\circ} \mathrm{C}$ or $0.05 \% /{ }^{\circ} \mathrm{C}$ of magnet temperature rise for ceramic and Alnico magnets, respectively. Demagnetization can be avoided by: limiting amplifier current, relaxing performance specifications, reducing the shaft load (moment of inertia, friction, viscosity), or selecting another motor.

If the motor selected has a long electrical time constant that is long compared to the specified response times, the motor will be unsuitable for the application. A general rule of thumb is that the motor electrical time constant should be not greater than one-fifth of the minimum allowable time required for speed changes. If the motor electrical time constant is a limitation, a different motor will have to be considered.

## 3. Power Supply Effects

If the power supply has already been specified, calculations should be made to determine if sufficient current and voltage are available. If the power supply has not been designed, the voltage and current requirement must be defined.

Maximum required current is defined by (5.3.8). Maximum torque $\mathrm{T}_{\mathrm{g} \text { max }}$ must be determined by examining the load and velocity profile to determine when the maximum combination of acceleration torque, friction torque, and viscous torque occur.

If insufficient current is available, the following options may be considered:

- Select a winding with a higher $\mathrm{K}_{\mathrm{T}}$;
- Couple the load through a gearhead to increase the torque delivered to the load;
- Select a larger motor with a higher $K_{T}$;
- Reduce the load by minimizing moment of inertia, friction, and viscosity.

The maximum required terminal voltage is given by:

$$
\begin{array}{r}
V_{\max }=\frac{T_{g \max }}{K_{T}^{\prime}} R_{h}+0.7395 K_{T}^{\prime} n_{\max } \\
{[V ; \text { oz-in, } \Omega, \text { oz-in/A, krpm }]} \tag{5.3.9}
\end{array}
$$

It may not be obvious if the maximum or minimum value of $\mathrm{K}_{\mathrm{T}}$ should be used to evaluate $\mathbf{V}_{\text {max }}$. Therefore, $\mathbf{K}^{\prime}{ }_{T}$ is intended to indicate whichever value of $\mathbf{K}_{\mathbf{T}}$ that
results in $\mathrm{V}_{\text {max }}$. Both maximum and minimum values of $K_{T}$ should be considered and the one selected which results in the maximum voltage.

### 5.4. APPLICATION OF MOTOMATIC SYSTEMS

The first problem the designer faces is the selection of a basic motor and control system. Next, he must choose a controller with the needed features. The system selection will principally be based on required speed and torque.

Applications in this category usually have three characteristics: (1) system acceleration and deceleration characteristics are not critical, i.e., they can be measured in time intervals of tenths of a second rather than in milliseconds; (2) speed should be held constant at the set value over a wide range of load torques; and (3) the speed should be controllable over a relatively broad range, say $10: 1$ to $1000: 1$. Applications fitting these categories are numerous; and while the transistor speed control solution may not always be as apparent to potential users as other drive methods, in sections 5.5 . and 5.6. we will present a variety of successful applications which demonstrate the versatility of the transistorized control system. While a speed controller for a silicon crystal puller may not seem likely to be similar to one for a blood cell separator, from the motor viewpoint they are just the same. Once your machine requirement is analyzed from the control system viewpoint, motor and control selection is considerably simplified.

To select the optimum motor and speed controller for your apparatus, you need to know the following:

1. Speed range required
2. Output torque required
3. Maximum ambient temperature
4. Other features needed (speed meter, torque meter, reversing switch, dynamic braking, 1 -turn or 10-turn speed control potentiometer, etc.)

### 5.4.1. SPEED RANGE

The speed range must be exactly determined. It is necessary to determine whether it is preferable to employ a direct-drive motor or to use a gearhead. For example, if the load speed range is 0.1 rpm to 10 rpm , it is apparent that a 100:1 ratio gearhead would change the motor speed range to 10 rpm to 1000 rpm . Better still, a 200:1 gearhead would place the motor speed range at 20 rpm to 2000 rpm - a region of better speed stability.

A brief explanation at this point will clarify how to select the operational speed range that will result in optimum speed stability and regulation. The speed stability and regulation properties of a speed control system depend primarily on the "signal-tonoise" level of the feedback voltage. In the upper portion of the speed range, the feedback signal is much larger than the noise voltages generated in the amplifier, wiring and tachometer. However, as the speed is decreased toward the lower end of the available speed range, noise signals tend to
become increasingly prominent. This is a physical fact which limits any analog feedback system.

Many times engineers consider using Motomatic systems to operate continuously in the lowest portion of the available speed range, simply because this would permit use of the output shaft in a direct-drive fashion without any gearing. While this may seem at first to be practical, the result would be the minimizing of available speed stability. If high speed stability and regulation are important, the proper solution is to insert a speed reduction between the motor and the load - thereby using a motor speed range which will yield best performance. Fig. 5.4.1 presents the speed-torque characteristics of a Motomatic E-650 speed control with regulation contours, showing that the optimum performance range is between 50 and 2000 rpm . This is the range in which one should strive to keep control applications, especially when high performance is a factor.

There are, of course, special applications where a primary machine operation must be performed in the very low end of the speed range, with occasional excursions into the high speed range. An example of this performance requirement is microfilm scanning, where a high speed search for the general area is followed by visual scanning in a very low speed mode - where any speed instability would be optically magnified, making visual scanning difficult. In such instances, design features are incorpo-


Fig. 5.4.1. Typical Motomatic E-650 speed-torque characteristics with regulation contours.
rated in the tachometer and amplifier to minimize instabilities due to ultra low speed operation.

### 5.4.2. OUTPUT TORQUE AND AMBIENT TEMPERATURE

For determination of torque requirements of the load, two questions must be asked:

1. Is the precise time which the motor takes to accelerate or decelerate the load a critical factor?
2. Does stopping and starting occur frequently?

If the answer to either of these questions is yes, then the torque required to accelerate the load must be calculated, and the system selected to provide this torque. If both
answers are no, then the maximum running torque is the rated torque required from the system.

Gear ratio selection must also be based on the required output torque. After the motor output torque figures have been adjusted for ambient temperature conditions (Fig. 5.4.2), the adjusted maximum torque can be multiplied by the gear ratio to get the theoretical output torque. The efficiency of gearheads varies between 60-70\% for worm gearheads and between 75-85\% for spur gearheads. We can arrive at a practical output torque figure by multiplying the theoretical torque by the efficiency figure. We must also consider the maximum torque capacity of the output gear. This is usually the limiting case in applications involving very high gear ratios.


Fig. 5.4.2. Temperature derating curve for Motomatic motor-generators.

For example, consider the case of an ElectroCraft E-650 motor rated at 5 lb -in with a 1000:1 gear ratio, and an efficiency figure of $80 \%$. The theoretical possible output torque would be calculated as follows:

$$
T_{p}=0.80 \times 10^{3} \times 5=4000 \mathrm{lb}-\mathrm{in}
$$

but the maximum allowed output torque shown in the selection guide for this gearhead in Tab. 5.4.1 shows a maximum allowed continuous torque of only $80 \mathrm{lb}-\mathrm{in}$; i.e., $1 / 50$ of the possible torque at the output shaft.

If such a hypothetical gearhead-motor combination were stalled on the output side, the continuous torque rating could be exceeded by a factor of 20 . In such a case, the gearhead would probably break, unless the torque limiter had been adjusted to hold the maximum input torque to a protective value.

E-550 MGHP (Precision Gearhead)

| Ratio | $10: 1$ | $30: 1$ | $50: 1$ | $100: 1$ | $500: 1$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Maximum <br> Output <br> Torque [lb-in] | 6 | 20 | 25 | 45 | 50 |

E-550 MGHD (Heavy-Duty Gearhead)

| Ratio | $5: 1$ | $10: 1$ | $50: 1$ | $100: 1$ | $200: 1$ | $500: 1$ | $1000: 1$ | $5000: 1$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Running <br> Torque [lb-in] | 3 | 6 | 25 | 50 | 80 | 80 | 80 | 80 |

E-650 MGHR (Right Angle Gearhead)

| Ratio | $5: 1$ | $10: 1$ | $20: 1$ | $30: 1$ | $40: 1$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Maximum <br> Output <br> Torque [lb-in] | 7.5 | 30 | 40 | 40 | 40 |

E-650 MGHS (Standard Spur Gearhead)

| Ratio | $50: 1$ | $100: 1$ | $200: 1$ | $500: 1$ | $1000: 1$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Running <br> Torque $[\mathrm{ib}-\mathrm{in}]$ | 27 | 80 | 80 | 80 | 80 |

Tab. 5.4.1. Motomatic gearheads data.

Another factor which has to be considered is the effect of inerital loads on the gearhead. Because of the high acceleration and deceleration potential of a Motomatic mo-tor-tachometer, it is relatively easy to induce high peak forces in the gearhead - at times high enough to break a gear tooth. If the application indicates that high inertia loads, input forces or overhung loads are apt to cause problems, contact ElectroCraft's Customer Engineering Department.

Figs. 5.4.3 and 5.4.4 illustrate motor speed ranges vs. torque for Motomatic systems with example reduction gear ratios.

Having determined the speed range and maximum torque requirements, the load characteristics can be superimposed on the system performance graphs (see Fig. 5.4.1) and the correct system selected.

### 5.4.3. SPEED REGULATION AND STABILITY

The terms "speed regulation" and "speed stability" can sometimes cause misunderstandings between motor manufacturers and system designers.

Speed regulation is defined as the change of motor speed due to a change in output torque. Thus, at Electro-Craft, a change of motor speed from 1000 rpm at no-load to 990 rpm at full torque is considered $1 \%$ speed regulation. This is a most important distinction. At Electro-Craft, we rate speed regulation as a percentage of "set speed". Most controller specifications define it as a percentage of "top speed". This distinction


Fig. 5.4.3. Operational contours for E-550 speed control system with various gear ratios.
becomes most important at the low end of the speed range. Thus, a 10 to 3000 rpm , $1 \%$ speed regulation Electro-Craft Motomatic controller at, say, 100 rpm may slow down to 99 rpm at full load torque. A conventional controller with a $1 \%$ speed regulation may slow down to 70 rpm ( 100 $\mathrm{rpm}-1 \%$ of 3000 rpm top speed). Almost the same specification; quite a difference in actual performance!

As shown in Fig. 5.4.1, these regulation figures vary with the operational speed
range. The regulation values for speed control systems should not be compared with open-loop motor speed regulation constant ( $\mathbf{R}_{\mathrm{m}}$ ) discussed in Chapter 2. The reason why the system speed regulation is so much better than the motor speed regulation is that the tachometer and the amplifier minimize the regulation effect by the feedback connections.

Speed stability is a term consisting of two basic parts: short-term stability, covering such disturbances as oscillation, "jitter" and electrical noise "pickup" induced velocity errors; and long-term drift velocity errors, usually due to amplifier input stage drift and temperature-induced tachometer gradient changes. The latter effect is discussed in section 2.11.

The short-term effects, such as oscillations and jitter, can either be caused by the control system itself, or be induced by the load or the coupling between load and motor. Motomatic control systems are compensated to exhibit good speed stability over a wide range of speeds, and over a wide range of load conditions. It sometimes happens that a large inertia load is coupled to a motortachometer through a compliant (springy) coupling or through gears with backlash, and the result is that some instability will be noticed at certain speeds. This problem can usually be corrected by minimizing the offending condition.

Systems requiring a high degree of speed stability over a very wide speed range may sometimes experience unacceptable instan-


Fig. 5.4.4. Operational contours for E-650 speed control system with various gear ratios.
taneous speed variations (ISV) due to the magnetic coupling between the motor and tachometer windings. These ISV's occur because the bifilar winding of the Motomatic motor-generator acts as an electrical transformer, superimposing disturbances originating in the motor winding on the tachometer output. These disturbances normally occur outside of the frequency response of the system and have little effect. However, there are critical speeds at which the disturbances can cause measureable ISV's. The remedy for this situation is to use a motor with a tachometer generator mounted on the same shaft, but isolated
electromagnetically from the motor winding. Such motors would be the E-576 as a substitute for the E-550 MG, and the E-676 as an alternative to the E-650 MG. This phenomenon is not usually significant, and can be ignored by $95 \%$ of users; but if a user believes that his system would be sensitive to such effects, he should contact the factory for advice.

The long-term drift is usually very small, since the Motomatic amplifier input stage has either temperature-matched differential input transistors or integrated circuits which serve to minimize drift.

The tachometer temperature coefficient errors are predictable, since for a given tachometer the maximum temperature coefficient is defined. When temperature conditions are known, it is a simple matter to predict the resulting errors. In cases where long-term errors cause problems, it is possible to provide air cooling of the tachometer, a heat radiator, or other means to minimize temperature effects.

### 5.4.4. MOTOMATIC SPEED CONTROL FEATURES

The outstanding standard features of Motomatic motor speed control systems are:

1. Wide speed range, $1000: 1$;
2. High speed regulation accuracy, up to $\pm 1 \%$ of set speed depending on model;
3. Smooth, low cogging output at all speeds;
4. Small physical size;
5. High reliability.

In order to provide the greatest flexibility in adapting Motomatic systems to individual requirements, a number of features are offered, some of which are standard and others optional. Product description found in sections 5.4.5., 5.4.6. and in Chapter 6. lists which models contain particular features. A general description of what these features provide is given below.

## Speed and Torque Readout

This is a meter which is calibrated to display either motor speed or output torque, as selected by a switch. This feature is supplied as standard on master controls and as an option for panel mounting of "open" controls.

## Dynamic Braking

This technique is used in cases where a motor must be decelerated at a faster rate than load friction can provide. The dynamic braking feature consists of a load resistor which is automatically switched across the motor armature by a transistor circuit anytime the motor speed exceeds the control command. For example, if the load moment of inertia in an application is high and the friction very low, deceleration time may prove excessive when the speed control command is reduced. With dynamic braking, the deceleration time will be vastly im-
proved; the system will follow commands both in acceleration as well as in deceleration.

## Reversing Option

This feature consists of a switch which permits the selection of clockwise, stop, or counterclockwise operation. In the stop mode, the motor is dynamically braked by insertion of a resistor across the armature.

## Adjustable Torque Limiting

This feature consists of a potentiometer which can be adjusted to provide a limit of the maximum output torque to any desired value. For example, if the load were such that it could be damaged by excessive motor torque, an adjustment of the torque limiter to a value below that in which damage could occur would provide the needed protection. The torque limiting circuit is completely automatic, and has a response time of a few milliseconds.

## Remote Control Input

The speed setting or command may, in some applications, need to be controlled automatically from an external source rather than by the operator. Most Motomatic systems have a remote control input which can serve either of two functions: (1) speed can be controlled from any sensor whose output is a DC voltage sufficient to cover the needed input range, and (2) the speed adjustment potentiometer or selector switch can be panel-mounted at a separate location
from the controller. Accompanying the remote input is a regulated voltage supply so that the remote input sensor can be as simple as a variable resistor, thermistor, photocell, linear potentiometer, etc.

### 5.4.5. SERIES E-550 MOTOMATIC CONTROLS

Fig. 5.4.5 shows an Electro-Craft E-550 control. The E-550 series controls have the following features:

## Basic specifications:

Torque 10 oz-in continuous, speed range 5-5000 rpm.

Master controls only:
Electronic tachometer: Indicates motor speed in revolutions per minute.

Electronic torquemeter: Indicates motor load torque in ounce-inches.


Fig. 5.4.5. Electro-Craft E-550 Master Control

Remote input: A provision for controlling motor speed by an external potentiometer or by a DC voltage derived from any source.

## Master and Standard controls:

Reverse-brake switch: Reverses motor rotation through a center stop-brake position.

## E-550 Front Control Panel

Power switch - this switch connects or disconnects the 115 VAC line for the control.

Power light - when this light is on, it indicates the power switch is in the ON position, and that the fuse and circuit breaker are normal.

Reverse-stop switch - this switch permits reversing of the motor through a center brake position. In the brake position, both the motor and generator leads are shorted through a current limiting resistor, bringing the motor to a stop in less than 0.5 s from full speed under no-load conditions.

Speed adjust control - this is a linear single turn potentiometer used to set speed between 0 and 5000 rpm .

Scale selector switch and meter - with the scale selector switch in the $0-5000 \mathrm{rpm}$ position, the meter will indicate within $5 \%$ the speed of the motor in revolutions per minute. With the scale selector switch in
the $0-12$ oz-in position, the meter indicates the motor output torque within $10 \%$ of reading.

## E-550 Rear Control Panel

Motor-generator connector - located in the lower left hand corner of the chassis.

Meter calibration control - this potentiometer is used to calibrate the tachometer to correctly indicate motor speed. To calibrate, use a strobe light locked to the line frequency at 3600 rpm and set motor speed at 3600 rpm . Loosen locknut on potentiometer, adjust meter to read 3600 rpm . Spot check at other speeds, and divide error if necessary.

Fuse - this is a 0.75 A slow blow fuse in series with the $A C$ line.

Circuit breaker - this is a 2 A circuit breaker in series with the motor designed to prevent prolonged overloads of the motor and control.

Remote input switch and jacks - with the command switch in the remote position, the remote control jacks are switched into the circuit and the front panel control is switched out. Speed can be controlled by: (1) remotely placed potentiometer 5 to $10 \mathrm{k} \Omega$; (2) a $0-20.5 \mathrm{VDC}$ signal with a 5 to $10 \mathrm{k} \Omega$ source impedance; (3) selector switch.

Remote control connections are shown in Fig. 5.4.6. Examples of control voltage/ speed relationships:


Fig. 5.4.6. Remote control mode connections for E-550 controls.
20.5 VDC input produces a motor speed of approximately 5000 rpm .

10 VDC input produces a motor speed of approximately 2500 rpm .

## E-550 Operating Instructions

1. Make sure POWER SWITCH is in the OFF position.
2. Insert the motor-generator plug into the jack on the rear panel.
3. Connect the control LINE CORD to any approved $115 \mathrm{~V}, 50 / 60 \mathrm{~Hz}$, grounded 3 -wire receptacle.
4. Turn the SPEED ADJUST CONTROL to the counterclockwise stop.
5. Place the COMMAND SWITCH (on the rear panel) in the NORMAL COMMAND position.
6. Place the SCALE SELECTOR in the $0-5000 \mathrm{rpm}$ position.
7. Place the REVERSE-STOP SWITCH in the STOP position.
8. Place the POWER SWITCH in the ON position. The red light just above the switch should light. If not, check the fuse and circuit breaker.
9. Place the REVERSE-STOP SWITCH in the $C W$ position and slowly rotate the SPEED ADJUST CONTROL clockwise. The motor shaft should rotate smoothly in a clockwise direction looking at the shaft end of the motor. The tachometer will show an increasing reading as the SPEED ADJUST CONTROL is rotated clockwise.
10. Place the REVERSE-STOP SWITCH in the CCW position. The motor shaft should turn in a counterclockwise direction, looking at the motor at the shaft end.
11. Place the SCALE SELECTOR in the $0-12$ oz-in position. As the load on the motor is increased, the torque meter will indicate an increasing reading.

Figs. 5.4.7 and 5.4.8 are outline drawings of the E-550 Master and E-550 Open Controls.

## Installation

When installing Electro-Craft motor-generators on any application, the following procedure should be carefully followed.

1. Mount securely on a rigid foundation.
2. Align all shafts accurately. The motorgenerator shaft and the driven machine shaft should be in line as near perfectly as possible. Failure to maintain proper alignment can result in excessive noise, overheating and part wear. Flexible couplings can be used to compensate for slight misalignment. Care must be used in selection


Fig. 5.4.7. Outline drawing, Electro-Craft E-550 Master Control.


Fig. 5.4.8. Outline drawing, Electro-Craft E-550 Open Control.
of couplings, as any compliance in the coupling material may cause the control to sense a false change in load requirements. This can cause the motor speed to oscillate about its setting.
3. Observe all limits listed in the specifications concerning maximum overhung loads, side thrust and end thrust.
4. For best operation, the motor-generator should be mounted in an area where air circulation is available.

## Maintenance

No lubrication is required, or should be used. If the motor-generator is equipped with a gearhead, lubrication instructions provided by Electro-Craft and/or the gearhead manufacturer should be followed carefully.

Under normal operating conditions, a 3000 hour brush life can be expected. If brushes are removed, they must be replaced in the same brushholder with the same orientation. If brushes are replaced, a 24 hour run-in period is recommended. Use only Electro-Craft brushes.

Disassembly of the motor-generator is not recommended. If disassembly is absolutely required, please contact Electro-Craft before proceeding.

### 5.4.6. SERIES E-650 MOTOMATIC CONTROLS

Shown in Fig. 5.4.9, Electro-Craft E-650 controls have the following features:

Basic specifications:
Torque 5 lb -in continuous; speed range 3-3000 rpm.

Master controls only:

Dynamic brake - this circuit allows more rapid deceleration of motor speed. It essentially places a short circuit across the motor winding during a down speed command. It is less effective at speeds under 100 rpm .


Fig. 5.4.9. Electro-Craft E-650 Master Control.

Torque limiting circuit - this circuit is, in effect, an electronic clutch. By limiting the current to the motor at any preselected level, the maximum torque can be adjusted to prevent damage to the motor, control and driven equipment in the event of a jam or stall.

Electronic tachometer - indicates motor speed in revolutions per minute.

Electronic torquemeter - indicates motor load torque in pound-inches.

Remote input - a provision for controlling motor speed by an external potentiometer or by a DC voltage derived from any source.

## Master and Standard controls

Reverse-brake switch - reverses motor rotation through a center stop-brake position.

## E-650 Front Control Panel

Power switch - this switch connects or disconnects the 115 VAC line for the control.

Power light - when this light is on it indicates the power switch is in the ON position, and that the fuse and circuit breaker are normal.

Reverse-stop switch - this switch permits reversing of the motor through a center brake position. In the brake position, both the motor and generator leads are shorted through a current limiting resistor bringing the motor to a stop in less than 0.5 s from full speed under no-load conditions.

Speed adjust control - this is a 10 -turn potentiometer, and is calibrated to indicate the percentage of top speed ( 3000 rpm ) within $3 \%$.

Scale selector switch and meter - with the scale selector switch in the $0-3000 \mathrm{rpm}$ position, the meter will indicate the speed of the motor in revolutions per minute. With the scale selector switch in the $0-5$ lb -in position, the meter indicates the motor output torque within $10 \%$ of reading.

## E-650 Rear Control Panel

Motor-generator connector - located in the rear lower left hand corner of the chassis. When the motor-generator is plugged into this connector, the lock should be turned so the connector cannot become disengaged through vibration or accident.

Fuse - this is a $5 \mathrm{~A}, 250 \mathrm{~V}, 3 \mathrm{AG}$ fuse in series with the circuit breaker and the $A C$ line.

Meter calibration control - this potentiometer is used to calibrate the tachometer to correctly indicate motor speed. To calibrate, use a strobe light locked to line frequency at 1800 rpm and set motor speed at 1800 rpm . Loosen locknut on the potentiometer and adjust meter to read 1800 rpm. Spot check at other speeds, and divide errors if necessary.

Torque control - this is the control for the electronic torque limiting circuit. If a load greater than the pre-selected level is applied, the motor and control cannot deliver additional torque, thus preventing damage to the driven equipment. Torque limiting can also be used to control acceleration. With the control in the minimum position,

## Mode



Remote potentiometer

Mode 2


Mode 3


Remote selector switch (any number of discreet speeds)

CAUTION: DO NOT GROUND REMOTE INPUT TERMINALS
Fig. 5.4.10. Remote control mode connections for E-650 controls.
the maximum torque that can be delivered is approximately $1.5 \mathrm{lb}-\mathrm{in}$. With the control in the maximum position, the control and motor-generator can deliver torque in excess of 5 lb -in. However, a load greater than 5 lb -in (red area) for a sustained period will result in damage to the control or mo-tor-generator, or both.

Remote input switch and .jacks - with the command switch in the remote position, the remote control jacks are switched into the circuit and the front panel control is switched out. Speed can be controlled by: (1) a remotely placed potentiometer of 30 to $50 \mathrm{k} \Omega$; (2) a $0-100 \mathrm{VDC}$ signal with a 30 to $50 \mathrm{k} \Omega$ source impedance; (3) the selector switch. Connections are shown in Fig. 5.4.10.

NOTE: In mode 2, speed stability is dependent on the stability of the remote control voltage. Maximum speed of the motor is also proportional to remote input voltage; e.g., a 100 VDC input voltage produces approximately 3000 rpm motor speed, and a 50 VDC input voltage produces approximately 1500 rpm. This can be changed by factory adjustment of the controller.

## E-650 Operating Instructions

1. Make sure the POWER SWITCH is in the OFF position.
2. Connect the motor-generator cable to the connector on the rear of the chassis.
3. Connect the control LINE CORD to any approved $115 \mathrm{~V}, 50 / 60 \mathrm{~Hz}$, grounded, 3 -wire receptacle.
4. Turn the SPEED ADJUST CONTROL fully counterclockwise.
5. Place the COMMAND SWITCH on rear panel in the NORMAL COMMAND position.
6. Place the SCALE SELECTOR in the $0-3000 \mathrm{rpm}$ position.
7. Place the REVERSE STOP SWITCH in the STOP position.
8. Place the POWER SWITCH in the ON position. The red light just above the power switch should light. If not, check the fuse located on the back of the chassis and reset circuit breaker.
9. Place the REVERSE-STOP SWITCH in the $C W$ position and slowly rotate the SPEED ADJUST CONTROL clockwise. The motor shaft should rotate smoothly in a clockwise direction looking at the shaft end of the motor. The tachometer will show an increasing reading as the SPEED ADJUST CONTROL is rotated clockwise.
10. Place the REVERSE STOP SWITCH in the CCW position. The motor shaft should turn in a counterclockwise direction, viewing the shaft end of the motor.
11. Place the SCALE SELECTOR in the $0-5 \mathrm{lb}$-in position. As the load on the
motor is increased, the torque meter will indicate an increasing reading.

Figs. 5.4.11 and 5.4.12 are outline drawings of the E-650 Master and Open Controls.


Fig. 5.4.11. Outline drawing, Electro-Craft E-650 Master Control.

## Installation

When installing Electro-Craft motor-generators on any application, the following procedure should be carefully followed.

1. Mount securely on a rigid foundation.
2. Align all shafts accurately. The motorgenerator shaft and the driven machine shaft should be in line as near perfectly as possible. Failure to maintain proper alignment can result in excessive noise, over-


Fig. 5.4.12. Outline drawing, Electro-Craft E-650 Open Control.
heating and part wear. Flexible couplings can be used to compensate for slight misalignment. Care must be used in selection of couplings, as any compliance in the coupling material may cause the control to sense a false change in load requirements. This can cause the motor speed to oscillate about its setting.
3. Observe all limits listed in the specifications concerning maximum overhung loads, side thrust and end thrust.
4. For best operation, the motor-generator should be mounted in an area where air circulation is available.

## Maintenance

No lubrication is required or should be used. If the motor-generator is equipped with a gearhead, lubrication instructions provided by Electro-Craft and/or the gearhead manufacturer should be followed carefully.

Under normal operating conditions a 3000 hour brush life can be expected. If brushes are removed they must be replaced in the same brushholder with the same orientation. If brushes are replaced, a 24 hour run-in period is recommended. Use only Electro-Craft brushes.

Disassembly of the motor-generator is not recommended. If disassembly is absolutely required, please contact Electro-Craft before proceeding.

### 5.5. APPLICATION EXAMPLES

In this section we will discuss a variety of applications of Electro-Craft Motomatic and other controls taken from our customer files. Some figures and specific details have been altered to protect proprietary information. These examples are set forth to illustrate the versatility of the controls, and to show how they have met requirements in applications varying from office machine drives to medical instrumentation controls.

We hope these "case histories" will help readers realize that applications of packaged speed control systems are limited only by the imagination of the potential users.

## OFFICE COPYING MACHINE

An office copying machine had a manuallyoperated mechanical speed control device, which provided exposure control for an infrared sensitive copy medium. In the initial design, the operator had to watch a temperature gage and continuously adjust the speed to match the varying temperature of the infrared source. The mechanical speed control had a reliability problem; the manual adjustment resulted in a high waste factor due to operator inattention.

The Electro-Craft E-500 motor-generator, and a modified version of our standard speed control system were selected for this application. This E-500 MG and control, as are any of the other Motomatic systems, is ideally suited for automatic response to changes in not only temperature, but light,
pressure, or any other parameter which can be converted to a resistance of voltage change.

Fig. 5.5.1 shows the modifications to the standard control which were necessary to meet this application's requirements. The thermistor is exposed to the same infrared source as the original and sensitized paper. The potentiometer $\mathbf{R}_{\mathbf{2}}$ is a manual speed adjust, and is set to achieve desired copy contrast. The resistor $\mathbf{R}_{\mathbf{3}}$ sets a minimum speed limit to prevent heat damage from overexposure. The design of the combination of thermistor and resistors in the sensor-command chain had to be coordinated with the desired response of the control.


Fig. 5.5.1. Example of a temperature-sensitive speed control input stage.

During the copying process, the increase in temperature in the exposure area is sensed by the thermistor, causing a decrease in its resistance. The resistance decrease causes the command voltage to rise, increasing copying speed. As the machine cools, the thermistor's resistance increases and the drive speed decreases. These interactions maintain the selected heat/speed ratio.

The use of the $\mathrm{E}-500$ system in the redesign improved the office copying machine in the following respects:

1. automatic response to temperature and speed command changes;
2. reliable exposure control, and increased customer satisfaction because of lower waste;
3. performance reliability was greatly increased;
4. reduced noise level;
5. moderate additional system costs.

## HYDRAULIC MOTOR DRIVES

A hydraulic motor utilized a system of pressurized hydraulic vanes to perform torque amplification, thereby providing a very powerful rotating shaft output. The output shaft was to follow very closely the speed and position of an input shaft requiring very low input torque. The customer wanted a suitable servomotor and control which could
command the hydraulic motor, giving it very broad speed range and optimum speed response.

The Motomatic E-550 system was selected for this application because its speed range more than met the manufacturer's needs, and the torque/moment of inertia ratio of the system permitted the responsive acceleration and deceleration needed by machine tool users. The E-550 MG and hydraulic motor are shown in Fig. 5.5.2.


Fig. 5.5.2. The E-550 MG utilized as an input drive for hydraulic motor.

In much the same way as the rotary hydraulic motor is a torque amplifier, the linear hydraulic motor is a force amplifier. The input motor shaft (in this case an E-650 MG) controls speed and displacement of the actuating arm. The unit shown in Fig. 5.5.3 is capable of applying a force of several hundred tons through a displacement of several meters. Quite clearly, reliability and broad speed range are the two key advantages of a closed-loop control system in this application.


Fig. 5.5.3. Linear hydraulic motor utilizing an E-650 MG as input drive. (Photo courtesy of Précision Mécarique Labinal, France)

## AUTOMATIC RETRIEVAL SYSTEM

A manufacturer of automated storage systems needed a high-response drive system to power the carriage in their $\mathbf{x}-\mathbf{y}-\mathbf{z}$ motion control arrangement. The customer


Fig. 5.5.4. The CONSERV-A-TRIEVE automated storage system. (Photo courtesy of Supreme Equipment and Systems Corp., Brooklyn, N.Y.)
selected the E-678 motor on the basis of its conformance to performance requirement.

Since this customer operates a circuit design facility, he chose to design a special servo circuit to fit the unique requirements for a wholly automated retrieval system. A photograph of the system is shown in Fig. 5.5.4.

## SPEED-TORQUE ANALYZER

A producer of custom-made gearing desired to measure accurately the friction torque of his gears. He provided his customers with data showing friction torque vs. speed, and on request produced friction data of the gearing at the customer's specified operating speed. This would allow his customers to take necessary measures to handle friction in their system designs. This application is typical of many situations where friction torque measurements are necessary. It can also be applied to the elimination of design errors due to inadequately stated data by providing static torque measurements and by monitoring shaft speed.

Using a Motomatic E-550 speed control system, his procedure for measuring any unknown torque is as follows:

1) with the motor unloaded, adjust motor speed to the desired setting by reading speed on the Motomatic speed meter.
2) with the motor operating at no load, record the torque reading on the Motomatic electronic torque meter. This reading will be the sum of internal motor friction torque and damping torque at the selected speed.
3) connect the unknown friction load and, again operating at the desired speed, record the reading on the torque meter.
4) the actual friction torque is the difference between the readings obtained in steps 2 and 3 above.

The E-550 speed control system gives both readout of speed and readout of the developed motor torque.

## ENERGY CHOPPER

The measurement of radiant energy requires the energy input to the sensor be chopped at a given frequency to separate the true signal from undesirable background and noise. A reference synchronization pulse is generated synchronously with the resultant chopped energy signal as shown in Fig. 5.5.5. In laboratory testing, where it is desirable to provide optimum chopping for a detector, the blade must be changed in order to cover a total frequency range of 0.65 Hz to 1 kHz . To fulfill this requirement with a minimum of operator effort, a multi-speed motor control was required.

The standard E-550 motor-generator and open control was selected for this application. The wide speed range (1000:1) of this control system made feasible the concept of a one plug-in selector card for the detector. With this selector card and its related chopper blade, the appropriate sensor detector instrument turning and mechanical chopping rate were automatically provided.

Utilizing the E-550 control system, the laboratory energy chopper was greatly improved over previous designs. Besides the feasibility of the one plug-in card for all adjustments, the system was improved in the following respects:

1) the wide speed range of the E-550 control system allowed a larger total frequency range to be scanned;
2) the accurate speed regulation of the control system (better than 0.5\%) improved accuracy of readings;
3) laboratory tests were repeatable with the E-550;
4) the long term speed stability of the E-550 provided accurate reading whether the laboratory instruments were operating for just minutes or for 18 or more hours a day;
5) the constant speed provided by the $\mathrm{E}-550$ system, which is independent of load torque variations and line variations, maintained a constant chopper frequency.


Fig. 5.5.5. Infrared energy chopper system.

This system, in effect a multispeed chopper, provides optimum chopping frequencies for virtually any type of detector.

Motomatic controls can be utilized in many other types of laboratory equipment, and will provide improved system performance as noted above. Applications quite similar to the above are: laboratory stirrers, industrial drilling machines, centrifuges and viscosimeters. Any application requiring high speed stability and regulation, and constant speed irrespective of load and line variations, can utilize Motomatic systems.

## LABORATORY STIRRER

A laboratory instrument manufacturer needed a wide range speed control system to
power a laboratory stirrer. The system should have high torque capacity and good regulation to accommodate a variety of viscosity levels and stirring speed requirements.

A Motomatic E-650 system was designed to fit the customer's requirements; the resulting design (Fig. 5.5.6) is a versatile, dependable laboratory stirrer which has found wide-spread use in laboratories.

In another case, a special automatic laboratory stirrer was designed to supply oxygen release at a controlled, adjustable rate for tissue-growing experiments.


Fig. 5.5.6. Application of E-650 standard and master controls for Servodyne Laboratory Stirrer. (Photo courtesy Cole-Parmer Instrument Co., Chicago, III.)

The relationship between oxygen release and stirring speed is nearly exponential, as shown in Fig. 5.5.7. The "oxygen feedback gain" would vary depending on which portion of the curve the system would operate. However, since the working range was limited, the gain change was only $4: 1$, which was within the stability margin of the system.

Since the oxygen release time constant was on the order of 100 s , it necessitated a special operational amplifier to provide feedback compensation in the "outer loop" to assure stability (Fig. 5.5.8). The "inner" velocity loop consisted of an E-550 Motomatic control.

## FILAMENT WINDING MACHINE

A customer wanted a versatile filament winding machine which would feature vari-


Fig. 5.5.7. Relationship between oxygen release and stirring speed in a tissue culture.
able mandrel speed, and also have a traverse mechanism with an adjustable traverse rate which would follow the varying mandrel speed in a proportional fashion.

The solution was a dual speed control arrangement shown in Figs. 5.5.9 and 5.5.10. The E-650 gearhead motor-generator drives the winding mandrel through a coupling. The E-650 control system supplies generator feedback voltage to the E-550 control as a reference voltage; thus the traverse drive is always "slaved" to the speed of the winding drive. The ratio between the two speeds is adjustable over a wide range, however, and the filament winder can accommodate a range of filament widths and spacing requirements.

This control method is adaptable to a host of other operations, requiring close ratio control of two variables.


Fig. 5.5.8. Setup for oxygen release control.

## REWINDER SPEED CONTROL

The application illustrated in Fig. 5.5.11 is a system used in textile fabrication for an operation called "center winding", but the method is usable for wire, fiber, paper, film, and any other material which must be rewound at a constant linear velocity.

Since the center winding process requires constant yarn speed, the shaft speed of the drive motor must be inversely proportional to the material roll diameter.

An E-650 control system connected as shown provides the needed performance. A gear and a magnetic pickup provide in-


Fig. 5.5.9. Dual motor filament winding machine arrangement.
dication of yarn velocity. The gear is mounted on a small roller which rides on the surface of the material roll. Pulses from the pickup are converted to a voltage


Fig. 5.5.10. Electrical diagram for the dual motor system of Fig. 5.5.9.
proportional to pulse rate (hence, to yarn speed). Comparing this voltage with the reference preset yarn speed, an error signal is obtained and fed to the DC control. The motor speed gradually decreases to maintain equality between the reference and the feedback voltage, thus holding yarn speed essentially constant during the roll buildup.

## MULTI-MOTOR CONTROL FOR INDUSTRIAL KNITTING MACHINES

A typical multi-motor control system shown in Fig. 5.5.12 includes power supplies, control circuits, safety circuits and motorgenerators to provide variable feed rates and sequences for feeding the material into knitting machinery.


The desired system was to provide adjustable output speeds in both incremental and continuous modes. High reliability and minimum downtime were also required.

Speed range and output torque requirements dictated that an Electro-Craft E-650 system be utilized. Off-the-shelf E-650 Motor-Generators with right angle gearheads (Fig. 5.5.13) were used. The control circuits are modified versions of off-theshelf items.


Fig. 5.5.12. Eight-motor knitting machine control.

Fig. 5.5.14 shows the circuit diagram of a typical multi-motor control system. The potentiometer R1 is the master speed adjust, and sets the base speed for all systems. Differences between voltages on the wiper of R1 and the output of G1 produce the error signal for speed alterations. In a multi-motor system all generators have a common connection to R1. A speed input adjustment causes speed changes in all systems. Resistors R2 and R3 from a voltage divider for the generator output. R2 is a gradient adjustment, setting a ratio between speed command signals from R1 and speed changes at the motor-generator shaft. The
gate switch ,S1, opens and closes the control amplifier input, starting and stopping the motor-generator. In addition, a junction point is provided in the speed adjust line for common sequences of all systems. The transistorized control amplifier provides the drive for the power output stage which controls the motor and a circuit breaker. The contacts of the circuit breaker are in series with a main breaker and a safety line which monitors the roving. If overloading occurs in the motor-generators or the power supply, or a break in roving occurs, the entire feed system will shut down.


Fig. 5.5.13. Electro-Craft motor with right angle gearhead.

Reliability in environmental extremes and minimum downtime are designed into the system by selection of high grade industrial components, assembled in easily removable modules. Modifications or additions to this scheme include meters to monitor speed and/or torque for each motor-generator, remote speed control, variable torque limiting, dynamic braking and reversing.


Fig. 5.5.14. Typical multiple motor control circuit.

Knitting, tension sensing winding, spooling, and pattern generating, are some of the many related applications which can be performed with Motomatic control systems.

## SEMICONDUCTOR MANUFACTURINGPHOTORESIST SPINNER

A critical phase in the manufacturing of semiconductors is the deposition of photoresist on a silicone wafer. Photoresist is a liquid which provides a photosensitive surface on the wafer. To assure proper density, the resist is deposited while the wafer is rotated at a high speed. Speed regulation is important, since the deposition thickness of photoresist is dependent not only on viscosity, but also on rotational speed.

Utilization of a Motomatic control system with active dynamic braking is especially helpful in applications requiring shaft velocity to closely follow speed command variations.

The transistorized dynamic brake requires no contacts or excitation. The counter emf of the motor is used to provide braking torque. This is accomplished by inserting a transistor circuit, which in effect shortcircuits the motor armature when the motor is "coasting". It is not active during normal speed control operation, nor when a speed increase command is entered. During these times it is isolated by a diode. When the speed command is decreased, the Motomatic control, for all practical purposes, turns off. In.effect, the motor becomes a generator with opposite polarity current flowing
in the external circuit, allowing the brake transistors to operate and short-circuit the motor. The effect of the dynamic brake is proportional to speed; it functions best at the higher speeds, where the photoresist spinner is required to operate.


Fig. 5.5.15. Portable heart-lung machine.

## BLOOD OXYGENATOR

A manufacturer of medical instrumentation needed a precise speed control system to drive blood pumps for an oxygenator. The system was designed to operate from 115 VAC line power, and is also capable of using 24 VDC battery power in emergencies.

Electro-Craft designed a special control system, adapting the Motomatic concept to this application. The control system can be
switched from AC line power to DC battery power in a fraction of a second, insuring uninterrupted service in case of power interruptions. The assembled heart-lung machine is shown in Fig. 5.5.15.

The reliability and versatility of ElectroCraft drive systems make them eminently suitable for numerous critical medical applications, such as blood and kidney pumps, respirators, infusion pumps, and many other demanding applications in the rapidly expanding medical electronics field.

## INFUSION PUMP

Infusion pumps are frequently used in medical laboratory experiments where accurate amounts of fluids are continuously added to a system under study.

Greater ease and speed of operation are some of the improvements attained by the laboratory equipment which incorporated a Motomatic motor, as shown in Fig. 5.5.16. The pump can control infusion fluid flows as low as $0.1 \mathrm{~cm}^{3} / \mathrm{h}$, or pump as fast as $10^{5} \mathrm{~cm}^{3} / \mathrm{h}$ with accuracies of $1 \%$. Other models can pump 12 different fluids simultaneously without any of the fluids coming into direct contact with any other fluid being pumped.

Exceptionally wide flow range, safety and reliability, combined with economy and flexibility, provided improvements over previous techniques. Improved precision and reliable, repeatable performance were desirable characteristics of this design using the Motomatic speed control system.


Fig. 5.5.16. Infusion pump. (Photo courtesy of Holter Company, a division of Extracorporeal Medical Specialties, Inc., King of Prussia, Pennsylvania)

## CRYSTAL PULLING MACHINE

In the silicon growing process, precise speed control for withdrawal rates of the crystal growth form the melt is required. Silicon ingots are first broken up and melted down in a crucible through high frequency induction heating. Then a seed of perfect crystalline structure silicon is touched to the surface of the molten silicon and the seed is slowly withdrawn (see Fig. 5.5.17).

Three motions are involved during this withdrawal time:

1. the crucible is rotated;
2. the seed is rotated in either the same or the opposite direction;
3. the seed is withdrawn at a very precisely controlled rate.


Fig. 5.5.17. Typical crystal pulling machine.

The Motomatic E-650 speed control system was selected for this application (Fig. 5.5.18) because of the following characteristics:

1. broad speed range;
2. fast system response;
3. the ability to provide constant withdrawal rates independent of the varying load.

The seed withdrawal rate determines crystal diameter; consequently, motor speed must be closely controlled. Indication of the melt junction temperature is obtained from
a photocell detector positioned on the projected image of the melt surface. Radiation is sensed from the melt surface at the line of intersection between the melt and the crystal. Because of the fast response of the E-650 system, it is capable of using this information for controlling crystal growth rate and diameter. Another beneficial feature of this system is that the speed rate maintained is independent of external influences such as line voltage variations and crystal weight.


Fig. 5.5.18. Crystal growing furnace. (Photo courtesy of Hamco, Rochester, N.Y.)


Fig. 5.5.19. An example of a feed rate controller.

## CONVEYOR FEED RATE CONTROL

To maintain constant density of bulk material carried on a constant speed conveyor, the feed rate of the material to the conveyor must be adjusted. Such controls may be utilized in the processing of natural and synthetic fibers, or in many related applications which require constant material density.

Fig. 5.5.19 illustrates a method to control the material density. An Electro-Craft E-650 Motomatic speed control system drives feed rolls which feed fiber stock from a storage bin to a conveyor. An array of photocells across the width of the conveyor senses the intensity of light transmitted
through the moving material from a source above the conveyor. The photocell output is converted to a voltage proportional to average light intensity. This voltage indicates material density, and is compared with a preset density reference voltage; the difference between the two voltages is the error signal which goes to the motor control. The error signal directs the motor to increase or decrease speed to maintain the requried density.

## REFLOW SOLDERING

In an industrial process the manual procedure required numerous soldering operations on a cylindrical object. The solder joints were not always reliable, and the
process was time-consuming. The manufacturer desired to eliminate these problems in order to increase production rates and improve reliability. A system was required which could position the work piece, solder, index to the next angular position, and repeat the process.

A Motomatic E-500 system was selected to drive both the solder positioning arm and the indexing axis. The arrangement is shown in Fig. 5.5.20.


Fig. 5.5.20. Motomatic E-500 dual control for automatic positioning, soldering and indexing.

Linear positioning through gearing is accomplished by the E-500 positioning the solder arm. An input "go" command is received by the motor control, which provides a position setting proportional to a reference signal. The motor position feedback comes from the angular position of a potentiometer. Once the object is in position, soldering is performed.

The next step is indexing, which is also performed by an E-500 system driving a worm gear. The index position is sensed by
a microswitch. This arrangement achieves automatic operation, and has both increased production and improved reliability.

## DRIVE FOR DISC MEMORY TESTING

In the testing of standard 14 -in computer memory discs, a disc is rotated at high speeds and checked for surface wobble.

The computer memory disc is pneumatically locked onto the air bearing spindle shown in Fig. 5.5.21. This unit is normally


Fig. 5.5.21. Universal air bearing spindle for holding 14 -in memory discs.
mounted to the top of a specially designed soft spring mounting (provided with suitable damping) in which the drive motor is suspended, as shown in Fig. 5.5.22. The drive motor is connected to the air bearing spindle by precision couplings to assure the spindle's accuracy - better than 2 millionths


Fig. 5.5.22. The Motomatic variable speed spindle drive system, featuring the $\mathrm{E}-650$ motor-generator and master control.
of an inch radially and axially. The surfaces of the chuck are ground to less than 10 millionths TIR. Testing requires high shaft speeds which are highly repeatable and closely regulated.

The E-650 speed control system provides a complete closed-loop system featuring a fast response E-650 motor-generator and the E-650 master controller, a transistorized velocity feedback amplifier incorporating a special low drift amplifier.

Under operating conditions, the E-650 provides a velocity feedback signal proportional to actual test speed, which is compared with the reference pre-set signal proportional to desired test speed. An error signal is generated if the two voltages differ.

By this means, the pre-set desired testing speed is accurately maintained. Overall speed regulation of the E-650 system is better than $1 \%$ over the greater part of the system dynamic range, thus providing very close regulation and test repeatability for this application.

## VARIABLE SPEED TAPE RECORDER

An automobile manufacturer wished to test vehicle suspension components by simulating a variety of road conditions at various speeds. The solution to this problem was to tape-record road surface information and play back the recording at the desired test speed. The output of this special tape playback unit (Fig. 5.5.23) would then drive a high-powered, electromechanical transducer, which would apply the proper force function to the parts under test.


Fig. 5.5.23. Variable speed tape recorder. (Photo courtesy of Précision Mécanique Labinal, France)

The capstan speed control chosen for this play-back was an E-550 system, which gives an effective test speed range of from 0.3 to
$300 \mathrm{~km} / \mathrm{h}(0.186$ to 186 mph$)$. In addition, the responsiveness of the E-550 system was adequate to simulate rapid acceleration and braking over the test road conditions.

## mOTOR TESTING

Modern production testing of permanent magnet DC motors requires two basic tests to determine armature resistance $\mathbf{R}$ and voltage constant $\mathrm{K}_{\mathrm{E}}$. The voltage constant $\mathrm{K}_{\mathrm{E}}$ can best be obtained by driving the motor under test with another motor (in effect, the motor under test is being run as a generator). The generated voltage and shaft velocity are simultaneously read and recorded.

In order to perform such testing, a constant speed drive system is required. The Motomatic E-650 system is employed for this application, as shown in Fig. 5.5.24. This system provides a direct readout of shaft speed (and torque, if required), while accurately regulating the speed. These systems maintain an overall speed accuracy of better than $0.5 \%$ over the greater part of the system dynamic range. The long term speed stability of the system leads to test repeatability, which is important for production testing. An additional benefit of this system is constant speed, independent of torque variations and line voltage fluctuations.

While the motor under test is being driven at a constant speed, the speed and generated voltage are recorded. The voltage constant is then obtained by the following relationship:

$$
K_{E}=\frac{E_{g}}{n}
$$

[V/krpm; V, krpm]

The motor torque constant $\mathrm{K}_{\mathrm{T}}$ can be determined by the measurement of the motor voltage constant which was just described. The torque constant is always related to the voltage constant, as shown in section 2.1.

The ease with which this measurement can be made, makes it the most suitable production acceptance test for both motor voltage constant and motor torque constant.


Fig. 5.5.24. The E-650 utilized for final motor testing at Electro-Craft Corp.

## TACHOMETER RIPPLE VOLTAGE TESTING

One of the important steps in testing precision tachometers is the ripple test. In the case of high performance tachometers having ripple voltages on the order of $1 \%$ peak-to-peak, the test setup must have constant speed drive arrangements with instantaneous velocity diviations of less than
$0.1 \%$ peak-to-peak. Such an arrangement used by Electro-Craft Corp. is shown in Fig. 5.5.25.

In this setup, a Motomatic E-650 master control provides power for the E-650 motortachometer which controls the speed of an inertia wheel driving the motor-tachometer combination under test. A rubber belt couples the drive motor to the flywheel. By a direct reading of the speed indicated by the electronic speed meter, and knowing the belt pulley ratio, the test speed can be calculated. Due to the variable speed feature of the Motomatic system, data can be gathered for numerous speed points.


Fig. 5.5.25. Precision tachometer test station at Electro-Craft Corp.

## TORQUE LIMITER

A manufacturer required a system which will accelerate a large inertial load without causing damage to the motor or control system.

Many of the Motomatic speed control systems feature transistorized, variable torque limiting. The torque limiter can be described as an "electronic clutch"; the motor can be stalled at a predetermined load torque and remain in this state indefinitely without damage to any component.

Acceleration of large inertial loads impose a heavy power demand on a motor. Once the load is rotating, power requirements fall off sharply. Utilizing a Motomatic system with variable torque limiting, a load of this nature can be accelerated using a much smaller motor (and control system) than would normally be required. The only sacrifice in performance would be an extension in acceleration time.

## PAPER WEB WIND-UP CONTROL

A system for the paper industry was required to control the rewind of paper from a large roll onto smaller rolls. The web must be continuously wound on the smaller rolls (see Fig. 5.5.26). A series of six spindles are evenly spaced on a turntable. When one spindle is fully wound, the turntable is indexed, bringing the next spindle into the winding position. The fully wound roll is removed, and a new core is loaded.

As the spindle is moved into the winding position, it must be accelerated up to desired initial speed. The web is then placed onto the core in a flying splice manner. During the winding cycle, the spindle speed must be controlled in a programmed manner


Fig. 5.5.26. Paper rewinding system, featuring Motomatic E-550 motor-generators and open chassis controllers.
to accommodate the roll buildup while maintaining peripheral speed equal to web speed.

The supply roll must be accelerated up to desired speed relatively slowly because of the large moment of inertia involved. Since it is desired to wind the smaller rolls during startup and slowdown times, the spindles of the smaller rolls must be programmed as a function of the master roll speed. The supply roll has a "dancer" arm storage to hold paper during spindle cycling.

In order to provide high dynamic response to match the desired acceleration and slow-
down characteristics of the velocity profile, a bidirectional Motomatic E-550 control was used. The open chassis controls were mounted on the turntable to avoid ship rings carrying low level signals for all motors.

## WELDING MACHINES

In a welding process the quality and uniformity of the weld is directly affected by the proper control of the welding wire feed rates, and by the relative motion between the work pieces and the welding joint.

Before the process was automated, a highly skilled operator would normally position
the welds and feed the welding wire. By providing an automatic, accurately controlled feed rate for weld wire, a proper quantity of material is always supplied to the welding joint. Once a setup was completed, the operator had only to load and unload the work pieces, as the control systems assumed a very high degree of uniformity, thus minimizing operator skill requirements.

Various Motomatic systems are used in these applications, depending on the torque and speed requirements of the welder. In Fig. 5.5.27, an end-to-end pipe welder employs a pair of E-550 systems. The first system governs wire feed, and the second system rotates the pipes with respect to the welder. A very high degree of repeatability from piece to piece is the result of utilizing this closed-loop control system.

The Motomatic closed-loop system automatically adjusts motor input voltage and current, so that constant speed is maintained even though the load may be varying. The integral tachometer senses the effect of load variations and controls the transistorized servo amplifier, which maintains constant speed. Thus, despite load variation from work piece to work piece, the Motomatic system will maintain the set speed within a close range, usually $1 \%$. Additionally, by providing a very smooth, stable motion for the work pieces, a highly uniform and repeatable welding operation was achieved.


Fig. 5.5.27. End-to-end automatic pipe welder. (Photo courtesy of Précision Mécanique Labinal, France)

Fig. 5.5.28 shows a larger welding machine, which requires the extra torque of an E-650 system.

Motomatic series E-650 speed controls were selected to drive both the carriage drive and the wire feed machanism for this application. The operator sets the feed rate and controls the welding function.

Fig. 5.5.29 shows an interesting extension of the welder control concept. An electron beam welder is operated along a linear direction, utilizing a "suction cup" technique to maintain the required vaccum. The exceptional requirements for broad speed
range and speed stability resulted in the selection of Electro-Craft's DC servomotor/ tachometers.


Fig. 5.5.28. Automatic welding machine utilizing Motomatic controls. (Photo courtesy of Guild Metal Joining Co., Bedford, Ohio)

## BIDIRECTIONAL POSITION CONTROL

In a data retrieval system, a DC drive system is required to move a search mechanism and position it accurately for data recovery. The system must be bidirectional, and may be required to operate on a 50\% accelerate/ decelerate duty cycle.

Positional accuracy of the drive system is limited by two variables: 1) the drift characteristics of the reference power supplies, and 2) the accuracy of the command and feedback signals. Control system selection had to be made on the basis of the ability to cope with these variables. Using the potentiometer supplied with the standard Motomatic E-550 BPC (Bidirectional Posi-
tion Control), system accuracy is approximately $0.1 \%$ per revolution or $0.36^{\circ}$.


Fig. 5.5.29. An electron beam welder. (Photo courtesy of Précision Mécanique Labinal, France)

The E-550 BPC system is a position control system employing a high gain, closed-loop electronic control and an E-550 motorgenerator. As shown in Fig. 5.5.30, the input voltage to this electronic control commands the motor to move the search mechanism to a specific position. Motor angular position is proportional to the command setting, with the position feedback coming from the angular position of a potentiometer. Actually, a potentiometer, or any other position feedback voltage source may be employed for similar applications. As the search mechanism is "homing in" on the desired position, velocity feedback is employed to control the travel rate to the commanded location.

Optional features of the E-550 bidirectional controller allow adjustment of the velocity profile parameters. For example, the re-


Fig. 5.5.30. Motomatic E-550 BPC block diagram.
sponse time can be adjusted to permit "soft" starting, and the travel rate can be set at any desired motor speed between 5 and 5000 rpm .

## MUSCLE EXERCISER-MEDICAL REHABILITATION DEVICE

An exerciser used in muscle rehabilitation requires an accurately calibrated velocity control with torque readout in order to monitor the patient's recovery. Muscle strength recovery can be precisely monitored and the patient's progress accurately measured through this novel device, shown In Fig. 5.5.31.

The rate of motion is set by the therapist. In the initial stage of rehabilitation, the patient merely follows the crank handle motion with his hands or feet. As he progresses, he may exert force to either retard or speed up the handle motion. The degree of muscle exertion can be read on a calibrated torque dial, and recorded on a separate device. The special features of this device have made this machine very useful


Fig. 5.5.31. Cybex Muscle Exerciser. (Photo courtesy of Technicon-Cybex, Inc., Tarrytown, N.Y.)
in scientifically controlled hospital rehabilitation programs.

The system utilizes an E-650 motor-generator as a prime mover with a special bidirectional amplifier controlling speed and providing positive or negative torque, as the patient requires.

## PUNCH PRESS FEED CONTROL

This application consisted of a clutch-brake operated material feed control for an automatic punch press. Due to the intermittent
demand and the incremental feed technique, instability of the feed system would occur. The clutch-brake system would wear, and require frequent adjustment.

A new system was designed, using an $\mathrm{E}-650$ speed control, commanded by a sensor arm to which a potentiometer was attached (Fig. 5.5.32). The redesigned system operates very smoothly, seeking an average velocity matching that of the punch press. This eliminates the jerky operation previously experienced, and prevents stretch damage to the material.


Fig. 5.5.32. Punch press feed control, utilizing a Motomatic E-650 speed control.

## CONNECTOR TEST STATION

A major manufacturer of electrical connectors wished to build a connector test station, which would perform tests on the insertion force of connectors over a repeated number of cycles. Since the speed at which the mating halves were joined had a large bearing on the force measured, it was found necessary to automate the contacting speed in order to obtain testing repeatability. At the same time it was recognized that the additional load of pin engagement would potentially cause a speed regulation problem; i.e., the larger the force required to mate the connector, the slower the drive system would tend to operate, thus complicating the problem.


Fig. 5.5.33. Connector test station. (Photo courtesy of AMP de France)

An E-550 system was selected which, coupled through a vertical ball screw (Fig. 5.5.33), gave the high degree of speed regulation needed to achieve repeatable results.

## BLUEPRINT MACHINE DRIVE

Diazo print machines ("blueprint" machines) traditionally have a manual speed adjustment which requires the operator to set the exposure time according to the transparency of the original material. The success of this process depends on the operator's judgment and skill, and the process often results in waste of both time and material.

A basic E-550 system, with added control circuitry, sets the machine speed accurately and automatically for consistently good quality reproductions. The purpose of the added circuitry is to generate a signal which is dependent on the original's transparency by generating a speed reference voltage, which in turn controls exposure time.

A photocell senses light passing from a mercury lamp through the original material to provide a measure of its transparency. The photocell provides a DC voltage proportional to the product of light intensity and transparency. This voltage is the speed reference to the motor drive. The control circuit stores it long enough to let a mercury lamp expose the paper. After the copy is made, a timing circuit reduces the reference voltage to revert the machine to idle speed until the next sheet of paper is inserted.

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Fig. 5.5.34. Film viewer setup.

## FILM SCANNING DEVICE

A customer wanted a wide range film scanning device, enabling the viewer to rapidly find the general area of interest, and then at a controlled low speed view the individual frames until the proper one is found and then stop. The film was stored on spools, and in the process of scanning and viewing, the film would be transferred from the supply spool to a take-up spool (Fig. 5.5.34).

A simple solution was achieved by using two E-550 motor-generators, driving each reel as shown in Fig. 5.5.34. However, since the reel sizes vary during a scan, the relationship between shaft speed and linear film speed varies in proportion to the reel size change. This would result in undesirable
control characteristics. An unique solution to this problem is illustrated in Fig. 5.5.35, where the two tachometers are electrically


Fig. 5.5.35. Tandem reel drive system.
connected in tandem. Since the reel motors and tachometers are mechanically connected through the film, a pseudo-linear relationship between speed command and film speed is achieved, so that comfortable manual control of film motion results.

## DRIVE FOR CHART RECORDER

The user wanted an infinitely variable speed paper drive for a strip-chart recorder. A speed range of $10000: 1$ was required, and the acceleration time at the highest chart speed was to be less than 0.15 s .

Electro-Craft proposed an E-576 motorgenerator, with a special "ultra-wide range" velocity amplifier with high peak power availability. The result was a chart drive with a capability of chart speed between $5 \mathrm{~m} / \mathrm{s}$ to $0.5 \mathrm{~mm} / \mathrm{s}$, and an acceleration time of less than 0.15 s from 0 to $5 \mathrm{~m} / \mathrm{s}$ chart speed.

## PAPER CUTTING CONTROL

In this application paper was unwound from a feed roll, then cut to size by a mechanical knife. The system used a clutch-brake combination which provided intermittent motion to position the paper properly at the cutting station. The clutch-brake system needed frequent maintenance and adjustment to maintain the required accuracy.

By replacing the clutch and brake with a Motomatic E-550 direct-drive system, the entire cutting operation was accelerated, cutting accuracy was improved, and maintenance was minimized.

In the direct-drive system, shown in Fig. 5.5.36, the motor is commanded to follow


Fig. 5.5.36. Paper cutting control utilizing a Motomatic E-550 MG and control for incremental motion.
an input trapezoidal velocity command. The incremental encoder produces pulse counts proportional to distance of paper travel. The counter has a preset number stored, representing the proper paper size. When a predetermined number of binary counts remains in the memory, a digital-to-analog converter will be switched into the velocity feedback path. Thus, the final stop position will be approached in a controlled velocity manner, and when the last count has been received, the "stop" command can be followed instantly with no overshoot or error.

This hybrid analog-digital system has numerous applications in industrial and scientific applications.

## BLOOD CELL SEPARATOR

The Celltrifuge ${ }^{\mathrm{TM}}$, a continuous flow experimental blood cell separator shown in Fig. 5.5.37, was developed in a program sponsored by the National Cancer Institute and IBM. The machine uses several Motomatic control systems powering eight pumps and a centrifuge to perform the task. The systems have been specially designed to provide ultrahigh reliability and very low ground leakage current for patient safety.


Fig. 5.5.37. Blood Cell Separator. (Photo courtesy of American Instrument Co., Silver Springs, Md.)

## ADJUSTABLE LATHE FEED RATE CONTROL

Fig. 5.5.38 shows an E-650 motor-generator adapted to replace the quick-change gears in a lathe application. The variable speed, remote control feature was used to continuously alter the feed rate in a complex industrial turning process, keeping tool


Fig. 5.5.38. Adjustable speed lathe carriage feed.
pressure constant in spite of varying material thickness.

## CONTOUR LATHE

The requirements for a contour lathe cutting tool position drive are high speed for slewing, low speed with high torque for the actual cutting operation, and a stable speed control over this range, with very good no-load to full-load speed regulation. A lathe manufacturer has found that an E-650 system met all these requirements and provided an economical and highly reliable solution, shown in Fig. 5.5.39.

## THREE-AXIS MILLING MACHINE CONTROL

Fig. 5.5 .40 shows an example of an industrial adaptation of Motomatic controls to a milling machine table drive.

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## THREE-AXIS MILLING MACHINE CONTROL

Fig. 5.5.40 shows an example of an industrial adaptation of Motomatic controls to a milling machine table drive.


Fig. 5.5.39. A contour lathe, model LM 350. (Photo courtesy of LEFEBRE et MARTIN, France)


Fig. 5.5.40. Automatic three-axis milling machine control.

In this case the system was designed to provide a variable speed, three-axis position control for repetitious, automatic, routine milling applications. The control system is based on the wide range Motomatic control, with an individually adjustable speed for each program sequence. The position sensors work against adjustable micrometer stops, with a dual feature; as the control system approaches the final stop position, it goes into a "final approach" speed - a low speed mode - from which the control system can repeatedly stop within known tolerances. Thus, the MOTOFEED ${ }^{\left({ }^{(1)}\right.}$ system holds a repeatability of $\pm 0.0005$ in in all three axes, with infinitely variable speeds from 0.1 to $42 \mathrm{in} / \mathrm{min}$. Using a specially designed gear reduction, the system can provide torque of 80 lb -in on the lead screw, even at the lowest speed, permitting heavy cuts at close tolerances. A uniquely designed clutch device can uncouple the drive system at any time, providing a manual operation feature without interference by motors or gears.


Fig. 5.5.41. A linear drive mechanism UNISLIDE, model B2515 CJ. (Photo courtesy of Velmex, Inc., Bloomfield, N.Y.)

## LINEAR DRIVE SYSTEM

A manufacturer of linear motion mechanisms needed a drive system with infinitely variable speed to drive the lead screw for its line of modular length, linear motion devices.

An E-550 MGHP Master Control System was selected for this application because of its versatility, wide speed range and torque limiting features. Fig. 5.5 .41 shows the finished linear drive mechanism, marketed under the trade name UNISLIDE (Velmex, Inc., Bloomfield, N.Y.).

## PHOTOTYPESETTER APPLICATION

In a typical phototypesetter (see Fig. 5.5.42) the font, or character set, is in the form of negative silhouettes on a rotating disc. The particular character to be set is selected by flashing a strobe light behind the disc at the appropriate time. The character image is then projected via a moving mirror on to a photosensitive surface where it is recorded. A minicomputer in the machine calculates the exact letter spacing, word spacing and hyphenations to achieve the correct line end justification. The mirror is then moved to cause the characters to be positioned correctly on the surface. One manufacturer uses an ElectroCraft L5000 series servo control with the Digital Positioning Option and an M1438 moving coil motor to drive the lead screw on which the mirror is mounted. Use of this system enables direct interface with the minicomputer. The performance of the
system is such that the motor and load can be moved and stopped through a typical step of 17 degrees in 5 ms and can make 90 such steps per second; positional accuracy is better than $\pm 1 / 2$ degree of rotation. This gives considerably faster operation than other systems.


Fig. 5.5.42. Phototypesetter Application

## DIAMOND SORTING SYSTEM

The system shown in Fig. 5.5.43 is designed to monitor the size of industrial diamonds and to automatically segregate them into ten (10) different categories. The diamonds ( 20 to 60 mils in length) are transported through the field of view of a Reticon line scan camera (first on the left), back illuminated using a Reticon light source (first on the right). The size is then displayed in mils on the microcomputer based controller (second from left) whose output is also transmitted to the motor controller (second from the right). An E576 servo motor (center) with ten bins on a rotary platform is then positioned to cause the various diamonds to be deposited into the appropriate bins.


Fig. 5.5.43. Reticon RS825 Diamond Selection System

## PRINTED CIRCUIT SOLDER FUSING MACHINE

The Research Inc. Solder Fusing System shown in Fig. 5.5.44 provides superior quality fusing on printed circuit board and multiple layer assemblies.

The solder fusing process converts the porous electroplated tin-lead on a circuit board into a homogeneous alloy with a strong intermetallic bond to the base copper.

The temperature achieved during the fusing process is critical to the formation of the bond. Since this temperature is to a large extent dependent on the length of time


Fig. 5.5.44. Printed Circuit Solder Fusing Machine. (Courtesy of Research Incorporated)
that the printed circuit is exposed to the heat source, the customer chose a Motomatic speed control to drive the conveyor which carries the circuit boards. This selection gave the advantage of smooth stable speed control over a wide range of speeds.

## COMPONENT MARKING SYSTEM

In this application a marking head had to be accurately synchronized with a conveyor chair carrying electronic components. The synchronization had to be such that the markings were accurately positioned on the component and that a minimum of components be missed during start up and shut down.

The solution is a P6200AP servo motor control, an E670/596 motor-gearhead and one of the PL100 series phase lock options. An analog tachometer generator mounted on the conveyor drive provides a reference signal which keeps the speed of the marking head approximately synchronized with that of the conveyor. The signals from the two pulse generators are fed into the phase lock system which adjusts the motor speed continuously to ensure that the speeds are matched identically and that the marker head is also correctly positioned so as to mark the component.

The resulting increased performance enables the marking of components to be increased in speed to about double that achieved by previous systems.

### 5.6. INCREMENTAL MOTION APPLICATIONS

### 5.6.1. INTRODUCTION

In this section we will discuss the principles of incremental motion operation, covering system dynamics and acceleration torques. We work through an example so that the reader will gain experience in calculating armature acceleration current, motor terminal voltage, motor power dissipation, maximum permissible repetition rate, cooling and slewing requirements, to aid in selecting a servomotor for his own application.

The application examples in this section illustrate the concepts brought forward in the preceding discussions of motor selection for incremental motion requirements. A comparison is made here between the rather tedious hand calculations with ElectroCraft computer solutions.

These application examples are drawn from Electro-Craft files, and serve to illustrate the motor and amplifier selection process. They should be of help to anyone who is contemplating a systems design. As we shall see, the computer design aids are of immense help in making a proper choice of system components. Electro-Craft Corporation Applications Engineering staff has available portable computer terminals, and can give you design help right in your office,
since we can connect the terminal to our computer through the telephone lines. If you require assistance in your servomotor selection, our staff stands ready to be of help.

The analysis forms shown in next pages give the reader the basic information, what data, and in which form, are necessary to be supplied by the customer and Electro-Craft staff for exact and complex calculation of a given application.

The velocity and displacement data of an analysis form should be described in more detail by means of velocity profile and displacement profile graphs, as shown in Fig. 5.6.1.


Fig. 5.6.1. Velocity and displacement profiles.




| Tachometer | Regulation |
| :---: | :---: |
| Gradient (Voltage Constant) $\qquad$ $\mathrm{V} / \mathrm{krpm}$ Ripple $\qquad$ V p-p max. at $\qquad$ rpm | Long Term Regulation $\qquad$ \% of set speed Short Term Accuracy $\qquad$ \% of set speed |

## Special Requirements

Ambient Temperature $\quad{ }^{\circ} \mathrm{C} \max ., \quad{ }^{\circ} \mathrm{C}$ min。
Other Ambient Characteristics: $\qquad$
Shaft Requirements (sketch); Mounting Face; Base (sketch)
Shaft Position: $\square$ horizontal $\square$ shaft up $\square$ shaft down
Spac Available: length___ in, width___ in, height_____ in
Forced Air Cooling Available: $\square$ Yes $\square$ No
Others (describe):

Customer $\qquad$ Date $\qquad$
Program Genr1*
Application $\qquad$ Contact Engineer
Line No.
50

51
Parameter
Input Datia Units $=$
Output Data Units =
Load Type =
Motor Data Required =
Peak Velocity (Run)
Slew Velocity
Total Displacement

## Units

British or Metric
British or Metric or Neither.
Lead Screw
Full Data or Temp. only
$\square \mathrm{cm} / \mathrm{s} \quad \mathrm{in} / \mathrm{s}$
$\square \mathrm{cm} / \mathrm{s} \quad \mathrm{in} / \mathrm{s}$
*Specify V1 or $S$, not both, equate other one to zero.

| 52 | T1 | Acceleration Time | $\square \mathrm{ms}$ |
| :---: | :---: | :---: | :---: |
|  | T2 | Run Time | $\square \mathrm{ms}$ |
|  | T3 | Deceleration Time | $\square \mathrm{ms}$ |
|  | T4 | Dwell \& Settle Time | $\square \mathrm{ms}$ |
| 53 | J1 | Lead Screw Moment of Inertia | $\square \mathrm{kg} \mathrm{m}^{2} \quad \square \mathrm{oz-in-s}^{2}$ |
|  | M | Carriage/Load Mass | $\square \mathrm{kg} \square^{\circ} \mathrm{Oz}$ |
|  | P | Screw Pitch | $\square$ turns/cm $\square$ turns/in |
|  | F1 | Friction Torque | $\square \mathrm{Nm}$ Oz-in |
|  | G | Gear Ratio | $\square: 1$ |
|  | T7 | Ambient Temperature (Max。) | $\square{ }^{\circ} \mathrm{C}$ |
| $\begin{aligned} & 54 \\ & \text { thru } \end{aligned}$ | M | Motor Model | $\square$ |
| 99 | K1 | Torque Constant | $\square \mathrm{oz-in} / \mathrm{A}$ |
|  | R2 |  | $\square \Omega$ |
|  | J2 | Armature Moment of Inertia | $\square \mathrm{oz-in-s}{ }^{2}$ |
|  | K8 | Damping Constant | $\square \mathrm{oz-in/krpm}$ |
|  | F2 | Static Friction Torque | $\square$ oz-in |
|  | R3 | Thermal Resistance | $\square{ }^{\circ} \mathrm{C} / \mathrm{W}$ |

Program Genr ${ }^{*}$ * Application

Customer $\qquad$ Date $\qquad$

Contact Engineer $\qquad$
Line No.
50

|  |  | Output Data Units $=$ | British or Metric or |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Load Type $=$ | Capstan |  |  |
|  |  | Motor Data Required $=$ | Full Data | or Temp. |  |
| 51 | *V1 | Peak Velocity (Run) | $\square \mathrm{cm} / \mathrm{s}$ | $\square$ | in/s |
|  | V2 | Slew Velocity | $\square \mathrm{cm} / \mathrm{s}$ | $\square$ | in/s |
|  | *S | Total Displacement | $\square \mathrm{cm}$ | $\square$ | in |

*Specify V1 or S, not both, equate other one to zero
52

53
T4
J1

R1
F1
G
T7 Ambient Temperature (Max)
54 M
thru
99
K1
R2
J2 Armature Moment of Inertia
K8

F2
R3

Acceleration Time
$\square \mathrm{ms}$

Run Time
$\square \mathrm{ms}$
$\square \mathrm{ms}$
$\square \mathrm{ms}$
T3 Deceleration Time

Units
British or Metric
British or Metric or Neither
Capstan
Full Data or Temp.on1y
$\square \mathrm{cm} / \mathrm{s} \quad \mathrm{in} / \mathrm{s}$
$\square \mathrm{cm} / \mathrm{s} \quad \mathrm{in} / \mathrm{s}$
$\square \mathrm{cm} \quad \square$ in

TI

Torque Constant
$\square o z-i n / A$
$\square \Omega$
$\square$ oz-in-s ${ }^{2}$
8
Damping Constant
$\square$ oz-in/krpm
Static Friction Torque
$\square$ oz-in
Thermal Resistance

Program Name: REEL 5*
Application: $\qquad$ Engineer:

Date: $\qquad$
Input Data (supplied by customer) :
At line 10 (1) V1 Operating tape speed (in/s)
(2) V2 Rewind speed (in/s)

Ac line 11 (1) $B \quad$ Tape width (in)
(2) R4 Min。 reel radius (no tape) (in)
(3) R5 Max. reel radius (full tape) (in)
(4) $X$ Max。 +/- motion of sense point (in)

At line 12 (1) J3 Moment of inertia, reel \& hub (oz-in-s ${ }^{2}$ )
(2) G Gear reduction (ratio) from motor shaft to reel
(3) N Number of tape loops
(4) F Friction torque of reel and tape (oz-in)
(5) T7 Tape tension (oz)

## Motor Data (supplied by ECC)

At line 20 (1) M 99

Motor model number (any number of motors can be input to program up to 80)
(2) R2 Motor resistance @ $25^{\circ} \mathrm{C}$ ( $\Omega$ )
(3) K1 Motor torque constant (oz-in/A)
(4) J2 Motor moment of inertia (oz-in-s ${ }^{2}$ )
(5) K8 Motor viscous damping constant (oz-in/krpm)
(6) W1 Motcr max. permissible armature temperature $\left({ }^{\circ} \mathrm{C}\right)$


Note: To assist ECC Applications Engineering in the selection of the best available motor, it is helpful if the customer can supply the following addition:1 information:

1. Max. ambient operating temperature for the system ( ${ }^{\circ} \mathrm{F}$ or ${ }^{\circ} \mathrm{C}$ )
2. Is forced-air cooling available?
3. Max. voltage and current available with proposed (or existing) power amplifier.

### 5.6.2. SELECTING THE PROPER SERVOMOTOR FOR AN INCREMENTAL MOTION SYSTEM

The most important class of servo systems in which Moving Coil Motors have been successfully applied is generally described as the intermittent motion or incrementer systems. In these applications, a low inertial load is required to be started and stopped rapidly and repeatedly, either in fixed increments, or in a random but programmable manner.

Typical examples are:

> Typical rate [steps/s]

| a) Feed mechanisms | $12-50$ |
| :--- | ---: |
| b) Line-by-line reading <br> and punching | $50-250$ |
| c) Incremental operation |  |
| in digital magnetic |  |
| tape transports |  |
| d) Punch card readers/ |  |
| punchers | $50-500$ |
| e) Magnetic head actuators | $1-25$ |

It may be seen intuitively in each example shown, that a low moment of inertia/high troque motor is required to successfully drive the load at the high repetition rates
involved. Moreover, it is frequently necessary to run the motor at high speeds in order to slew through large shaft angular displacements. For example, document feed devices are required to step the paper for a predetermined number of lines, followed by a jump from page to page. Since all motion time is lost time, the slewing operation must be completed at the highest possible speed compatible with drive servo capability.

## Principles of Operation

The incremental drive is basically a highperformance DC velocity servo system, as shown in block diagram form in Fig. 5.6.2. The motor is fitted with a standard DC tachometer, and is driven by a DC power amplifier.


Fig. 5.6.2. Block diagram of an incremental servo system.

Incremental motion is obtaıned by operating the servo system in a non-linear mode, and is easily explained by reference to the waveforms in Fig. 5.6.3. Suppose that with the system at rest, we apply a step voltage to the input. Since the motor cannot attain any speed in zero time, there is no counter voltage from the tachometer.


Fig. 5.6.3. Incremental motion waveforms.

Therefore, the effect of the step input signal is to produce a large error signal, a-b, at the amplifier input. The error signal exceeds the saturation level of the amplifier.

As a result, the full power supply voltage, c-d, is applied to the motor and acceleration begins following the line e-g. The rising tachometer voltage reduces the error
signal during this time until the error signal falls within the acceptance band of the amplifier, and the system stabilizes in the constant velocity mode g-h. Only a nominal current, $\mathbf{I}$, is required to supply the friction and load torque.. The operating speed will be such that tachometer voltage is very nearly equal to the input voltage.

If now the input voltage step is terminated at the time $t_{p}$, the process reverses itself. The input change now produces a large error signal $\mathbf{i - j}$ in the opposite polarity causing a deceleration current $\mathbf{I}_{\mathrm{d}}$. The re rulting torque, in turn, drives the velocity back to zero along the slope $\mathbf{h}-\mathbf{k}$ which is nearly identical to the slope $\mathbf{e}-\mathbf{g}$, unless the friction torque is significant.

An analysis of the motion performance capabilities of an incrementer gets down to the equations of classical mechanics, with the acceleration values as the critical param eter.

In applications of this type, the incremental motion is almost wholly dissipative; i.e., little useful work is extracted from the motor since virtually the entire input energy is dissipated as heat. Such systems are therefore power limited in the sense that perfor mance is largely determıned by the maximum armature dissipation capability. The calculations are, therefore, aimed at deter mining the power dissipation at "worst case" operating conditions.

## System Dynamics

It should already be apparent that the in cremental servo is a sophisticated piece of
electromechanical engineering. In particular, the motor must operate satisfactorily in three quite distinct modes: a high speed start-stop mode, a good speed regulation mode for constant rates, and a high rewind speed mode.

Let us first look at the basic dynamic equation for motor output torque in an incremental motion application.

$$
\begin{equation*}
T_{g}=K_{T} I_{a}=J \frac{d \omega}{d t}+T_{f}+D \omega \tag{5.6.1}
\end{equation*}
$$

From this we can see that first of all we must know what $\omega(\mathrm{t})$, the velocity profile, is. Let us assume that $\omega(\mathrm{t})$ is an incremental step motion which is repeated with a certain frequency - a profile many practical systems would be required to follow. In the velocity profile of Fig. 5.6.4, $\omega_{1}$ is the running velocity, and the angle of increment is $\theta=\int_{0}^{\mathbf{t}^{T}} \omega(\mathrm{t}) \mathrm{dt}$. The acceleration

Fig. 5.6.4. Velocity profile of a typical incremental motion system.

$$
\begin{equation*}
t_{T}=t_{g}+t_{s} \tag{5.6.2}
\end{equation*}
$$

This is the condition which we now will investigate.

It is first necessary to establish the tape acceleration necessary to insure that the inter-record gap will not be exceededina single start-stop cycle. Let total displacement $d=d_{g}+d_{s}=0.375$ in, allowing sufficient margin for overshoot, etc., then:

$$
d_{g}=d_{s}=\frac{0.375}{2}=0.1875 \mathrm{in}
$$

Assuming the acceleration of the tape remains constant during the time $\mathrm{t}_{\mathbf{g}}$, we have:

$$
\begin{equation*}
\mathrm{a}=\frac{\mathrm{v}^{2}}{2 \mathrm{~d}_{\mathrm{g}}}=\frac{75^{2}}{0.375}=15000 \mathrm{in} / \mathrm{s}^{2} \tag{5.6.3}
\end{equation*}
$$

The acceleration time $\mathrm{t}_{\mathrm{g}}$ is given by:

$$
\begin{equation*}
t_{g}=\sqrt{\frac{2 d_{g}}{a}}=\sqrt{\frac{0.375}{1.5 \times 10^{4}}}=5 \mathrm{~ms} \tag{5.6.4}
\end{equation*}
$$

The total motion time is therefore $\mathbf{t}_{\mathbf{T}}=10 \mathrm{~ms}$ when $t_{g}=t_{s}$.

## Acceleration Torque

Since the capstan radius $r_{c}=1 \mathrm{in}$, then the capstan angular acceleration is

$$
\begin{align*}
a & =\frac{a}{r_{c}}=\frac{15000 \mathrm{in} / \mathrm{s}^{2}}{1 \mathrm{in}} \\
& =15000 \mathrm{rad} / \mathrm{s}^{2} \tag{5.6.5}
\end{align*}
$$

Acceleration torque is given by the familiar expression:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{a}}=\mathrm{J}_{\mathrm{T}} a \tag{5.6.6}
\end{equation*}
$$

where

$$
J_{T}=J_{m}+J_{L}
$$

( $\mathrm{J}_{\mathrm{T}}$ is the sum of the moments of inertia of the motor and the load, including tachometer)

Unfortunately, there is no direct analytic method whereby we can put in the numbers and come out with the motor model number for the solution. In fact, Electro-Craft's computerized analysis simply starts with the lowest cost servomotor available and works up the list until a motor is found which meets all requirements, since the equations apply to the entire performing system including the motor and load. Therefore, the analysis is an automated process of elimination based on acceptable performance and lowest possible cost.

After several attempts, we find that a suitable motor for operation in this type of capstan application is the Electro-Craft $\mathrm{M}-1400$, which has the following parameters:

Moment of inertia $J_{m}=0.002 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}$
Torque constant $\mathrm{K}_{\mathrm{T}}=12 \mathrm{oz}-\mathrm{in} / \mathrm{A}$
Armature Resistance $R=1.6 \Omega$

Total system moment of inertia is therefore:
Motor armature $\quad 0.002$ oz-in-s ${ }^{2}$
Capstan (typical) 0.002

Tachometer (typical) 0.0005

$$
\mathrm{J}_{\mathrm{T}}=0.0045 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
$$

The system acceleration torque becomes:

$$
\begin{aligned}
\mathrm{T}_{\mathrm{a}} & =4.5 \times 10^{-3} \times 1.5 \times 10^{4} \\
& =67.5 \mathrm{oz}-\mathrm{in}
\end{aligned}
$$

The deceleration torque is, for all practical purposes, equal and opposite, since friction is generally negligible in systems of this type.

## Armature Acceleration Current

The armature peak current $\mathbf{I}_{\mathrm{pk}}$ is simply obtained from the expression:

$$
\begin{equation*}
I_{p k}=\frac{T_{a}+T_{f}}{K_{T}} \tag{5.6.7}
\end{equation*}
$$

where $T_{a}$ is the acceleration torque and $T_{f}$ is the total system friction torque.

As we have noted, static friction in systems of this type is very small. Also, in a welldesigned DC servomotor the damping loss can be neglected. Hence:

$$
I_{p k}=\frac{67.5}{12}=5.6 \mathrm{~A}
$$

## Motor Terminal Voltage

The maximum terminal voltage V is required at the end of acceleration interval and may be calculated from the standard equation:

$$
\begin{equation*}
V=R I_{a}+K_{E} n \tag{5.6.8}
\end{equation*}
$$

In this case we must consider the "hot" resistance of the motor armature ( $20 \%$ increase) so that

$$
\begin{aligned}
\mathrm{R} & =1.2 \times 1.6 \Omega=1.92 \Omega \\
\mathrm{I}_{\mathrm{a}} & =\mathrm{I}_{\mathrm{pk}}=5.6 \mathrm{~A} \\
\mathrm{~K}_{\mathrm{E}} & =\frac{\mathrm{K}_{\mathrm{T}}}{1.3524}[\mathrm{~V} / \mathrm{krpm} ; \mathrm{oz}-\mathrm{in} / \mathrm{A}] \\
& {[\text { see }(2.1 .24)] }
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{K}_{\mathrm{E}} & =8.873 \mathrm{~V} / \mathrm{krpm} \\
\mathrm{n} & =716 \mathrm{rpm}
\end{aligned}
$$

Substituting into (5.6.8) we get

$$
V=17.1 \mathrm{~V}
$$

Thus, a suitable pulse amplifier required to drive this capstan servomotor should be capable of delivering a peak current pulse of 6 A at approximately 20 V in the described repetition mode.

## Calculation of Motor Power Dissipation

We will now proceed to calculate motor power dissipation for the example at hand.

The maximum repetition rate is chosen as the basis for "worst case" calculation. Although the power dissipation during the velocity mode portion of the velocity profile is very small compared to the acceleration/deceleration dissipation, it has been included in the calculation below. In some examples, it can be safely ignored.

The peak acceleration/deceleration current was calculated to be 5.6 A. The power dissipation - heat loss - in any device is given by $R \mathbf{I}_{\text {RMS }}^{2}$. We can calculate the RMS value of the current as follows (refer to Fig. 5.6.5):

$$
\begin{equation*}
I_{\text {RMS }}=\sqrt{\frac{\Sigma l_{i}^{2} t_{i}}{\Sigma t_{i}}} \tag{5.6.9}
\end{equation*}
$$

For the current profile of Fig. 5.6.5 we get

$$
\mathrm{I}_{\mathrm{RMS}}=4.02 \mathrm{~A}
$$

Armature heat loss is therefore

$$
\begin{aligned}
P_{L} & =R I_{R M S}^{2}=1.92 \times 4.02^{2} \\
& =31.03 \mathrm{~W}
\end{aligned}
$$

In order to see whether the motor can handle this power dissipation, we can check the temperature rise of the uncooled ElectroCraft M-1400 motor under the conditions calculated above.

The M-1400 has the thermal resistance

$$
\mathrm{R}_{\mathrm{th}}=2.4^{\circ} \mathrm{C} / \mathrm{W}
$$

The motor armature temperature rise will then be

$$
\begin{equation*}
\Theta_{r}=P_{L} \quad R_{t h}=74.47^{\circ} \mathrm{C} \tag{5.6.10}
\end{equation*}
$$

Assuming a maximum ambient temperature of $40^{\circ} \mathrm{C}$ (and a maximum safe armature temperature of $155^{\circ} \mathrm{C}$ ), we will reach a final armature temperature of:

$$
\begin{equation*}
\Theta_{\mathrm{a}}=\Theta_{\mathrm{A}}+\Theta_{\mathrm{r}}=114.47^{\circ} \mathrm{C} \tag{5.6.11}
\end{equation*}
$$

which is safely inside the maximum temperature limit of the armature.

If we were to use forced-air cooling, the thermal resistance would decrease drastically , and the armature temperature would be much lower than the foregoing example shows. Conversely, much higher dissipation rates could be accommodated.


Fig. 5.6.5. Current profile for a typical incrementing application.

## Slewing Requirement

One of the most interesting features of the incrementer servo - as distinct from the stepper motor - is its ability to perform large displacements at very high slewing rates without the need to worry about un-
stable velocity regions or programmed ac-celeration-deceleration. In the system under discussion, for example, line printers are required to skip large areas of printout paper as, for example, when printing out on standard size forms. Check writing is a typical example.

The problem reduces to one of allowing the servo drive motor to accelerate to much higher slewing velocities (typically a few thousand rpm) and controlling motion so that the system will be decelerated to a stop within permissible displacement tolerances.

A typical velocity versus time plot of a line printer application is shown in Fig. 5.6.6. We consider the system acceleration as being more or less linear; and, with a sufficiently high amplifier voltage and good power supply regulation, the mechanism and paper can be brought up to $100 \mathrm{in} / \mathrm{s}$ in less than 25 ms .


Fig. 5.6.6. Velocity vs. time plot for a line printer servo slewing through large displacements.

## Conclusion

Electro-Craft Moving Coil Motors are ideally suited for use as incremental motion drive
servomotors, because they have inherently good acceleration (high torque-to-moment of inertia ratios). Of equal significance in practical systems is the economy of using direct-drive DC servo systems with their low maintenance costs and high reliability.

Extensive life testing of the Electro-Craft Moving Coil Motors over a five-year period has indicated that there is no observable deterioration in the performance of the motors when they are operated within specification limits of pulse current and armature dissipation. These factors should be given very serious consideration when specifying Moving Coil Motors for use in incremental motion and high acceleration drives.

The Electro-Craft concept of the Moving Coil Motor undoubtedly has considerable development potential, especially in industrial equipment where the performance limits of mechanical indexing or clutchbrake devices are approached. It is well worth the design engineering effort required to investigate the potential performance improvement - sometimes an order of magnitude improvement in motion time of which the Moving Coil Motor is now capable. Nevertheless, it is important to realize that the total system's concept must be considered for optimum performance. Electro-Craft is unique in being the only manufacturer of low inertia motors with an equivalent capability in the electronic servo control field. Many customers have benefited from our wide experience in control systems design.

### 5.6.3. TAG PRINTER FEED DRIVE

## System Requirement

The drive mechanism positions sales tags incrementally during printing and coding operations. The tags are driven by a 0.75 in diameter wheel which is driven directly by the motor. The tags are to be positioned 0.167 in in 21 ms and remain at rest for 29 ms . In the worst case, the cycle is repeated continuously. The power supply is limited to 10 V and 4 A . Roller moment of inertia is 0.001 oz-in- $\mathrm{s}^{2}$, and load friction torque is 10 oz-in. The maximum ambient temperature is $65^{\circ} \mathrm{C}$.

## Motion Profile

Since there are no restrictions on the motion profile expect for motion time, a trapezoidal velocity profile with equal acceleration, run and deceleration times will
be selected to minimize power dissipation as illustrated in Fig. 5.6.7.

The basic data and variables of the system are:
roller radius $r=0.375$ in
linear displacement per step $\mathrm{x}=0.167$ in
$\theta_{1}$ angular displacement during acceleration time $\mathrm{t}_{1}$
$\theta_{2}$ angular displacement during run time $\mathrm{t}_{2}$
$\theta_{3}$ angular displacement during deceleration time $\mathrm{t}_{3}$
$\theta_{\mathbf{T}}=\theta_{1}+\theta_{2}+\theta_{3}$ total step angular displacement

$$
\omega_{\mathrm{o}}=\text { run velocity }
$$



Fig. 5.6.7. Velocity profile of a tag printer.

For a trapezoidal velocity profile

$$
\begin{aligned}
t_{1} & =t_{2}=t_{3}=0.007 \mathrm{~s} \\
\theta_{1} & =\theta_{3}=\frac{\omega_{o} t_{1}}{2} \\
\theta_{2} & =\omega_{o} t_{1} \\
\theta_{T} & =2 \omega_{0} t_{1}=\frac{x}{r} \\
\omega_{0} & =\frac{x}{2 r t_{1}}=\frac{0.167}{2 \times 0.375 \times 0.007} \\
& =31.81 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

## Load Calculations

The next step in the system analysis is to compute the load at the motor shaft with the following inertia and friction values:

$$
\begin{aligned}
& J_{L}=0.001 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \\
& T_{\mathrm{L}}=10 \mathrm{oz}-\mathrm{in}
\end{aligned}
$$

We can calculate the load torques during time intervals $t_{1}, t_{2}$ and $t_{3}$ and also the RMS load torque as follows:
a) load torque during time $t_{1}$

$$
T_{L 1}=\frac{J_{L} \omega_{0}}{t_{1}}+T_{L}=14.54 \mathrm{oz}-\mathrm{in}
$$

b) load torque during time $t_{2}$

$$
T_{L 2}=T_{L}=10 \text { oz-in }
$$

c) load torque during time $t_{3}$

$$
T_{L 3}=\frac{J_{L} \omega_{o}}{t_{3}}-T_{L}=-5.46 o z-\mathrm{in}^{*}
$$

d) RMS load torque (for $t_{1}$ thru $t_{4}$ )

$$
\begin{equation*}
\left(T_{L}\right)_{R M S}=\sqrt{\frac{\Sigma T_{L i}^{2} t_{i}}{\Sigma t_{i}}} \tag{5.6.12}
\end{equation*}
$$

## Motor Selection

The initial selection of a first motor candidate is based on educated guesswork. When the results of the first selection are examined, the second or third choices (if needed) are usually close to the optimum. In this case, a high-performance Moving Coil Motor is not required because acceleration rates are relatively low. Therefore, we can accept a certain mismatch between the conventional motor armature moment of inertia and the relatively low load moment of inertia in order to choose the most economical motor for the application. Then we can also expect that the RMS generated torque required will be 1.5 to 2 times greater than the RMS load torque. We will try a Motomatic E-540 with the following parameters:

$$
\begin{aligned}
& K_{T}=10.02 \mathrm{oz}-\mathrm{in} / \mathrm{A} \\
& \mathrm{R}=1.55 \Omega @ 25^{\circ} \mathrm{C}
\end{aligned}
$$

[^3]\[

$$
\begin{aligned}
& \mathrm{J}_{\mathrm{m}}=0.0038 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \\
& \mathrm{D}=0.1 \mathrm{oz}-\mathrm{in} / \mathrm{krpm} \\
& T_{\mathrm{f}}=3 \mathrm{oz}-\mathrm{in} \text { (maximum) } \\
& R_{\mathrm{th}}=5.0^{\circ} \mathrm{C} / \mathrm{W}
\end{aligned}
$$
\]

We will first go through a hand calculation of the motor and amplifier requirements, and then follow with a computer analysis of the same problem.

The torque values $\mathrm{T}_{\mathrm{L} 1}, \mathrm{~T}_{\mathrm{L} 2}$, and $\mathrm{T}_{\mathrm{L} 3}$ are functions of the load conditions only - no attention has yet been paid to the effect of a motor and its associated friction and inertia contributions. In the following section we will now include the constants of the selected motor to get values for the generated torque, which is the total torque required to run the motor and the load under specified conditions.

Generated torque required during time $\mathbf{t}_{\mathbf{1}}$

$$
\begin{align*}
T_{1} & =\frac{\left(J_{m}+J_{L}\right) \omega_{o}}{t_{1}}+T_{L}+T_{f} \\
& =34.81 \mathrm{oz}-\mathrm{in} \tag{5.6.13}
\end{align*}
$$

Damping torque is neglected here since motor speed is low and the damping constant is small.

Generated torque required during time $\mathrm{t}_{2}$

$$
\begin{equation*}
T_{2}=T_{L}+T_{f}=13 o z-i n \tag{5.6.14}
\end{equation*}
$$

Generated torque required during time $\mathrm{t}_{3}$

$$
\begin{align*}
T_{3} & =\frac{\left(J_{m}+J_{L}\right) \omega_{o}}{t_{3}}-T_{L}-T_{f} \\
& =8.81 \mathrm{oz}-\mathrm{in} \tag{5.6.15}
\end{align*}
$$

Total RMS generated torque required (for $\mathrm{t}_{1}$ thru $\mathrm{t}_{4}$ )

$$
T_{R M S}=\sqrt{\frac{\Sigma T_{i}^{2} t_{i}}{\Sigma t_{i}}}
$$

$$
\begin{equation*}
=14.29 \mathrm{oz}-\mathrm{in} \tag{5.6.16}
\end{equation*}
$$

## Armature Temperature Calculation

Temperature rise [see (5.3.7)]

$$
\begin{equation*}
\Theta_{\mathrm{r}}=\frac{\mathrm{R} \mathrm{R}_{\mathrm{th}} \mathrm{I}_{\mathrm{RMS}}^{2}}{1-\left.\mathrm{R} \mathrm{R}_{\mathrm{th}} \psi\right|_{\mathrm{RMS}} ^{2}} \tag{5.6.17}
\end{equation*}
$$

where

$$
\mathrm{I}_{\mathrm{RMS}}=\frac{\mathrm{T}_{\mathrm{RMS}}}{\mathrm{~K}_{\mathrm{T}}}=\frac{14.29}{10.02} \cong 1.43 \mathrm{~A}
$$

Thus,

$$
\begin{aligned}
\Theta_{r} & =\frac{1.55 \times 5 \times 1.43^{2}}{1-1.55 \times 5 \times 0.00393 \times 1.43^{2}} \\
& =16.9^{\circ} \mathrm{C}
\end{aligned}
$$

Remembering that $\Theta_{A}$ was established at $65^{\circ} \mathrm{C}$, we get

$$
\Theta_{a}=\Theta_{A}+\Theta_{r} \cong 82^{\circ} \mathrm{C}
$$

Thus, we notice that the biggest contributor to motor heat is the ambient temperature, but the motor temperature is well below the maximum limit of $155^{\circ} \mathrm{C}$.

## Amplifier Requirement

During time $\mathrm{t}_{1}$

$$
\begin{align*}
& I_{1}=\frac{T_{1}}{K_{T}}=\frac{34.81}{10.02}=3.474 \mathrm{~A} \\
& V_{1}=R_{h} I_{1}+K_{E} n_{o} \tag{5.6.18}
\end{align*}
$$

where the "hot" resistance

$$
\begin{aligned}
R_{h} & =R\left(1+\psi_{c u} \Theta_{r}\right) \\
& =1.55(1+0.00393 \times 57) \\
& =1.9 \Omega
\end{aligned}
$$

Note that $\Theta_{r}$ must be the temperature rise above $25{ }^{\circ} \mathrm{C}$ (which is the temperature at which R is established).

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{E}}=\frac{\mathrm{K}_{\mathrm{T}}}{1.3524}=7.409 \mathrm{~V} / \mathrm{krpm} \\
& \mathrm{n}_{\mathrm{o}}=9.5493 \quad \omega_{\mathrm{o}}=303.8 \mathrm{rpm}
\end{aligned}
$$

Thus,

$$
V_{1}=8.85 \mathrm{~V}
$$

During time $\mathrm{t}_{2}$

$$
\begin{aligned}
& \mathrm{I}_{2}=\frac{\mathrm{T}_{2}}{\mathrm{~K}_{\mathrm{T}}}=\frac{13}{10.02} \cong 1.3 \mathrm{~A} \\
& \mathrm{~V}_{2}=\mathrm{R}_{\mathrm{h}} \mathrm{I}_{2}+\mathrm{K}_{\mathrm{E}} \mathrm{n}_{\mathrm{o}}=4.73 \mathrm{~V}
\end{aligned}
$$

During time $\mathrm{t}_{3}$

$$
\begin{aligned}
& \mathrm{I}_{3}=\frac{\mathrm{T}_{3}}{\mathrm{~K}_{\mathrm{T}}}=\frac{8.81}{10.02}=0.88 \mathrm{~A} \\
& \mathrm{~V}_{3}=\mathrm{R}_{\mathrm{h}} \mathrm{I}_{3}+\mathrm{K}_{\mathrm{E}} \mathrm{n}_{\mathrm{o}}=3.92 \mathrm{~V}
\end{aligned}
$$

Thus, we can stay within the power supply limitation of 10 V and 4 A .

Next, we will show the same analysis performed by a computer program developed by Electro-Craft Corporation. Fig. 5.6.8 shows the input data sheet, displaying not only input data but also the symbols accompanying it on the printout. These symbols may vary somewhat from the ones used throughout this book, due to the absence of suitable symbols on the printer.

The computer program (Fig. 5.6.9) simplifies the calculation chores and allows easy comparison of power dissipation and power supply demand by various motor models.

## Worst Case Motor Analysis

In the following example we have evaluated the tag printer application power dissipation and amplifier requirement, using the worst case torque constant and armature resistance. Assume $\mathrm{K}_{\mathrm{T}}$ to be $8 \%$ lower than nominal and R to be $8 \%$ higher than nominal for this evaluation. The result for this situation is given in Fig. 5.6.10. We can see that the armature temperature rose by $6{ }^{\circ} \mathrm{C}$, and that the voltage and current requirements rose less than $10 \%$ from the values computed using nominal motor constants. The results shown here may not seem dramatic, but this is due to the relatively low motor power dissipation in this application. In cases where the motor is operating close to its dissipation limit, it is very important to perform the worstcase analysis to prevent unexpected dissipation problems from limiting the operational performance of a system.

(NOTE: Lines 10 and 11 define the velocity profile: it may be necesiary to satisfy a given value for 's'; however, before entering the data into the computer, it is a preferred method to define the velocity profile in terms of velocity and time values.)

At line 12 (1) Jl, Load inertia, oz.-in.-sec. ${ }^{2}$
(2) Rl, Drive roll or capstan radius
(3) F, Friction Torque at load shaft, oz. -in.
(4) G, Reduction gear, motor to load
(5) $\mathrm{T}_{\mathrm{a}}$, Ambient temperature ${ }^{\circ} \mathrm{C}$.

| .001 |
| :---: |
| .375 |
| 10 |
| 1 |
| 65 |

(NOTE: Line 12 defines the load conditions.)
Input Data (supplied by ECC) :
At line 20 (1) M, Motor Model No.
(2) Kl, Motor torque constant (oz.-in./amp.)
(3) R2, Motor armature resistance (at $25^{\circ} \mathrm{C}$. )
(4) J2, Motor inertia (oz.-in.-sec. ${ }^{2}$ )
(5) K8, Motor viscous damping constant

(oz.-in./KRPM)
(6) $T_{F}$, Friction Torque oz.-in.
(7) Rth. Thermal Resistivity ${ }^{\circ} \mathrm{C} . /$ watt

| 3 |  |
| :--- | :--- |
| 5 |  |

(NOTE. To assist ECC Applications Engineering in the selection of the best available motor, it is helpful if customer can supply the following additional information:

1. Max. ambient operating temperature for the system ( ${ }^{\circ} \mathrm{F}$. or ${ }^{\circ} \mathrm{C}$.)
2. Is air cooling available?

Max. voltage and current available with proposed (or existing)
power amplifier.
Fig. 5.6.8. Incremental servo computer analysis for tag printer feed drive.
(Not corrected for this third edition)
$F(N E L O C I T Y(F A D / S E C)=31.9$

```
T1=7 SR=0.b\angle2 ACC, RFI/S+Q = 4552
T2 = 7 S2 = 0.084
T3=7 S3=0.042 LEC, FHL/:92= LSS2
[WFLI TIME = 29 T@TAL DISPLACEMENT =0.167
MQTER FLN KPM = 3E4
MAX REF FATE (S-S CYCLES/SEN) = 20
AMEIENT OPEFATING TEMP (LEGREES C) = 65
J1=E.O& F1 = 0.375 F1= 10 G = 1
JMOTOR MODEL 5 S& ARNATURE TENF (LECREES () = % .
K1=10.6: F2=1.55 ,F% =0.0038 K8=0.1 F2= = F3=4
ARMATURE F(WER LISSIPATIUN ( WATTS) = 3.9
RNS CLPFENT (ANPS) = 1.4
TEFNINAL FESISTANCE AT N.AX TEMP (6HMS) = 1.91
ACC TUKRL! (lN-NZ) = 34.9
I-ACC = % V-FCO=8.9
-FIN=1.: V-HIN= 2.7
I-LFC=%.9 V-LFC=%.9
```

Fig. 5.6.9. Computer printout of tag printer feed drive analysis. (Not corrected for this third edition)


```
T2 = 7 S% = &.k&&
T3 = 7 S3 = 0.K&2 [EC, FAL/S+2 = <.52
DWELL TINE = 29 TQTAL LISFIACEMFNT = 6.167
MOTOR RLN LFN = 302
MAX REF FATE (S-S (YCLES/SEC) = ck
ANBIENT EFFRFTING THNF (IEGREES L) = 65
```



```
MATOR MOLEL #S\angleO AFNATLFF THMP (LEEFEES () = 90
```



```
ARMATLKE FOWER LISSIFFTION (VATIS) = 5.1
IMS (URRENT (AMPS) = 1.\epsilon
TERMINAI RESISTANCF AT NIAX TEMP (VHNT) = 2.09
ACC TOROLF (IN-EZ) = 349
I-ACC = 3.8 V-ACC = 1k
I-FUN = 1.4 V-RUN = 4
I-IFC = 1 V-LFC=<:1
```

Fig. 5.6.10. Tag printer drive worst case analysis.

### 5.6.4. CONVEYOR DRIVE SYSTEM

## System Requirements

A motor must be selected to mate with existing drive components. The conveyor and load must be accelerated from rest to $200 \mathrm{ft} / \mathrm{min}$ in 0.5 s , driven at constant speed for 14 s and decelerated to rest in 0.5 s . For


Fig. 5.6.11. Drive train mechanism for conveyor drive system.


Fig. 5.6.12. Motion profile for conveyor drive system.
worst case analysis, the cycle is repeated continuously. The drive train description and motion profile are shown in Figs. 5.6.11 and 5.6.12, respectively.

## Load Calculations

The first step in the system analysis is to calculate the load at the motor shaft. In this case, all the inertial loads were not known by the customer, but mechanical dimensions and weights of each were furnished. From this data, the moments of inertia can be calculated. All moments of inertia are calculated here as equivalent moments of inertia at the motor shaft.

Note: In this example, the mass $m$ of individual parts of the drive mechanism is calculated in units [oz-s $2 / \mathrm{in}$ ], not in [slug] (see description of the British system of units in the Appendix A.2.). This is more convenient because the moments of inertia, $J$, are calculated in units [oz-in-s ${ }^{2}$ ] and no conversion factors have to be used. The mass $m$ is thus calculated from given weight $w$ [ oz ] as follows:

$$
\begin{equation*}
m=\frac{w}{g} \quad\left[o z-s^{2} / i n ; o z, i n / s^{2}\right] \tag{5.6.19}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{g}= 386.0878 \mathrm{in} / \mathrm{s}^{2} \text { is the standard gravi- } \\
& \text { tational acceleration. }
\end{aligned}
$$

Load moments of inertia are as follows:

## 1. Reducer

$$
\mathrm{J}_{1}=0.04 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
$$

(manufacturer's list value)

$$
N=10-\text { gear ratio }
$$

2. Sprocket \#1

$$
\begin{aligned}
& r_{2}=2 \mathrm{in}-\text { radius } \\
& \mathrm{w}_{2}=2 \mathrm{lb}=32 \mathrm{oz} \\
& \mathrm{~m}_{2}=\frac{\mathrm{w}_{2}}{\mathrm{~g}}=0.0829 \mathrm{oz}-\mathrm{s}^{2} / \mathrm{in} \\
& \mathrm{~J}_{2}=\frac{\mathrm{m}_{2} \mathrm{r}_{2}^{2}}{2 \mathrm{~N}^{2}}=0.00166 \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}}^{2}
\end{aligned}
$$

3. Sprocket \#2

$$
\begin{aligned}
& r_{3}=6 \mathrm{in} \\
& \mathrm{w}_{3}=20 \mathrm{lb}=320 \mathrm{oz} \\
& \mathrm{~m}_{3}=0.829{\mathrm{oz}-\mathrm{s}^{2} / \mathrm{in}}^{2\left(\mathrm{~N} \frac{r_{3}}{r_{2}}\right)^{2}}=0.0166 \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}}^{2}
\end{aligned}
$$

4. Chain

$$
\begin{aligned}
& \mathrm{w}_{4}=24 \mathrm{lb}=384 \mathrm{oz} \\
& \mathrm{~m}_{4}=0.995 \mathrm{oz}-\mathrm{s}^{2} / \mathrm{in} \\
& \mathrm{~J}_{4}=\frac{\mathrm{m}_{4} \mathrm{r}_{2}^{2}}{\mathrm{~N}^{2}}=0.0398 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
\end{aligned}
$$

5. Drive Shaft

$$
r_{5}=1.5 \mathrm{in}
$$

$w_{5}=80 \mathrm{oz}$

$$
m_{5}=0.207 \mathrm{oz}^{2} \mathrm{~s}^{2} / \mathrm{in}
$$

$$
J_{5}=\frac{m_{5} r_{5}^{2}}{2\left(N \frac{r_{3}}{r_{2}}\right)^{2}}=0.000259 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
$$

6. Drive Wheels (4 pcs)

$$
r_{6}=8 \text { in }
$$

$$
\left.\begin{array}{l}
w_{6}=20 \mathrm{lb}=320 \mathrm{oz} \\
m_{6}=0.829 \mathrm{oz}-\mathrm{s}^{2} / \mathrm{in}
\end{array}\right\} \text { one wheel }
$$

$$
J_{6}=4 \frac{m_{6} r_{6}^{2}}{2\left(N \frac{r_{3}}{r_{2}}\right)^{2}}=0.118 \mathrm{oz}-\cdot \mathrm{in}-\mathrm{s}^{2}
$$

7. Load

$$
\mathrm{w}_{7}=100 \mathrm{lb}=1600 \mathrm{oz}
$$

$$
m_{7}=4.144 \mathrm{oz}-\mathrm{s}^{2} / \mathrm{in}
$$

$$
J_{7}=\frac{m_{7} r_{6}^{2}}{\left(N \frac{r_{3}}{r_{2}}\right)^{2}}=0.295 \text { oz-in-s }{ }^{2}
$$

## 8. Drive Chains (2 pcs)

$$
\left.\begin{array}{l}
\mathrm{w}_{8}=150 \mathrm{lb}=2400 \mathrm{oz} \\
\mathrm{~m}_{8}=6.216 \mathrm{oz}-\mathrm{s}^{2} / \mathrm{in}
\end{array}\right\} \text { one chain }
$$

$$
J_{8}=2 \frac{m_{8} r_{6}^{2}}{\left(N \frac{r_{3}}{r_{2}}\right)^{2}}=0.884 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}
$$

Total load moment of inertia

$$
J_{L}=J_{1}+\cdots+J_{8}=1.395 \mathrm{oz}-{\mathrm{in}-\mathrm{s}^{2}}^{2}
$$

## Load Friction Torque

Next, we calculated the friction torque at the motor shaft. $\mathrm{T}_{\mathrm{Lf}}^{\prime}=40 \mathrm{lb}-\mathrm{ft}$ is the load friction torque measured at the drive shaft. Therefore, load friction torque at the motor shaft is given by

$$
T_{L f}=\frac{T_{L f}^{\prime}}{N \frac{r_{3}}{r_{2}}}=1.333 \mathrm{lb}-\mathrm{ft}=256 \mathrm{oz}-\mathrm{in}
$$

## RMS Load Torque

We now have sufficient information to calculate the RMS load torque at the motor shaft. This can be computed from torque values during $t_{1}, t_{2}$ and $t_{3}$.

## Load torque during time $\mathrm{t}_{\mathbf{1}}$

Load torque during $\mathrm{t}_{\mathbf{1}}$ is acceleration torque plus load friction torque:

$$
T_{1}=T_{a 1}+T_{L f}
$$

where

$$
T_{a 1}=J_{L} \frac{\omega_{o}}{t_{1}}
$$

and

$$
\omega_{0}=\frac{v_{0}}{r_{6}} N \frac{r_{3}}{r_{2}}
$$

$$
v=200 \mathrm{ft} / \mathrm{min}=40 \mathrm{in} / \mathrm{s}
$$

Then

$$
\begin{aligned}
& \omega_{\mathrm{o}}=150 \mathrm{rad} / \mathrm{s} \\
& \mathrm{~T}_{\mathrm{a} 1}=418.5 \mathrm{oz} \cdot \mathrm{in} \\
& \mathrm{~T}_{1}=674.5 \mathrm{oz} \cdot \mathrm{in}
\end{aligned}
$$

## Load torque during time $\mathrm{t}_{\mathbf{2}}$

Load torque during time $t_{2}$ is simply the load friction torque

$$
T_{2}=T_{L f}=256 o z-i n
$$

## Load torque during time $\mathrm{t}_{3}$

Load torque during time $t_{3}$ is the difference between acceleration and friction load torques (see the velocity profile in Fig. 5.6.12):

$$
T_{3}=T_{a 1}-T_{L f}=162.5 \mathrm{oz}-\mathrm{in}
$$

RMS load torque is then given by

$$
\left(T_{L}\right)_{R M S}=\sqrt{\frac{\Sigma T_{i}^{2} t_{i}}{\Sigma t_{i}}}=277.87 \cong 278 \mathrm{oz}-\mathrm{in}
$$

## Motor Selection

In this system, acceleration torques contribute a relatively small part to the RMS load
$\qquad$
Application: CONVEYOR DRIVES 4STEMContact (Engineer): $\qquad$
Velocity Profile: Telephone: $\qquad$


Problem Input Data (supplied by customer):
At line 10 (1) V1, Forward/Reverse relocity, ips
(2) V2, Rewind/Slew velocity, ips


At line 11 (1) Tl , Acceleration time, mS
(2) T2, Run time, mS
(3) T3, Deceleration, mS
(4) T4, Dwell time, ms

(NOTE: Lines 10 and 11 define the velocity profile: it may be necessary to satisfy a given value for 's'; however, before entering the data into the computer, it is a preferred method to define the velocity profile in terms of velocity and time values.)

At line 12 (1) Jl, Load inertia, oz.-in.-sec. ${ }^{2}$
(2) Rl, Drive roll or capstan radius
(3) F, Friction Torque at load shaft, oz.-in.
(4) G, Reduction gear, motor to load
(5) T , Ambient temperature ${ }^{\circ} \mathrm{C}$.

(NOTE: Line 12 defines the load conditions.)
Input Data (supplied by ECC) :
At line 20 (1) M, Motor Model No.
(2) Kl, Motor torque constant (oz.-in./amp.)
(3) R2, Motor armature resistance (at $25^{\circ} \mathrm{C}$.)
(4) J2, Motor inertia (oz.-in.-sec. ${ }^{2}$ )
(5) K8, Motor viscous damping constant (oz.-in./KRPM)
(6) $\mathrm{T}_{\mathrm{F}}$, Friction Torque oz. - In .
(7) $\mathrm{R}_{\text {th }}$. Thermal Resistivity ${ }^{\circ} \mathrm{C}$./watt

| 702 |  |
| :---: | :---: |
| 26.9 |  |
| .43 |  |
| .15 |  |
| 7 |  |


(NOTE: To assist ECC Applications Engineering in the selection of the best available motor, it is helpful if customer can supply the following additional information:

1. Max. ambient operating temperature for the system ( ${ }^{\circ} \mathrm{F}$. or ${ }^{\circ} \mathrm{C}$.)
2. Is air cooling available?
3. Max. voltage and current ava lable with proposed (or existing) yower amplifier.

Fig. 5.6.13. Analysis form for conveyor drive system motor application. (Not corrected for this third edition)
torque. Therefore, it is estimated that a motor capable of providing the calculated acceleration torque and delivering in excess of 278 oz-in RMS torque will be adequate. A Motomatic E-702 seems to be a logical choice.

Motor parameters:

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{T}}=26.9 \mathrm{oz}-\mathrm{in} / \mathrm{A} \\
& \mathrm{R}=0.43 \Omega @ 25^{\circ} \mathrm{C} \\
& \mathrm{~J}_{\mathrm{m}}=0.15 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \\
& \mathrm{D}=7 \mathrm{oz}-\mathrm{in} / \mathrm{krpm} \\
& \mathrm{~T}_{\mathrm{f}}=15 \mathrm{oz}-\mathrm{in} \text { (maximum) } \\
& \mathrm{R}_{\mathrm{th}}=1.6^{\circ} \mathrm{C} / \mathrm{W}
\end{aligned}
$$

Motor Performance Calculation - a computer input data sheet is shown in Fig. 5.6.13, and the corresponding solution printout appears as Fig. 5.6.14. For examples of the calculations, hand computations will also be carried out below, permitting a comparison to be made.

```
RLN VFLGCITY (FAL/SEC) = 150
T1 = 500 ACC, RAL/ST2=300
12=14000
T3 = 50R ACC, RAL/ST2= = 300
WWLL TIME = G
JMAX REP RPTE (S-S CYCLES/SEC) = .67
AMEIENT BPERATING TEMP (LEGRFES C) =&S
M=1.4 F1 = 25G
```



```
AFMATURF PQWER LISSIPATION (WATTS) = 84.2'
FMS CLRRENT (AMFS) = 11.3
TERMINAL RESISTANCF AT MAX TEMP (EHMS) . 65
ACC TERELE (IN-CZ) = 741
I-ACC = 27.5 V-ACC = 46.6
I-KLN = 10.4 V-RLN = 35.3
I-LEC=7 V-LLC = 33.2
```

Fig. 5.6.14. Conveyor drive system motor selection printout.

## Manual Calculations

## Generated RMS Torque

The required motor torques in time intervals $t_{1}, t_{2}$ and $t_{3}$ can be calculated from load torque data by adding the motor parameters $J_{m}, D$ and $T_{f}$. The individual generated torques then become

$$
\begin{aligned}
& T_{1}=741 \mathrm{oz}-\mathrm{in} \\
& \mathrm{~T}_{2}=281 \mathrm{oz}-\mathrm{in} \\
& \mathrm{~T}_{3}=189 \mathrm{oz}-\mathrm{in}
\end{aligned}
$$

and the required RMS generated torque is $\mathrm{T}_{\text {RMS }}=305.3 \mathrm{oz}-\mathrm{in}$.

RMS value of motor current is then

$$
\mathrm{I}_{\mathrm{RMS}}=\frac{\mathrm{T}_{\mathrm{RMS}}}{\mathrm{~K}_{\mathrm{T}}}=\frac{305.3}{26.9}=11.35 \mathrm{~A}
$$

## Amplifier Requirements

Current and voltage requirements during times $t_{1}, t_{2}$ and $t_{3}$ must be calculated based on temperature stabilized armature resistance.

Armature temperature rise $\Theta_{r}$ is given by

$$
\begin{equation*}
\Theta_{r}=\frac{R R_{t h} I_{R M S}^{2}}{1-\left.R R_{t h} \psi\right|_{R M S} ^{2}} \tag{5.6.17}
\end{equation*}
$$

The E-702 motor has

$$
\begin{aligned}
& \mathrm{R}=0.43 \Omega \\
& \mathrm{R}_{\mathrm{th}}=1.6^{\circ} \mathrm{C} / \mathrm{W}
\end{aligned}
$$

so that

$$
\Theta_{r}=136{ }^{\circ} \mathrm{C}
$$

The motor resistance at operating temperature is then

$$
R_{h}=R\left(1+\psi \Theta_{r}\right)=0.66 \Omega
$$

a) During time $\mathrm{t}_{1}$

$$
\begin{align*}
& \mathrm{I}_{1}=\frac{\mathrm{T}_{1}}{\mathrm{~K}_{\mathrm{T}}}=\frac{741}{26.9}=27.55 \mathrm{~A} \\
& \mathrm{~V}_{1}=\mathrm{R}_{\mathrm{h}} \mathrm{I}_{1}+\mathrm{K}_{\mathrm{E}} \mathrm{n}_{\mathrm{o}} \tag{5.6.18}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{E}}=\frac{\mathrm{K}_{\mathrm{T}}}{1.3524}=19.9 \mathrm{~V} / \mathrm{krpm} \\
& n_{\mathrm{o}}=9.5493 \times 10^{-3} \omega_{\mathrm{o}}=1.432 \mathrm{krpm}
\end{aligned}
$$

so that

$$
V_{1}=46.7 \mathrm{~V}
$$

b) During time $\mathrm{t}_{2}$

$$
\begin{aligned}
& \mathrm{I}_{2}=\frac{\mathrm{T}_{2}}{\mathrm{~K}_{\mathrm{T}}}=\frac{281}{26.9}=10.45 \mathrm{~A} \\
& \mathrm{~V}_{2}=R_{h} \mathrm{I}_{2}+K_{E} n_{0}=35.4 \mathrm{~V}
\end{aligned}
$$

c) During time $\mathrm{t}_{3}$

$$
\begin{aligned}
& \mathrm{I}_{3}=\frac{\mathrm{T}_{3}}{\mathrm{~K}_{\mathrm{T}}}=\frac{189}{26.9}=7.03 \mathrm{~A} \\
& \mathrm{~V}_{3}=\mathrm{R}_{\mathrm{h}} \mathrm{I}_{3}+\mathrm{K}_{\mathrm{E}} \mathrm{n}_{\mathrm{o}}=33.14 \mathrm{~V}
\end{aligned}
$$

Motor temperature

$$
\Theta_{a}=\Theta_{r}+\Theta_{A}=136+25=161^{\circ} \mathrm{C}
$$

Notice that armature temperatures, $161^{\circ} \mathrm{C}$ and $160{ }^{\circ} \mathrm{C}$ (calculated by computer - see Fig. 5.6.14) exceed the temperature rating of the motor. Let's evaluate the next larger unit available, a model E-703 with the following characterics:

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{T}}=24.5 \mathrm{oz}-\mathrm{in} / \mathrm{A} \\
& \mathrm{R}=0.26 \Omega @ 25^{\circ} \mathrm{C} \\
& \mathrm{~J}_{\mathrm{m}}=0.2 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \\
& \mathrm{D}=10 \mathrm{oz}-\mathrm{in} / \mathrm{krpm} \\
& \mathrm{~T}_{\mathrm{f}}=15 \mathrm{oz}-\mathrm{in} \text { (maximum) } \\
& \mathrm{R}_{\mathrm{th}}=1.2^{\circ} \mathrm{C} / \mathrm{W}
\end{aligned}
$$

The computer solution is given in Fig. 5.6.15.

```
RLN UFLNCITY (RAE/SEC)=150
T% =500 ACC RAD/ST2 = 30%
T2 = 14QQ
T2 = 5QG ACC, RAL/S+2 = 3QK
L4.FLL. TIME = &
MFX KFF KATF (S-S CYCLES/SEC) = .47
FNBIENT (PEFATING TENF (DFGREES C) =2S
N! = 1.L F1 = 25%
NTVFNGLFL ATE3 ARMFTHKE TEMF (LECRKLSC) = 87
```



```
AFMATLFE FQWER LISSIPATIGN (KATTS) = 51.8
NTS CURFFNT (AMPS) = 12.7
TFRMINAL. FESISTANCE AT MAX TEMP (OHMS) =0.32
ACC TOFQLE (IN-UZ) = 758.2
1-ACCC=36.9 V-ACC=36
I-RLN = 11.6 V-FUN=29.7
```

Fig. 5.6.15. Re-run of the conveyor drive system problem, applying the $\mathrm{E}-703$ motor.
Note that in the printout of Fig. 5.6.15 the armature temperature ( $87{ }^{\circ} \mathrm{C}$ ) is well within the temperature rating of the motor. This is due to the added stack length of the E-703, which provides a comfortable margin for temperature consideration.

Therefore, the Motomatic E-703 can be used in this application, meeting all system requirements without overheating.

### 5.6.5. COMPUTER TAPE TRANSPORT

 REEL MOTORThe function of a reel motor in a computer tape system is to hold the amount of tape stored in the buffer bins constant (see Fig. 5.6.16). The reels are wound and unwound as required by the motion of the drive roll or capstan motor. Rotational speed and direction of the reel motors is controlled by the amount of tape hanging in the buffer bin.

The tape transport under consideration has 32 photocell sensors placed on the wall of
the buffer bin. They produce a voltage which is proportional to the tape loop position in the buffer bin. As the tape moves up and down in the buffer bin, a greater or lesser number of photocells are shaded. If a sensor is shaded by tape, it turns a switching circuit off. Summing the voltages produced by the switching circuits yields a voltage signal proportional to tape position in the bin (Fig. 5.6.17). When the position signal is positive, the reel drive functions so as to provide the buffer bin with tape. When it is negative, the reel drive functions so as to remove tape from the bin.


Fig. 5.6.16. Computer tape transport reel drive system.

The computer run for an application of this type is presented in Figs. 5.6.18 and 5.6.19. The results show that the motor


Fig. 5.6.17. Voltage vs. tape position in the bin of Fig. 5.6.16.
selected, under specified conditions, will achieve a maximum armature temperature of $108.5^{\circ} \mathrm{C}$, with an armature dissipation of nearly 70 W .

```
"EFI. SERVO ANALYSIS COMFZIMENI. IF FLEGIKO: MAF + CORH
- ORNAKC OFEHAIING SPEEL (1FS)= ??S
YFA|NO SPEED (IFS)= 5AO
    MOTOR MOOFL NH. E-T&?
            WFFI EMPIT KO REFi FULI KO KEWINO:
\begin{tabular}{|c|c|c|c|}
\hline MAX MOTOR SPEED (RPM) & 459.334 & 229.667 & 1837.33 \\
\hline AKMATURE SISSIPATION (WATIS) & 35.8157 & \(69.3 € 68\) & \\
\hline ainsatuke temr (DEG C) & 67.9797 & 108.45 & \\
\hline GLCEL TORQUE (OZ-IN RMS) & 259.349 & 342.276 & \\
\hline ACCEL CURRENT (ANHS KNS) & 6.17497 & 8.14943 & \\
\hline
\end{tabular}
INPUT DATE CHECK
AMBIENT TENF = 25
\(\sim 4=2 \cdot f \quad 85=5 \cdot ? \quad \mathrm{~J} 1=4 \cdot 355 \mathrm{~J} 2=1 \cdot 2\)
\(G_{1}=1 \times 1=14 \quad N 1=1+1=4 \quad 17=8\)
MOTOR DATA FOR MOLEL NC. E-7A3
```



Fig. 5.6.18. Computer printout of tape transport reel motor analysis.

Reel Servo Analysis

Program Name: REEL 5*
Application:

Customer: $\qquad$
Engineer: $\qquad$
Date: $\qquad$ Telephone: $\qquad$
Input Data (supplied by customer):
At line 10 (1) $\mathrm{Vl}=$ Operating tape speed, ips
(2) $V 2=$ Rewind speed, ips


At line 11 (1) R4= Empty Reel Radius
(2) R5 = Full Reel Radius
(3) $\mathrm{Xl}=\mathrm{Max}$. overall motion of sense point (Max. +point to max. - point in inches)
(4) Jl= Empty Reel Inertia
(5) J2= Full Reel Inertia

| 2.6 |
| :---: |
| 5.2 |
| 14.0 |
| .355 |
| 1.2 |

At line 12 (1) Gl= Gear Reduction Ratic (from motor to reel)
(2) $\mathrm{Nl}=\mathrm{No}$. of Tape Loops
(3) Fl= Friction of Reel and Tape
(!) T7= Tape Tension
(5) $\phi 1=$ Ambient remperatire $\left({ }^{\circ} \mathrm{C}\right)$


Mutor Data (supplied by ECC):
At line 20
through 99 (1) MS= Motor Model No. (Any number of motors can be input to program up to 80) (Enter Motor No. Using This Format: E-703)
(2) $\mathrm{Kl}=$ Motor Torque Constant oz-in/amp
(3) $\mathrm{R} 2=$ Motor Resistance at $25^{\circ} \mathrm{C}$ (ohms)
(4) J3= Motor Ineriia (oiss)
(5) K8= Damping Constant (oz-in/KRPM)
(6) F2 $=$ Friction Torque (oz-in)
(7) R3= Thermal Resistance

|  |
| :---: |
| $E-703$ |
| 42 |
| .79 |
| .2 |
| 10 |
| 15 |
| 1.2 |

NOTE: TO assist ECC Applications Engineering in the selection of the best available motor, it is helpful if customer can supply the following additional information:

1. Max. ambient operating temperature for the system
( ${ }^{\circ} \mathrm{F}$. or ${ }^{\circ} \mathrm{C}$.)
2. Is air cooling available?
3. Max. voltage and current available with proposed (or existing) power amplifier.

Fig. 5.6.19. Computer analysis input for the tape transport reel motor application. (Not corrected for this third edition)

### 5.7. APPLICATION OF P6000 SERIES SERVO MOTOR SYSTEMS

The P6000 systems are unique among the spectrum of servo motor controls currently on the market. Their uniqueness stems from the "total system" approach used during their conception and design. The design features incorporated in the series result in a wide range of highly flexible modules which can be interfaced with each other to enable the construction of custom systems tailored exactly to the needs of the user. Electro-Craft has had over 10 years of experience designing custom systems for specific applications; the know-how built up over those years has enabled the incorporation of many unique and previously unavailable features into the P6000 series.

### 5.7.1.GENERAL DESCRIPTION OF <br> THE P6000 RANGE OF SERVO MOTOR CONTROLS

The heart of P6000 series is a fast response Pulse Width Modulated fully regenerative DC amplifier. The amplifier uses a fixed switching frequency of 20 kHz which eliminates the annoying audible noise generated by other types of PWM amplifiers. The series incorporates the following standard features:

Dual Stage Current Limit - Independently adjustable peak current and average current limits, together with full adjustment of the peak current limit duration. These adjustments can be made internally or from remotely mounted potentiometers.

Regenerative Energy Protection - The amplifiers are fully protected against the build-up of electrical energy in the power supply caused when high inertia loads are decelerated slowly. Units not having this kind of protection are susceptible to catastrophic failures under certain load conditions.

Crowbar Protected Power Supply - This feature makes the amplifier short circuit proof. If either output terminal on the amplifier is shorted to the other or to ground, or if the amplifier operation is disrupted by component failure or misuse, an SCR in the power supply is triggered blowing the power supply fuse and discharging the energy stored in the storage capacitors. This feature protects the output transistors and prevents catastrophic failure. Switching amplifiers not having this feature can fail in a literally "explosive" manner.

Amplifier Clamps - The amplifier has built-in clamping to prevent output surges during application or removal of AC power. Other circuits also provide independent clamps for clockwise and counterclockwise rotation for uses as end of travel limits. These clamps can be actuated by logic signals or contact closures.

Separate Power Supply - The power supply which enables operation from standard AC lines can be separated from the amplifier unit and mounted independently, or if desired, the amplifier can be purchased without the power supply and
used in conjunction with existing unipolar DC supplies.

Flexibility - The amplifier has provision for up to three plug-in option p.c. cards. One of these is permanently used for the servo summing amplifier; the other two can be used to connect a wide range of standard or custom options. A complete list of options can be found in section 5.7.2.

Serviceability - The amplifiers have been designed to meet the maintainability requirements of both the "minimum downtime user" and the user who is working on a tight budget and cannot keep a stock of spare assemblies. For the "minimum downtime user", all major subassemblies and p.c. cards can be replaced by removing a maximum of four screws in each and unplugging the assembly. In fact, since all input/ output connections are made through plugs and sockets, if preferred, the entire amplifier can be replaced in minutes.

For the user who prefers to troubleshoot and repair rather than replace, all test points are brought out to a 14 -pin test point array for easy location; current loops are provided on the output stages to enable use of oscilloscope current probes; and all p.c. cards are hinged for access to components without removal.

Dual Axis Amplifiers - All models of the 6000 series are available in a dual axis configuration with two amplifiers sharing one power supply. This arrangement gives
maximum efficiency and economy saving as much as $20 \%$ on the cost of a dual axis system.

### 5.7.2.STANDARD OPTIONS

This list represents all of the standard options for the P6000 series which were available at the publication date (August 1975). More standard options are planned, and in addition custom options to meet your exact needs are available.

## R1000 Ramp Generator

The R1000 option permits the selection of one of two preset adjustable acceleration rates and one of two preset adjustable deceleration rates. The adjustments for ramp slopes are independent of each other and the different preset slopes can be selected by switch contacts or logic signals.

The R1000 also permits the selection of one of two preset adjustable motor speeds or a speed set by an external analog signal, thus the option can be used for digital selection of speed.

The ramp generator does not affect the closed-loop frequency response of the system to load disturbances.

The option is useful in any application where the acceleration and deceleration need to be controlled in a repeatable manner.

## PL1000 Phase Lock System

The phase lock option enables speed control with zero long term speed drift and synchronization of a motor with any other moving part.

This option is particularly suited to applications where the speed of the motor forms a reference, such as chart drives or scanning drives. The system is also ideally suited to component marking applications where a motor driving a marking head has to be accurately synchronized with components on a moving conveyor.

## DPS 6000 Digital Positioning System

The DPS 6000 is digital position system option which can be ordered with the standard P6000 Pulse Width Modulated servomotor controller series or the L5000 linear servo amplifiers. The unique design of the DPS 6000 and the high performance of the P6000 series controllers combine to provide a positioning system that can achieve higher accuracy and faster step times than stepper motors, clutch/brakes and other widely used systems.

A typical 200 step per revolution system will give single step times of 15 ms , including settling and slew speeds of 10,000 steps/second - even with load moments of inertia as high as 0.05 oz-in-s ${ }^{2}$ - without loss of position.

Positional accuracy is typically in the order of $0.5^{\circ}$, even with high disturbing torques.

Greater accuracy can be obtained by using systems having higher resolutions without deterioration of high speed slew and large step performance. Holding torques are on the order of 250 oz-in peak, 140 oz-in continuous.

The system accepts parallel binary or BCD step size inputs which enables direct control from thumb wheel switches and direct operation from minicomputers, eliminating the need for a translator/indexer or other interface logic.

An 8 bit speed input is provided so that the speed of the motor during stepping can be controlled. This feature, or the alternative analog speed input, enables the DPS 6000 to be used for contouring as well as point to point positioning applications.

For specifications see Chapter 7.

### 5.7.3.STANDARD DRIVE PACKAGES

The SD series of Standard Drive Packages consist of specially matched combinations of Electro-Craft servo motors and servo controllers. Both the motor and control are manufactured by Electro-Craft and are covered by a single warranty. Thus ElectroCraft can offer total system responsibility.

All of the SD series can be fitted with any of the standard or custom plug in options.

Full specifications of the SD series can be found in Chapter 7.

### 5.7.4. AMPLIFIER INSTALLATION

Portions of the amplifier and power supply produce heat which must be vented away from the components. Thus it is important to allow sufficient space around and above the amplifier for proper air circulation. Electro-Craft recommends at least 1 in clearance above and on both sides of the amplifier and power supply. If the amplifier is located in a completely enclosed cabinet, louvers or vents must be provided on each side near the fan. In addition to this, enough space should be allowed in front of the amplifier so that the logic control motherboard may be tilted down for servicing. For this to be done, 5.5 in of clearance should be provided in front of the amplifier. Another 2 in of clearance should be left behind the power supply to allow replacement of fuses and disconnection of rear panel connectors.

Holes have been provided in the chassis rails to secure the amplifier and power supply to a sub-chassis or other surface. If the power supply is being mounted remotely and the rails are not being used, the holes in the chassis may be used. These components should be secured using \#10-24 machine bolts of sufficient length.

### 5.7.5. CONTROL ADJUSTMENTS

## Motherboard Controls

The controls that regulate the current limiting circuitry of the amplifier are located on the motherboard. Because the
peak current adjustment will affect the average current adjustment, the peak current control should be set first. Unless a model P6042 Tektronix DC current probe is available, the peak current adjustment should be set by percent of rotation. The P6100 peak current range is between 3.5 and 15 A . Thus, when the control is fully clockwise, the peak current limit will be 15 A , while one-half rotation will allow only 9.25 A , or one-half the difference between the upper and lower limits. The P6200 amplifier should be adjusted using a range between 7 and 25 A , and then following the same procedure. The same method may be used when setting the peak current duration, or the time to which the current limiter will allow peak current. The range of adjustment is between 0.01 s and 0.1 s . With the control set at maximum clockwise rotation, the peak current duration will be 0.1 s . A central position will allow about one-half this time.

The next step is to set the average current limit. This setting will not effect the peak current limit. Set the control fully counterclockwise. With a DC ammeter in series with either motor lead, operate the motor under a stall condition. Slowly turn the average current limit control clockwise until a current reading is obtained which is considered the safe maximum continuous current of the motor to be used with the amplifier.

An alternate and more precise method of current adjustment would be to make use
of the Test Point number 6 on the Motherboard. This point is the output of the current sensing amplifier and will give a voltage reading proportional to the amplifier current. An oscilloscope can be carefully connected to this point and the motor current set using the following proportion:

$$
\begin{array}{ll}
\text { P6200 - } & 0.2 \text { Volts }=1 \mathrm{Amp} \\
\text { P6100 - } & 0.4 \text { Volts }=1 \mathrm{Amp}
\end{array}
$$

## Summing Amplifier

The gain control on the summing amplifier (plug-in card) should be turned as far clockwise as is practical to insure stable operation. This will provide maximum gain. Set the input signal level control fully counterclockwise. After this is done, set the balance control so that there is no rotation of the motor. Then adjust the input level control for the proper speed range for the particular application in which the system is to be used. This must be done in conjunction with the external signal speed command. See section 5.7.7. for further details.

### 5.7.6. CUSTOMER CONNECTIONS

## Motor Tachometer Hook-up

The 6-pin connector located at the rear panel of the power supply is used to interface the motor and tachometer with the amplifier. If the amplifier is purchased without a power supply the connector
will be located on the rear of the amplifier chassis. To ensure that the motor is properly phased, the following procedure should be followed.

1. Connect a DC voltmeter to the motor leads. Set the meter on the 10 V range.
2. As viewed from the motor shaft end, rotate the motor shaft clockwise (CW) by hand.
3. Observe the voltmeter deflection and determine and mark the positive ( + ) lead of the motor.
4. Repeat steps 2 and 3 with the voltmeter connected to the tachometer leads.
5. Connect the motor and tachometer to the amplifier as indicated
(Figure 5.7.1)
CW Rotation with Positive Command (into summing amplifier)

| Motor positive (+) | pin 1 |
| :--- | :--- |
| Motor negative (-) | pin 4 |
| Tach positive ( + ) | pin 6 |
| Tach negative ( - ) | pin 3 |

Counterclockwise (CCW) Rotation with Positive Command

| Motor positive (+) | pin 4 |
| :--- | :--- |
| Motor negative ( - ) | pin 1 |
| Tach positive $(+)$ | pin 3 |
| Tach negative ( - ) | pin 6 |

If either the motor housing or a tach shield must be grounded, this can be done by


Fig. 5.7.1. Motor Connector as viewed from rear panel.
making the following connections to the six pin motor connector:

$$
\begin{array}{ll}
\text { Motor Housing Ground } & \text { pin } 5 \\
\text { Tachometer shield } & \text { pin } 2
\end{array}
$$

## Control Signal Connections

The connections necessary to provide for signals to control the operation of the motor are made through the 36 -pin connector located on the back panel of the power supply or amplifier chassis, if the power supply is not used or is used remotely. Some of the inputs are designated as auxiliary inputs. These auxiliary inputs are connected to one of the optional plug-in card connectors on the motherboard but they may be used to interface with other cards by the use of jumper wires on the motherboard. The functions of the inputs shown in Fig. 5.7.2 are as follows:

Pin 1: External Command, input to the summing amplifier to drive the motor. This input is not used with the ramp generator.

Pin 2: External Command Common, the common to be used in conjunction with the command signal. This point is common to the power supply and may be used as the common for any low level signal.

Pin 3: Inhibit, A Drive removes drive signal from one side of the output stage bridge. To use the inhibit, connect a normally closed switch between pin 3 and pin 5 . When the switch opens, the motor will stop rotating in one direction (the motor does not brake to stop). If the inhibit is not used, pins 3 and 5 (A Drive) and pins 4 and 6 ( $B$ Drive) must be jumpered together.

$$
\begin{aligned}
& \mathrm{O}^{33} \mathrm{O}^{29} \mathrm{O}^{25} \mathrm{O}^{21} \mathrm{O}^{17} \mathrm{O}^{13} \mathrm{O}^{9} \mathrm{O}^{5} \mathrm{O}^{1} \text {. } \\
& \mathrm{O}^{34} \mathrm{O}^{30} \mathrm{O}^{26} \mathrm{O}^{22} \mathrm{O}^{18} \mathrm{O}^{14} \mathrm{O}^{10} \mathrm{O}^{6} \mathrm{O}^{2} \\
& \mathrm{O}^{35} \mathrm{O}^{31} \mathrm{O}^{27} \mathrm{O}^{23} \mathrm{O}^{19} \mathrm{O}^{15} \mathrm{O}^{11} \mathrm{O}^{7} \mathrm{O}^{3} \\
& \mathrm{O}^{36} \mathrm{O}^{32} \mathrm{O}^{28} \mathrm{O}^{24} \mathrm{O}^{20} \mathrm{O}^{16} \mathrm{O}^{12} \mathrm{O}^{8} \mathrm{O}^{4}
\end{aligned}
$$

| $\mathrm{O}^{33} \mathrm{O}^{29} \mathrm{O}^{25} \mathrm{O}^{21} \mathrm{O}^{17} \mathrm{O}^{13} \mathrm{O}^{9} \mathrm{O}^{5}$ | $\mathrm{O}^{1}$ |
| :--- | :--- |
| $\mathrm{O}^{34} \mathrm{O}^{30} \mathrm{O}^{26} \mathrm{O}^{22} \mathrm{O}^{18} \mathrm{O}^{14} \mathrm{O}^{10} \mathrm{O}^{6}$ | $\mathrm{O}^{2}$ |
| $\mathrm{O}^{35} \mathrm{O}^{31} \mathrm{O}^{27} \mathrm{O}^{23} \mathrm{O}^{19} \mathrm{O}^{15} \mathrm{O}^{11} \mathrm{O}^{7}$ | $\mathrm{O}^{3}$ |
| $\mathrm{O}^{36} \mathrm{O}^{32} \mathrm{O}^{28} \mathrm{O}^{24} \mathrm{O}^{20} \mathrm{O}^{16} \mathrm{O}^{12} \mathrm{O}^{8}$ | $\mathrm{O}^{4}$ |

$\mathrm{O}^{33} \mathrm{O}^{29} \mathrm{O}^{25} \mathrm{O}^{21} \mathrm{O}^{17} \mathrm{O}^{13} \mathrm{O}^{9} \mathrm{O}^{5} \mathrm{O}^{1}$
$\mathrm{O}^{34} \mathrm{O}^{30} \mathrm{O}^{26} \mathrm{O}^{22} \mathrm{O}^{18} \mathrm{O}^{14} \mathrm{O}^{10} \mathrm{O}^{6} \mathrm{O}^{2}$
$\mathrm{O}^{35} \mathrm{O}^{31} \mathrm{O}^{27} \mathrm{O}^{23} \mathrm{O}^{19} \mathrm{O}^{15} \mathrm{O}^{11} \mathrm{O}^{7} \mathrm{O}^{3}$
$\mathrm{O}^{36} \mathrm{O}^{32} \mathrm{O}^{28} \mathrm{O}^{24} \mathrm{O}^{20} \mathrm{O}^{16} \mathrm{O}^{12} \mathrm{O}^{8} \mathrm{O}^{4}$
$\mathrm{O}^{33} \mathrm{O}^{29} \mathrm{O}^{25} \mathrm{O}^{21} \mathrm{O}^{17} \mathrm{O}^{13} \mathrm{O}^{9} \mathrm{O}^{5} \mathrm{O}^{1}$
$\mathrm{O}^{34} \mathrm{O}^{30} \mathrm{O}^{26} \mathrm{O}^{22} \mathrm{O}^{18} \mathrm{O}^{14} \mathrm{O}^{10} \mathrm{O}^{6} \mathrm{O}^{2}$
$\mathrm{O}^{35} \mathrm{O}^{31} \mathrm{O}^{27} \mathrm{O}^{23} \mathrm{O}^{19} \mathrm{O}^{15} \mathrm{O}^{11} \mathrm{O}^{7} \mathrm{O}^{3}$
$\mathrm{O}^{36} \mathrm{O}^{32} \mathrm{O}^{28} \mathrm{O}^{24} \mathrm{O}^{20} \mathrm{O}^{16} \mathrm{O}^{12} \mathrm{O}^{8} \mathrm{O}^{4}$

Fig. 5.7.2. Control Signal Connector as viewed from rear panel.

Pin 4:Inhibit, $B$ Drive removes drive signal from the other side of the output stage bridge. When pin 4 is open, the motor is prevented from turning in one direction.

Pin 5: Inhibit Common provides the signal to be used with the inhibit inputs. When the $A$ and $B$ Drive inhibits are connected to the inhibit commons, the output stage drive signals are turned off whenever the regulated power supply is below a predetermined point. This prevents damage to the output transistors during turn-on and turn-off of the amplifier.

Pin 6: Inhibit Common is the same point as Pin 5 and provides the signal for the B Drive Inhibit.

Pin 7: Logic Clamp, A Drive, if a TTL logic " 1 " is applied to this point the A Drive is clamped or inhibited.

Pin 8: Logic Clamp, $B$ Drive, if a TTL logic " $I$ " is applied to this point the Drive is clamped or inhibited.

Pin 9: Reference Voltage, +15 Volts, provides a regulated voltage through a 1,000 ohm, $1 \%$ resistor that may be used with a control potentiometer for a drive signal.

Pin 10: Reference Voltage, $\mathbf{- 1 5}$ Volts, provides the same feature as Pin 9 but with a negative voltage.

Pin 11: Reference Voltage Common is the common point to be used in conjuction with the reference voltage or any other low level signal.

Note: The following are used only with the Ramp Generator option.

Pin 12: Ramp Generator External Input, allows for control of the motor speed through the Ramp Generator with an external analog signal.

Pin 13: Slope B Select selects the B slope by application of a TTL logic " 0 " or a switch closure to ground.

Pin 14: Slope A Select provides the same functions as Pin 13 for the A Slope.

Pin 15: Ramp Number 2 Initiate selects one of two àdjustable final motor speeds. This is controlled by applying a logic " 0 " or a switch closure to ground.

Pin 16: Ramp Number 1 Initiate, performs the same function as Pin 15 for Ramp Number 1.

Pins 17-36: are used for the DPS Option or any special inputs that may be required for specialized applications. If these inputs are not required they will not be wired into the harness.

NOTE: Pin numbers and pin configurations may vary depending on the particular options used. Verify the above connections by consulting the instruction manual.

### 5.7.7. SUMMING AMPLIFIER COMPENSATION

The components on the summing amplifier card must be selected for the type of motor and load conditions that are expected to be used with the amplifier. Assuming that this has already been done at the factory, the adjustments on the card will allow for proper compensation for an efficiently operating servo system. The first step in compensating the summing amplifier is to adjust the controls to a starting point that will ensure a stable, damped response. This can be done by adjusting the controls according to the following table. Refer to Fig. 5.7.3 for control location.


Fig. 5.7.3. Summing Amplifier Adjustments.

## Offset Adjustment

With the amplifier turned on and the motor connected, adjust R1 so that the motor shaft does not turn. This will adjust for any amplifier offset. R4 has previously been set to minimum so that external input signals will have no effect.

## Test Instrument Connection

Connect an oscilloscope to the output of the tachometer at pin 10 of the summing amplifier card. The ground of the scope should be connected to the point marked "GND" on the lead of the electrolytic capacitor C39 near the summing amplifier. Connect a DC voltage source of appropriate magnitude at pin 7 of the control signal connector. (The ramp generator, if one is being used, should not be plugged in at this time). The signal common of the voltage source should be connected to pin 8. This voltage source should be a step function (square wave generator), capable of being switched on and off.

## Servo System Compensation

The objective of this compensation is to get a high DC gain (R6 and R3 CW as far as possible) consistent with stability,
and a wide bandwidth ( R 2 CW as far as possible). The waveforms shown in Fig. 5.7.4 are typical of those obtained during the adjustment procedure.


Fig. 5.7.4. Waveforms and Corresponding Corrective Action.

The "ideal" waveform should be the end result of the servo system compensation. This, of course, should be done with the proper load conditions that are to be used with the motor. The following procedure for adjustment is recommended for best results.

1. Turn R4 CW far enough to drive the motor and tachometer to achieve about a 10 V deflection on the oscilloscope (or any level consistent with the end application of the motor).
2. Turn R3 CW as far as possible without causing a sustained oscillation.
3. Turn R5 CCW until an underdamped waveform is obtained.
4. Correct the overshoot by turning R2 CW.
5. Repeat steps 3 and 4 until a waveform as close to ideal as possible is obtained.
6. Turn R 6 CW in about $10^{\circ}$ increments while adjusting R2 and R5 to compensate for any under or overshoot until the best response is achieved. (Note: As R6 is turned CW the motor speed will reduce. This may be corrected by turning R4 CW).
7. The input command can then be scaled for the correct output speed
per volt by adjusting R4. If the potentiometer does not allow enough adjustment, a new range can be obtained by replacing the resistor R21. A larger resistance will require a higher input voltage to produce a given speed, and conversely a smaller resistance will require a lower input voltage.

This procedure should make it possible for individuals with little experience to get a working system. However, there are difficulties such as "gear backlash", or "dead band" which may be encountered, for which this procedure may not be adequate. In these cases it may be necessary to design a special compensation scheme to fit the application. When using a ramp generator, the basic summing amplifier should be adjusted for a slight under-damping or slight overshoot to assure sharp corners on the ramp. If the ramp generator is being used it should now be inserted into the correct option connector (with power to the amplifier off) and the set-up procedure for the ramp generator performed. Refer to RAMP GENERATOR ADJUSTMENT for the correct procedure.

### 5.7.8. RAMP GENERATOR ADJUSTMENT

Before the ramp generator can be adjusted, the summing amplifier must be properly compensated. The following procedure assumes that this has already been done. The input command signal to the control connector will have to be moved from the summing amplifier input to the proper input
of the ramp generator unless the on-board ramp amplitude signal is used, in which case only a logic input signal is required. The ramp generator is designed to provide linear ramp inputs to the command input of the summing amplifier to provide the required servo system velocity profile. Both the ramp amplitudes and ramp slopes are independently adjustable. Two discrete (adjustable) ramp amplitudes plus zero (adjustable) are initiated by two TTL inputs to the board. Two other TTL inputs select one of two positive slopes or one of two negative slopes, each individually adjustable over a decade range. Ranges may be changed by changing the integrating capacitor to obtain ramp slopes ranging from $10 \mathrm{~V} / \mathrm{ms}$ to $0.667 \mathrm{~V} / \mathrm{s}$.
(Refer to Fig. 5.7.5.)

## 1. Ramp Initiate

One of the two ramp amplitudes (final values) can be selected by a TTL low signal on either pin 11 or pin 12. The final ramp amplitudes are individually adjustable via R1 and R2 on the ramp generator board. R2 adjusts overall gain. Jumpers on the ramp generator board allow the selection of either a 0 to +10 V ramp or a 0 to -10 V ramp on either input. A typical ramp sequence is as follows. A ramp is initiated by a TTL low on either pin 11 or pin 12 (a low on both is an undefined state). The output will ramp at the selected slope to a final amplitude and polarity determined by pots R29 and R32 and their appropriate jumpers. A high signal on both


Fig. 5.7.5. Ramp Generator Adjustments.
inputs will cause the unit to ramp to zero. Zero is offsettable $\pm \mathrm{mV}$ by R .

## 2. Slope Select

A TTL low on pin 10 selects one set of ramp slopes, one positive slope adjusted by R16 and one negative slope adjusted by R20. A TTL low on pin 9 selects a second independent set of slopes; the positive slope adjusted by R14 and the negative By R27. A TTL high on both pin 10 and pin 9 is not normally used. It will cause the ramp integrator to hold the value it was at
when both inputs went high until the integrator drifts due to input bias currents. A TTL low on both inputs is not defined.

## 3. Adjustment Procedure

With a low on pin 10 or pin 9 and both pin 11 and pin 12 high, adjust R7 for 0 VDC at pin 1 (ramp generator output). Apply a low to pin 12 and adjust R29 for the correct ramp amplitude. Release the low on pin 12 and apply a low to pin 11 and adjust R32 for the other ramp amplitude. Ramp polarities are selected by jumpers. To adjust ramp slopes apply a low to pin 10, apply a periodic low to pin 11 or pin 12 (whichever causes a negative ramp) and adjust R16 for the correct slope. With a periodic low on pin 11 or pin 12 (whichever causes a positive ramp) adjust R20 for the correct slope. Release the low on pin 10, apply a low to pin 9 , and repeat the above adjusting R14 and R27 for the correct slope. Adjust overall gain pot R2 if necessary. Ranges may be changed by changing integrating capacitor(s). If polarized capacitors are used, they must be placed back to back i.e. (+) to (+) or ( - ) to ( - ). The currents adjusted by R16, R14, R20 and R27, vary between 10 and $100 \mu \mathrm{~A}$. The ramp slope is given by

$$
\frac{d V}{d t}=\frac{I}{C}[V, s ; \mu A, \mu F]
$$

An external voltage command, such as a square wave input, may be used with the ramp generator by feeding this signal into
pin 7 of the ramp generator connector. This input is normally connected to pin 12 of the control connector.

### 5.7.9.IN CASE OF DIFFICULTY

The best procedure for troubleshooting is to isolate the source of the problem and then replace the section that is at fault. Try to determine what is the most likely thing to cause the difficulty. This troubleshooting guide consists of only the most common problems. It may be necessary to refer to two or more areas to correct a problem in some cases. Time spent analyzing a problem is time well spent. As you gain experience working with the amplifier and power supply, it will become very easy to isolate the source of any problem.

Never overlook the obvious, no matter how ridiculous it may seem. The following is a list of items that should be considered before looking to more difficult possibilities.

1. Line cord unplugged.
2. Circuit or connectors loose.
3. Fuses removed or blown out.
4. Shorted or open wires.
5. Defective motor or tachometer.
6. No power at AC outlet.

The following "Troubleshooting Guide" should help solve almost any difficulty that may arise in the operation of the amplifier.

### 5.7.10. TROUBLESHOOTING GUIDE

Amplifier and fan inoperative, no DC voltage present.

## POSSIBLE CAUSE

1. Line cord not plugged in.
2. No power at AC outlet.
3. Line fuse at back panel defective. NOTE: If the line fuse continues to blow out, one of the rectifiers on the rectifier bracket may be shorted. Disconnect transformer secondary and check rectifier for short using an ohmmeter.
4. Loose connection to transformer or rectifier bracket.
5. Defective rectifier on rectifier bracket.
6. Burned out or defective crowbar fuse.

CAUTION: If the crowbar fuse continues to burn out, this may indicate a fault in the output stage. It should be determined if there is a defective component or shorted output before reapplying power to the amplifier. The problem may be isolated by disconnecting the power connector to the amplifier, replacing the fuse and reapplying power to the power supply. If the crowbar does not fire, this indicates the source of the problem is in the amplifier.
2. Loose or unplugged connector on output stage circuit board.

1. Speed command signal missing.

Amplifier inoperative, fan works and DC voltage present at both filter capacitor and the stage, motor shaft stiff in both directions of rotation.

## POSSIBLE CAUSE

2. External command connector loose or disconnected.
3. External command level control potentiometer located on summing amplifier, set at minimum.
4. Clamp circuit activated due to loss of plus or minus 15 V supply voltages.
5. Open connection between inhibit output and either or both of the inhibit inputs.

NOTE: At this point it would be beneficial to isolate the problem to either the output or the motherboard. To do this, it is recommended that an oscilloscope be used to check the test points on the motherboard. Start at the drive to the output stage. If these signals match those in Table 5.7.1., the problem is in the output stage. Continue comparing signals and note all differences. Then consult the Electro-Craft service representative. When spare parts are available, the isolated problem component should be replaced.

Table 5.7.1. Test Point Waveforms

| TEST POINTS |  |  |
| :---: | :---: | :---: |
| NO. | DESCRIPTION | WAVEFORM |
| 1 | Current Limit Amp | Proportional to I |
| 2 | B Trip Point | -0.5V |
| 3 | IC Power Supply | $-15 \mathrm{~V}$ |
| 4 | Power Supply Comm. | OV |
| 5 | IC Power Supply | +15V |
| 6 | Current Diff. Amp | Proportional to 1 |
| 7 | Peak Current Mono. | $4 \mathrm{PK} 1-\sqrt{30} \mathrm{~V}$ P-P |
| 8 | Tri. Gen. Drive | $\int \quad \begin{aligned} & 12 \mathrm{~V} \mathrm{P}-\mathrm{P} \\ & 20 \mathrm{kHz} \end{aligned}$ |
| 9 | Tach Input |  |
| 10 | A Trip Point | +0.5V |
| 11 | Triangle Generator |  |
| 12 | No Connection |  |
| 13 | Output Drive A | $\square\left[\begin{array}{l}25 \mathrm{~V} \mathrm{P-P} \\ 20 \mathrm{kHz}\end{array}\right.$ |
| 14 | Output Drive B | $\int \begin{aligned} & 25 \mathrm{~V} \text { P-P } \\ & 20 \mathrm{kHz}\end{aligned}$ |
| N.B. | $I=$ Amplifier Output |  |

## Chapter 6

## Brushless DC Motors

### 6.1. INTRODUCTION

Since the performance of brushless DC motors is intimately tied to the commutation of current in motor windings, it may be appropriate to briefly review the commutation of conventional DC permanent magnet motors. The main reason for dividing a conventional DC motor winding system into segments is to minimize the effect of winding inductance as it relates to the turn-on and turn-off behavior of current in a segment. A secondary reason for the choice of the appropriate number of winding segments is to control the torque ripple. Thus we find that a fractional horsepower DC motor may have anywhere from 7 to 32 commutator bars per armature, and an integral horsepower motor below 5 to 10 hp may have less than 100 bars, and larger versions may have more than 100 bars per armature. The transient event of a coil commutation is generally well hidden inside the rotating armature of a conventional DC motor. The commutation event is not usually viewed on an oscilloscope screen, except as seen from outside the motor brush connections - the reason being that it is not easy to reach proper internal test points in a rotating structure. Much of the design effort in such motors, then, has gone into proper brush and commutator choice, and the results have been viewed from the observer's point by recording emitted radiation, brush and commutator heating, and erosion of com-
mutation surface or brush surface. A great variety of brush compositions have enabled designers to cope with the widely varying operational and environmental conditions that exist in today's technology.

As we contemplate the desirability of using semiconductor devices for commutation of a DC permanent magnet motor we are faced with a set of new problems, and a simple translation of the brush-type motor designed to a brushless type is not practical. For example, if we take a 16 bar, two-pole DC motor and were to substitute semiconductor devices for the brush and commutator assembly to achieve the same function, we would end up with 32 power transistors, two of which would be conducting at any one given time. It would then result in a very inefficient use of semiconductors - a utilization factor of about $6 \%$. Obviously the problem has to be solved in a new way to achieve cost effective performance.

Before we discuss brushless motor principles, let us remember the varying design constraints which conventional DC motors have to face and the varying applications which exist today. Examples of the variety of performance requirements are:

[^4]- bidirectional speed-control motor systems
- unidirectional speed-control motor systems able to handle over-running loads
- torque motors able to operate at stalled conditions
- low inertia servo motors able to handle acceleration torque demands one order of magnitude larger than the continuous torque
- servo motors with low torque-ripple specifications, in either low inertia or high inertia versions

The listing could go further, but these examples illustrate the unique design
demands which exist for brush type DC motors.

The same conditions hold for brushless DC motor systems, and therefore the following discussion of brushless DC motors will treat some of the more common configurations which serve their intended purposes in a cost-optimal way.

Although the following describes brushless DC motors controlled by transistor systems, the reader should realize that thyristors (SCR's) can as well be used for the same purpose, although the turn-off of the latter may be complicated due to the unique characteristics of the thyristor.

### 6.2. DEFINITION OF A BRUSHLESS DC MOTOR SYSTEM

Since the control circuits for a step motor or frequency controlled AC motor at first glance may appear to be similar to some brushless DC motor controllers, it is appropriate at the outset to clarify the distinctions between them. If we first define what a brushless DC motor system characteristic should be, we can then realize the differences between that type of system and the others.

A brushless DC motor system should have the torque-speed characteristics of the conventional DC permanent magnet motor.

Fig. 6.2.1 illustrates the torque-speed characteristics of a conventional DC motor. By our definition above, the brushless DC motor should have the same basic characteristics. From these basic relationships we can derive mathematical expressions which enable us to apply simple mathematical analysis and achieve solutions to servo system problems. These conditions are discussed in chapter 2, and apply generally to brushless DC motor systems.


Fig. 6.2.1. Typical speed-torque characteristic for a conventional permanent magnet DC motor. Armature voltage is held constant.


Fig. 6.2.2. Typical speed-torque characteristic for a step motor system. Excitation voltage is held constant, step frequency varied.

If we examine a step motor system speedtorque characteristic (Fig. 6.2.2) we observe non-linear relationships between the two variables. The discontinuous part of the curve is due to a resonant oscillatory mode inherent in the motor design. The discontinuous nature of the step motor low-speed performance limits its usefulness in velocity control applications. Thus the step motor is a unique device, which differs from a brushless DC motor in fundamental performance aspects.

The AC motor characteristic in Fig. 6.2.3 shows an entirely different relationship between speed and torque. The useable


Fig. 6.2.3. Typical speed-torque curves of a frac-tional-horsepower polyphase induction motor. Excitation voltage and frequency is held constant.
part of the curve lies in the region between 90 and $100 \%$ of synchronous speed - and the synchronous speed depends on the excitation frequency. The controllable AC motor system depends on a variable frequency, variable AC voltage, which has to be coordinated with the shaft velocity to produce a controlled "slip frequency" current in the rotor windings. Because the rotor-stator structure can be considered
a transformer, it does not work well at low frequencies (which corresponds to low shaft speeds). This is a fundamental difference between the AC motor and the brushless DC motor, since in the latter torque is produced by the interaction of a magnetic field produced by a permanent magnet rotor, and magnetic field due to a DC current in a stator structure.

### 6.3 PRACTICAL SOLUTIONS TO BRUSHLESS COMMUTATION

The first general comment about brushless motors and controllers is that the design philosophy should point to a motor design which would require the least number of power semiconductor switches to adequately meet the performance requirements. The second assumption is that wherever possible one should use permanent magnet materials in order to eliminate the need for slip rings in the rotor assembly. In a brushless DC motor it is usually most practical to provide a stator structure as shown in Fig. 6.3.1 where the windings are placed in an external, slotted stator. The rotor consists of the shaft and a hub assembly with a magnet structure. The picture shows a two-pole magnet. For contrast Fig. 6.3.2 shows the equivalent cross-sectional view of a conventional DC motor where the permanent magnets are situated in the stator structure and the


Fig. 6.3.1. Cut-away view of brushless DC motor assembly.


Fig. 6.3.2. Cut-away view of a conventional permanent-magnet DC motor assembly.
rotor carries the various winding coils. We can see that there are significant differences in winding and magnet locations. The conventional DC motor has the active conductors in the slots of the rotor structure, and in contrast, the brushless DC motor has the active conductors in slots in the outside stator. The removal of heat produced in the active windings is easier in the brushless DC motor, since the thermal path to the environment is shorter. Since the permanent magnet rotor does not contribute any heating, the result is that the brushless DC motor is a more stable mechanical device from a thermal point of view.

In spite of the advantages discussed above, there are cases where a brushless DC motor uses the configuration shown in Fig. 6.3.1, but then the roles of the two parts are reversed so that the permanent magnet outside structure rotates and the wound lamination part is the stator. The result of such a design is a high rotor moment of inertia, which sometimes is of interest to the user
who needs a high mechanical time constant in order to filter any existing torque ripple. In such cases the thermal advantage of the design in Fig. 6.3.2 is not utilized.

In order to illustrate the similarities and differences between the conventional and brushless DC motor systems, we can see sketches of the two in Fig. 6.3.3 and 6.3.4. In Fig. 6.3.3 we have the system elements of a conventional DC motor and control. The connections between the rotor windings and the commutator are shown in an oblique sketch for the sake of clarity. A bidirectional controller and driver stage is shown together with a power supply and servo compensation and interlock logic.

The equivalent brushless DC motor system is shown in Fig. 6.3.4, where the main differences are seen to be static windings, permanent magnet rotor, four transistors instead of the two found in the conventional motor, and a shaft position encoder. The latter generates logic signals which control the commutation of the windings. As we shall see, many different commutation configurations exist, and next we will examine the most commonly encountered models.

One of the simplest practical brushless DC motor circuits is shown in Fig. 6.3.5 (SEE NOTE IN FIGURE) This is a "half-wave" control circuit with a conduction angle of


Fig. 6.3.3. Essential parts of a conventional DC motor servo control.


Fig. 6.3.4. The essential parts of a brushless DC motor control.
$120^{\circ}$ el. As we see in the accompanying diagram each winding is used one third of the time and the logic control of the system is rather simple. The speed and torque output of the motor can be controlled by varying the power supply voltage $\mathbf{V}_{\mathbf{s}}$. In the lower part of the diagram we see the same system with a reversed torque. The torque reversal is achieved not by reversing the power supply voltage as in a conventional DC motor, but instead by shifting all logic functions $180^{\circ} \mathrm{el}$. This example illustrates one of the basic differences between brush type and brushless type DC motors.

The example just given ignores one very important point in brushless motor com-
mutation performance, namely the handling of the inductive transient current in each winding as it is commutated. The circuit shown in Fig. 6.3 .5 would experience a forward voltage breakdown of each transistor as its conduction would be discontinued. This is due to the voltage produced by the stored energy in each winding. Such breakdown conditions can be tolerated in low-power systems, where the stored energy is low. However, if any significant amounts of current and voltage are handled in such a system, a point is reached where breakdown conditions would cause damage to the semiconductor junctions. Therefore other methods are employed to assure proper commutation of the inductive energy in each winding.

NOTE: The diagram of Fig. 6.3.5 shows the "torque function" [ $T(I)$ ] of each winding. This function shows that winding's contribution of torque at various shaft angles when a constant current I is flowing in the winding. The unit value of torque over the conduction angle then becomes the torque constant $\mathrm{K}_{\mathrm{T}}$ of the winding.



Fig. 6.3.5. A three-phase, half-wave brushless motor controller.


Fig. 6.3.6. Controller configuration, current wave-form and logic sequence for a two-phase 4-pole brushless DC motor.

Instead of applying the remedy for the inductive energy problem to the circuit we have just reviewed, let us instead for the moment look at another type of circuit where the commutation transient concept can be easily visualized without having to worry about other problems which exist in the circuit of Fig. 6.3.5 (which we will discuss later).
(See Fig. 6.3.6 on preceding page 6-9)
If we turn to Fig. 6.3.6, which shows a two-phase brushless motor using two power supplies, $+\mathrm{V}_{\mathrm{s}}$ and $-\mathrm{V}_{\mathrm{s}}$, we note that we now have four power transistors and four diodes. Each half of the circuit controls its own winding, and the two are essentially independent from each other. The accompanying diagram shows phase currents and logic signals for one direction of operation. The current waveforms in the diagram are intended to by typical of real-time response of current at a given shaft velocity. The diagram, therefore, shows not only current response with respect to rotor position, but also current versus time at a given shaft velocity. If we, for instance, look at current $\mathrm{I}_{\mathrm{O} 1}$ we see that it has an exponential initial increase to a steady state value which is maintained until the $90^{\circ}$ position has been reached. Then $\mathbf{Q 1}$ is switch to the OFF condition. The stored energy is now dissipated through the power supply and returned through diode D2, and the exponential decline is shown in $\mathrm{I}_{\mathrm{D} 2}$, while the current rise now is progressing in Q3. Thus there is a continuous torque production maintained in the motor as one stage is turned off and the next is turned on.

The diodes provide an ideal transient path for the stored conducted energy. The energy is either stored in one of the filter capacitors or transferred to the other winding, depending on the logic sequence. If fast recovery diodes are used, the commutation event is carried out without any significant RFI emission. An oscilloscope picture of the current waveforms in a twopole full-wave brushless motor is shown in Fig. 6.3.7. The pictures were taken during continuous motor operation at a torque level of $80 \mathrm{oz}-\mathrm{in}$ and 3600 rpm .


Fig. 6.3.7. Current waveforms in the phase windings of a four-pole, two-phase, full-wave brushless motor. Motor running at 3600 rpm , delivering 80 oz -in. Power supply voltage $\pm 30 \mathrm{~V}$.

Fig 6.3.8 shows an extended version of the fwo-phase full-wave brushless motor system. The winding configuration is based on a "star" connection stator arrangement where each winding is oriented $120^{\circ}$ from the other. The six transistors are connected to the end points of each stator leg, and thus form a three-phase full-wave motor control. This system differs from the previous one in that conduction is always


Fig. 6.3.8. Controller configuration, torque function, and logic sequence for a three-phase, full-wave brushless DC motor.
continuous in one leg when the other is being commutated. We can see that when Q1 is energized between 0 and $60^{\circ}, \mathbf{0 5}$ is also conducting, and current is thus flowing from point $A$ to point $B$. In the next sequence ( $60-120^{\circ}$ ) 06 is energized and the current will then flow from point $A$ to point $C$. In the meantime the current through leg B declines to zero by conduction through D2. The conduction angle per phase is $120^{\circ}$, as compared to $90^{\circ}$ in the two-phase circuit. Since this circuit requires only a single power supply it tends to be more compact than the two-phase circuit; and further, since $67 \%$ of the available windings are used at any one time compared to $50 \%$ in the former circuit, the three-phase circuit is more efficient than the two-phase circuit of Fig. 6.3.6.

We can now return to the half-wave threephase circuit in Fig. 6.3.5, which was shown without any means for handling the stored inductive energy. If we were simply to provide a diode path for each winding, such as shown in Fig. 6.3.9a typical of relay coil circuits - we would


Fig. 6.3.9a. A conventional "fly-back" diode arrangement.
find that the inductive energy is well handled by the diode arrangement. But because the diode is located across the coil and not across the transistors as in Figures 6.3.6 and 6.3.8, we find that the back EMF will also flow in the diode circuit when the polarity of the back EMF is reversed $\left(180-360^{\circ}\right.$ in coil 1). In order to prevent current flow due to the back EMF during this "off" period of each cycle, one has to let diode conduction occur to a voltage source which has a voltage at least two times the power supply voltage. One arrangement accomplishing this is shown in Fig. 6.3.9b. As the conduc-


Fig. 6.3.9b. A biased "fly-back" diode arragement.
tion of $\mathbf{Q 1}$ ceases, the winding inductance will generate a voltage equal to $2 \mathrm{~V}_{\mathrm{s}}$, which will allow the inductive transient to be dissipated in the power supply circuit. But because the diode is biased, when the back EMF is reversed and could normally cause current flow, it cannot overcome the bias, and therefore no current flows. This type of circuit will thus allow for proper handling of the stored inductive energy, but at the expense of extra components and power dissipation in the bias supply.

The controller circuits we have seen so far represent the basic forms which may be encountered. Often due to special requirements, other variants of these circuits will be used. For instance in Fig. 6.3.10 we have a four-phase, half-wave controller.

This circuit features a $90^{\circ}$ conduction angle and is at times used where low torque ripple is important. While it would seem that the two-phase full-wave circuit in Fig. 6.3.6 basically fulfills the same purpose, the reason for using this circuit may



Fig. 6.3.10. Four-phase, half-wave circuit.
be that it gives similar performance with a single power supply and does not need cross-fire prevention interlocks. It can therefore be used in relatively low power circuits where cost is important and where
the fly-back current problem can be handled in a simple way.

Fig. 6.3.11 shows a three-phase full-wave circuit with a grounded neutral. This circuit


Fig. 6.3.11. Three-phase, full-wave circuit with grounded neutral.
offers some advantages when a constant current drive method is necessary. The symmetrical location of transistors with respect to ground offers advantages in the linear amplifier driving each transistor pair. We notice that at any given time two stages are conducting simultaneously although commutation occurs at alternate angular intervals. The significance of this is that the load current is shared between two transistors, which may affect the choice of their characteristics. On the other hand, due to the dual power supply, each transistor has to withstand the sum of the two supply voltages. This is in contrast with the three-phase circuit shown in Fig. 6.3.8, where two transistors conduct at any given time, but they are always in series. Therefore no load sharing occurs but the transistors see only one supply voltage instead. Returning to Fig. 6.3.11, we should be aware of another potential problem, namely that the dual supply type circuits (this includes the circuit in Fig. 6.3.6) are susceptible to power supply "pump-up" during certain operational conditions such as start and stop conditions driving high inertial loads, especially if pulse-width or pulse-frequency modulation
is utilized. It is also true that such pump-up situations may occur in the single supply type circuits, but they are somewhat easier to control in this respect.

If we now turn our attention to the output stage in Fig. 6.3.12, we see a six-phase fullwave circuit with grounded neutral. The timing diagram for this circuit is shown in Fig. 6.3.13, and we notice a $120^{\circ}$ conduction angle and that four transistors are conducting at any one time. We thus have excellent load sharing, and since commutation occurs every $30^{\circ}$, we have very low torque ripple conditions.

In addition, since the windings are divided into six independent coils, the electrical time constant is lower than in the equivalent three-phase circuit. Therefore, this type of motor performs excellently in ultra high performance requirements, rivaling that of low inertia moving coil motors. The price for this, of course, is that 12 power transistors are used in the output stage circuit. On the other hand, since there is load sharing between four transistors at any one time, the ratings for each transistor can


Fig. 6.3.12. Six-phase, full-wave commutation circuit with grounded neutral, and two power supplies.


Fig. 6.3.13. Torque function and logic sequence for unidirectional operation of six-phase, full-wave circuit of Fig. 6.3.12.
be held at a lower level than for the equivalent three-phase circuit.

Other variants of these circuits can be used, such as a pair of three-phase full-wave star or delta connected three-phase bridges, each displaced $30^{\circ}$ el from the other. The resultant system would then use a single power supply, but two independent commutation circuits.

The foregoing examples illustrate the large variety of circuit possibilities for commuta-
tion control of brushless DC motors. If we recall the statements at the beginning of this discussion citing the various application requirement which DC motors - and therefore brushless DC motors - encounter, we can realize that there is no single "good" circuit which can economically serve all purposes. Since the functional aspects of a brushless DC motor system are also intimately tied into the system cost, we have to keep in mind that an independent choice of the best motor and control has to take many factors into account.

### 6.4. TORQUE GENERATION BY VARIOUS CONTROLLER CONFIGURATIONS

We have now discussed some general concepts in the area of brushless motor commutation without specific regard for the control of torque and proper utilization of available options in the magnetic circuit. We can distinguish two principal control configurations which are used in today's brushless motors. The first method is based on the use of a sinusoidal torque generation scheme, and the second utilizes a trapezoidal torque generation principle.

### 6.4.1.THE SINUSOIDAL CONTROL SCHEME

This system uses a magnet and field coil arrangement which produces a torque function due to current in each of two coils which has a sine-cosine relationship as shown in Fig. 6.4.1. The torque contribu-


Fig. 6.4.1. Torque function of a sinusoidal type two-phase brushless DC motor.
tion by each coil is:

$$
\begin{array}{ll}
\text { Coil 1 } & \mathrm{T}_{1}=\mathrm{I}_{\mathrm{m} 1} \mathrm{~K}_{\mathrm{T}} \sin \theta \\
\text { Coil 2 } & \mathrm{T}_{2}=\mathrm{I}_{\mathrm{m} 2} \mathrm{~K}_{\mathrm{T}} \cos \theta \tag{6.4.2}
\end{array}
$$

where
$T_{1}$ and $T_{2}$ are instanta-
neous torque values
$\mathrm{K}_{\mathrm{T}}$ is the torque constant of each winding

The current in each coil is controlled as a function of shaft angle as follows:

$$
\begin{align*}
& I_{m 1}=I \sin \theta  \tag{6.4.3}\\
& I_{m 2}=I \cos \theta \tag{6.4.4}
\end{align*}
$$

with I being the amplitude of desired motor current. The total output torque is

$$
\begin{equation*}
T_{o}=T_{1}+T_{2} \tag{6.4.5}
\end{equation*}
$$

and combining (6.4.1) thru (6.4.4) we get

$$
T_{0}=K_{T} I\left(\sin ^{2} \theta+\cos ^{2} \theta\right)
$$

and since

$$
\sin ^{2} \theta+\cos ^{2} \theta=1
$$

we finally arrive at

$$
\begin{equation*}
T_{0}=K_{T} I \tag{6.4.7}
\end{equation*}
$$

and we have a motor which has essentially no torque ripple and has linear characteristics similar to a conventional DC motor. However, the generation of sinusoidal, amplitude controlled currents requires a complex sine-cosine generator (resolver)
either by analog or digital design, and a linear amplifier. In addition, certain compromises must be made in the motor design to achieve the sinusoidal torque relationship, which does not give optimum motor efficiency. Thus this method has limited use because of technical and economic reasons.

### 6.4.2.TRAPEZOIDAL TORQUE FUNCTION

The other control method is based on a trapezoidal torque function such as shown in Fig. 6.3.8 (see this illustration in the previous discussion). The principle in this case is to design the motor to provide a trapezoidal torque function and to utilize the unvarying portion of the torque function to produce useful torque. One has to choose a conduction angle and a number of coils such that the resultant torque is independent of shaft position and the motor output torque is then directly proportional to the input current. The torque function in Fig. 6.3.8 has a trapezoidal shape. Proper use of modern high coercive force magnets makes such torque functions possible. While the torque in some motors does not quite conform to this simple form, the relationship is sufficiently close to be useful for purposes of analysis. Thus, for example, if a controlled current $I$ is allowed to flow from $A$ to $B$ for $0-60^{\circ}$ and from $B$ to $A$ for $180-240^{\circ}$, and if other coils will similarly contribute torque at the intermediate angular intervals, we then conclude that the resultant torque is

$$
\begin{equation*}
\mathrm{T}_{\mathrm{o}}=\mathrm{K}_{\mathrm{T}} \mathrm{I} \tag{6.4.8}
\end{equation*}
$$

where $\mathrm{K}_{\mathbf{T}}$ is the value of torque per unit current at the flat portion of the trapezoid. Consideration has to be given to the rise and fall times of the current in the various coils, so that $T_{0}$ is essentially constant.

The two basic control principles discussed above are most commonly used in brushless motor systems today. The sinusoidal control scheme requires that both the magnetic circuit design and the control current will have sinusoidal shapes to fulfill the motor function. On the other hand, the trapezoidal torque function scheme requires that the magnetic flux is distributed such that the torque function is near constant over the desired conduction angle. Since the controller can commutate the motor windings with square wave control inputs, the control circuit can be of digital nature. This fact makes the control logic for the trapezoidal system rather simple. In addition, this type of circuit utilizes current only when it can be optimally effective. Therefore the trapezoidal control scheme tends to yield lower motor losses than the sinusoidal scheme. Hence, the trapezoidal commutation system is generally preferred for high performance, high power control systems.

### 6.5. COMMUTATION SENSOR SYSTEMS

Several methods are available today for the angular position sensing system. The most commonly used methods are Hall effect sensors, electro-optical sensors, and radio frequency (RF) sensors.

The Hall effect sensing system utilizes a sensor which detects the magnitude and polarity of a magnetic field. The signals are amplified and processed to form logic compatible signal levels. The sensors are usually mounted in the stator structure, where they sense the polarity and magnitude of the permanent magnet field in the air gap. The outputs of these sensors control the logic functions of the controller configuration to provide current to the proper coil in the stator, and the system can provide some compensation for armature reaction effects which are prominent in some motor designs. One drawback with such a location of the angular position sensor is that it is subject to stator temperature conditions, which may at times be rather severe in high performance applications. It is not unusual, for instance, to allow a winding temperature to reach 160-180 ${ }^{\circ} \mathrm{C}$ for peak load conditions. Such a temperature may affect the Hàl effect switching performance, and can therefore be a system performance limitation.

The Hall effect device can, of course, be located away from the immediate stator structure and may use a separate magnet for angular sensing. In such a case the sen-
sor is not necessarily subject to the severe operating conditions mentioned above; but on the other hand, neither does it compensate for armature reaction problems.

The second alternative for angular sensing is the electro-optical switch, most commonly a combination of a light emitting diode (LED) and a phototransistor. A shutter mechanism controls light transmission between the transmitter and the sensor. The sensor voltages can be processed to supply logic signals to the controller. The electro-optical system lends itself well to generation of precise angular encoding signals.

The third angle sensing method (radio frequency sensing) is based on inductive coupling between RF coils. Several varieties of such devices have been used with varying degrees of circuit complexity. The angular sensing accuracy of such devices depends on several design factors, which may limit their use in high performance systems. In addition, their switching time (which depends on oscillator frequency and sensor circuit damping) may cause switching time delays which can be undesirable at higher shaft speeds.

The three systems discussed have their own advantages and drawbacks, and the requirements of each application usually dictate which is best suited.

### 6.6. POWER CONTROL METHODS

So far we have discussed commutation control without reference to power control of the brushless DC motor. One method of power control is to vary the supply voltage to the commutation system. An example of this method is shown in Fig. 6.6.1. The six switching transistors will control commutation at the proper angular intervals, and the series-connected power transistor will handle velocity and current control of the brushless motor. This can be accomplished either by linear (Class A) control or by pulse-width or pulse-frequency modulation. If directional control is needed, the commutation sequence must be adjustable 0 or $180^{\circ}$ el. In effect, then, we have a series regulator controlling the power supply voltage for a switching stage commutation controller.

Another way of controlling voltage and current in the brushless motor is to let the commutation transistors control the motor current by either linear control means or by pulse-width or pulse-frequency modulation. Such a control method results in better utilization of available semiconductor devices, but proper attention must be paid to power dissipation in the controller stage. In the case of linear transistor control the control stage must be operated in a constant current mode rather than constant voltage mode, since otherwise the transistor stage held at a zero output voltage would tend to conduct back EMF induced currents during the inactive part of the cycle, causing a viscous damping
effect which is detrimental to motor


Fig. 6.6.1. Series regulator control of power for three-phase brushless DC motor.

The pulse-width or pulse-frequency control scheme is well suited for control of voltage and current to a brushless motor. Since logic circuitry is already in place capaiole of switching the appropriate transistors on and off, the implementation of such control is possible. The switching rate has, of course, to be compatible with a proper current form factor (see the discussion of pulse-width modulations and form factor in section 3.3), and also has to be within the switching capability of the power transistors so as not to cause undue dissipation losses during the transistor turn-off and turn-on times. An example of the variation in current form factor at two different pulse-width frequencies is shown in Fig. 6.6.2. Although the two oscilloscope pictures are taken at two different motor speeds, the improvement in current form factor is readily apparent. In addition, proper attention has to be focused on safety interlocks to prevent "cross-firing" between adjacent transistor stages. Such switching schemes are based on the principles discussed in Chapter 4, and many of the advantages (and problems) discussed
in this chapter apply equally well to the switching type commutation controller for brushless DC motors.


Fig. 6.6.2. Examples of poor current form factor (upper) and improved current form factor (lower) in a three-phase, full-wave brushless DC motor winding. Output torque $=2250 z-\mathrm{in}$.

Any high performance brushless DC motor will require some form of current limit control, either to protect the controller stage or to protect the magnetic circuit. With either of the controller schemes discussed above it is easy to apply such current limit control. However, the switching type controller can sustain current limit conditions without significant circuit power dissipation - as compared to a linear type of control system, where
the excess power is dissipated in the power transistors. The reasons given above point clearly toward pulse-width or pulse-frequency modulation as a superior control scheme for brushless DC motors where any significant amount of DC power is being controlled. Since the intention of this chapter on brushless motors is to familiarize the reader with the various parts of the brushless motor system, and not to be a design manual for brushless motors and controls, it would be beyond the scope of this chapter to go into detail of the variety of logic schemes, power control circuits, and current limit options which are available today.

Power control systems used in ElectroCraft brushless DC motors utilize pulsewidth modulation controls for all size motors except ultra-high performance, low inertia motors for incremental motion, where constant-current techniques are used. All systems are provided with current limit circuits, some of which work on a multi-level, time based current-limit principle. Thus the motor control systems can provide extremely high acceleration rates and yet have adequate safety provisions for moderate or long term overload.

### 6.7. MOTOR CONSTANTS

Since the same basic principles are used in both the conventional permanent magnet DC motor and the brushless DC motor, it would seem that the basic motor constants also would be the same. This is true if we use some caution in applying the constants.

A simplified electrical equation of a brushless DC motor is:

$$
\begin{equation*}
V=I R+L \frac{d I}{d t}+K_{E} \omega \tag{6.7.1}
\end{equation*}
$$

where

I is the sum of the phase currents.
$\mathbf{R}$ is the resistance of a phase winding
$L$ is the inductance of a phase winding
$K_{E}$ is the voltage constant of a phase winding over the conduction angle
$\omega$ is the angular velocity of the motor shaft

These relationships hold well for the common brushless motor structures, although there are lower order effects due to mutual
inductance between windings, overlapping conduction angles and unequal rise and fall times of current (due to differing charge and discharge paths). For practical applications eq. (6.7.1) is adequate, however.

The dynamic equation for a motor coupled to a load is:

$$
\begin{equation*}
K_{T} l=\left(J_{m}+J_{L}\right) \frac{d \omega}{d t}+D+T_{f}+T_{L} \tag{6.7.2}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{K}_{\mathbf{T}}= & \text { torque constant of motor } \\
& \text { winding }
\end{aligned}
$$

$J_{\mathrm{m}}=$ motor moment of inertia
$J_{\mathrm{L}}=$ load moment of inertia
D = viscous damping coefficient
$\mathrm{T}_{\mathrm{f}}=$ motor friction torque
$T_{L}=$ load friction torque
In a brushless DC motor $\mathrm{T}_{\mathrm{f}}$ is small, usually only due to bearing drag, the viscous damping coefficient is also very small, and both items can usually be ignored in dynamic performance calculations. (For derivation of the motor transfer function see section 2.3.3.)

### 6.8. BRUSHLESS DC TACHOMETERS

DC analog output tachometers are often necessary in high performance servo applications, where they provide velocity feedback for speed control purposes or servo system stability. The requirements of a brushless DC tachometer are the same as for a brush-type DC tachometer: an output voltage proportional to shaft velocity and the polarity corresponding to the direction of the shaft rotation. The quality of the tachometer depends on the stability of the voltage constant and the magnitude and frequency of voltage ripple.

Digital-type tachometers can be used successfully where the controlled motor speed range is in a region where the pulse rate is high enough to provide a sufficient servo bandwidth after the required filtering following D/A conversion. However, when servo control is required over a wide range of speeds, down to a stop position, then rate information is required over the entire range; therefore the digital tachometer is often not sufficient for servo system operation.

The Electro-Craft brushless DC tachometer is based on a permanent magnet rotor and a multi-coil stator structure, which is commutated by a MSI (medium scale integra-
tion) circuit. The output voltage of the system behaves exactly as the brush-type DC tachometer. The ripple voltage depends on tachometer design, but can be controlled to a level better than 1\%. Depending on the rotor moment of inertia and the temperature stability required, ceramic, alnico or rare-earth materials are used to provide the flux for the tachometer. The block diagram of the system is shown in Fig. 6.8.1. When the brushless DC tachometer is mounted directly on the motor shaft, the motor shaft encoder output can also be used for tachometer commutation. Since the motor and tachometer rotors are both located on the same shaft, a very stiff mechanical structure is realized. Thus the unavoidable torsional resonances can be kept at very high frequency levels, where they do not influence servo system behavior.


Fig. 6.8.1. Brushless DC tachometer system.

### 6.9. EXAMPLES OF BRUSHLESS MOTORS

In Fig. 6.9.1 we see a picture of a high performance incremental motion brushless DC motor, Electro-Craft Corporation Model 17. This motor was designed to be light in weight in addition to having low armature moment of inertia. The rotor configuration includes a four-pole magnet structure using rare-earth magnet materials. The performance parameters are shown in Table 6.9.1. Shown are two alternative connections, 3 -phase and 6 -phase full-wave systems. Slightly better characteristics are achieved by the 6 -phase connection, because of better winding utilization.

The system performance is shown in Fig. 6.9.2, where we see an example of the output current of one "constant current" stage during operation in a velocity mode with 75 oz -in torque at 500 rpm .

We see one complete commutation cycle of one of the six stages. The current is kept constant during the positive and negative commutation cycles, each $120^{\circ}$. In this system four phases are always condücting. Since each phase contributes one-fourth of the torque, and the current per phase is 3.6 A, we can calculate the torque constant:

$$
\mathrm{K}_{\mathrm{T}}=\frac{75}{4 \times 3.6}=5.2 \mathrm{oz}-\mathrm{in} / \mathrm{A}
$$

In order to demonstrate incremental motion performance, the servo motor was connected to a load with $\mathrm{J}_{\mathrm{L}}=0.5 \times 10^{-3}$


Fig. 6.9.1. Electro-Craft Corporation Brushless DC Motor Model 17.
oz-in-s ${ }^{2}$, and the system was compensated for the added inertia. The amplifier was adjusted for a peak current limit of 3.8 A per phase (this is the continuous current rating of the motor, with 5 cfm of air cooling). The measured system servo bandwidth with the load was 270 Hz .

Fig. 6.9.3 shows a torque-speed characteristic of the motor-tachometer in a closedloop velocity mode. The current limit causes the fall-off of speed as the torque requirements exceed $70 \mathrm{oz}-\mathrm{in}$, and at


Fig. 6.9.2. Current in one phase of six-phase low inertia servo motor in a "constant current" drive mode. Constant velocity of 500 rpm at $\mathrm{T}=75 \mathrm{oz}-\mathrm{in}$.

TABLE 6.9.1. Preliminary Motor Characteristics of Model 17 Brushless
DC Motor

| Parameter | Unit | Connection |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 3 phase | 6 phase |  |
|  |  |  | -1 | -2 |
| $K_{T}$ (per phase) | oz-in/A | 9.6 | 5.3 | 10.6 |
| $K_{E}$ (per phase) | V/krpm | 7.5 | 4.1 | 8.2 |
| $R$ (per phase) | $\Omega$ | 5.4 | 2.7 | 10.8 |
| $\tau_{\mathrm{e}}$ (per phase) | $\mu \mathrm{s}$ | 700 | 350 | 350 |
| $\mathrm{J}_{\mathrm{a}}$ | oz-in-s ${ }^{2}$ | . $65 \times 10^{-3}$ | $.65 \times 10^{-3}$ | . $65 \times 10^{-3}$ |
| T cont (free air) | oz-in | 30 | 30 | 35 |
| T cont (12' $\times 12^{\prime \prime}$ heat sink) | oz-in | 45 | 45 | 50 |
| T cont (5 cfm@ $0.2 \mathrm{in} \mathrm{H}_{2} \mathrm{O}$ ) | oz-in | 60 | 60 | 70 |
| T max | oz-in | 120 | 120 | 140 |
| T max $/ J_{a}$ | $\mathrm{rad} / \mathrm{s}^{2}$ | 185,000 | 210,000 | 210,000 |
| Max. winding temp. $=160^{\circ} \mathrm{C}$ |  |  |  |  |
| Motor dimensions: <br> Diameter 2.75 in Length 3.0 in <br> Weight 16 oz |  |  |  |  |

speeds above 3000 rpm the effects of power supply voltage limits are evident.

In order to assess the worst-case output torque ripple characteristics, a measurement of torque output of a stall motor was made under various constant current
conditions. The torque was measured by a strain gage torque arm, as the motor housing was slowly rotated in a mechanical "dividing head". An X-Y recorder plotted the varying torque as one complete electrical revolution was made (Fig. 6.9.4). The torque ripple under these conditions was


Fig. 6.9.3. Torque-speed characteristics of a brushless motor-tachometer in a closed-loop velocity mode.
measured to be less than $\pm 5 \%$ of average output torque. Since this motor was designed as a low inertia motor, it does not necessarily represent the best attainable torque ripple performance in brushless DC motors. It may reflect the trade-off between low rotor moment of inertia and torque ripple.
A demonstration of incremental motion performance characteristics of Model 17 is displayed in Fig. 6.9.5, where the system is excited with a larger command signal. Thus
we see the motor velocity switched from +5850 rpm to -5850 rpm . The linear portion of the velocity trace shows the acceleration, in this case limited by the current limit of 3.8 A per phase. If we measure the slope, we obtain

$$
\frac{\mathrm{d} \omega}{\mathrm{dt}}=\frac{1220 \mathrm{rad} / \mathrm{s}}{19 \times 10^{-3} \mathrm{~s}}=64 \mathrm{krad} / \mathrm{s}^{2}
$$

which is the acceleration rate at the set current limit of 3.8 A and a load moment of inertia of

$$
J_{L}=0.5 \times 10^{-3} \text { oz-in-s }{ }^{2}
$$



Fig. 6.9.4. Static torque ripple test of Mod. 17, six-phase brushless DC motor and constant-current type controller.


Fig. 6.9.5. Constant acceleration at a current of 3.8 A , and a load moment of inertia of $\mathrm{J}_{\mathrm{L}}=.5 \times 10^{-3} 0 \mathrm{oz-in-s}^{2}$.

A cross-sectional view of a brushless DC motor-tachometer is shown in Fig. 6.9.6. The motor rotor, tachometer rotor and the shaft encoder shutter are all mounted on the same shaft, which contributes to a high torsional stiffness between all elements. A special brushless DC motor designed for
constant speed applications is shown in Fig. 6.9.7. The motor is intended for use in disc memory systems, where it drives the discs at a constant speed of 3600 rpm and a torque of $50 \mathrm{oz}-\mathrm{in}$. The run-out of the shaft end is held to a tolerance of less than 50 millionths of an inch.

An example of a two-phase, full-wave controller for a 0.25 hp brushless DC motor is shown in Fig. 6.9.8. The circuit board contains analog velocity loop summing amplifier, pulse-width modulation circuit, commutation logic, driver circuits and power output stage. DC power supply is not included.

Fig.6.9.9 shows an example of a brushless DC motor, displayed with two optional stator assemblies of different stack lengths
and a typical ceramic magnet rotor assembly. The commutation encoder is located in the rear portion of the housing assembly.


Fig. 6.9.6. Brushless DC motor-tachometer.


Fig. 6.9.7. Brushless DC motor disc memory spindle drive package.


Fig. 6.9.8. Complete two-phase, full-wave ampli-fier-controller for a $1 / 4 \mathrm{HP}$ brushless DC servo motor.


Fig. 6.9.9. Brushless DC motor housing together with two different stator assemblies and a ceramic magnet rotor.

### 6.10. SUMMARY

Brushless DC motors can be used for a variety of applications. The advantages of state-of-the-art magnet materials have made possible brushless DC motor designs which have very high torque-to-inertia ratios. Since commutation is performed in circuit elements external to the rotating parts, there are no items which would suffer from mechanical wear, except the motor bearings. The brushless DC motor will, therefore, have a life expectancy limited only by mechanical bearing wear, and the reliability of the electronic controller.

The brushless DC motor can be controlled with very efficient amplifier configurations. In cases of severe environmental conditions the controller can be located remotely from the motor. The control system can easily interface with digital and analog inputs, and is therefore well suited for incremental motion (see the following discussion 6.11) and for phase-locked speed control systems. The motor and control have a lower level of radio frequency emission than the conventional DC motors and controls.

The brushless DC motor controller has, in general, a more complex configuration than the controller for an equivalent conventional DC motor, but may be similar in size and complexity to a closed-loop step motor controller. The commutation sensor system can in some cases provide some incremental information of shaft angle for incremental motion applications, but usually an en-
coder system is added to the commutation system to suit the application.

Table 6.10.1 shows a brief comparison of the conventional DC motor and the brushless DC motor. Due to the many variants available of the two kinds, the comparison has to be very general. However, it may give more insight into the basic charaacteristics of the two.

## TABLE 6.10.1.

A brief comparison between conventional DC motors and brushless DC motors

## CONVENTIONAL DC MOTORS

GOOD POINTS:

1. Controllability over a wide range of speeds.
2. Capable of rapid acceleration and deceleration.
3. Convenient control of shaft speed and position by servo amplifiers.

PROBLEM AREA:

1. Commutation (brushes) causes wear and electrical noise.
This problem can be kept under control by selection of best materials for each application.

## BRUSHLESS DC MOTORS

GOOD POINTS:

1. Controllability over a wide range of speeds.
2. Capable of rapid acceleration and deceleration.
3. Convenient control of shaft speed and position.
4. No mechanical wear problem due to commutation.
5. Better heat dissipation arrangement.

## PROBLEM AREA:

1. Requires more semiconductor devices than the brush type DC motor for equal power rating and control range.

### 6.11.STEP MOTORS VERSUS BRUSHLESS DC MOTORS IN DIGITAL POSITION SYSTEMS

### 6.11.1. INTRODUCTION

Step motors are, in their proper market, very useful and economical. The designer of a system only has to send a given number of pulses to a controller and the motor will move an appropriate number of steps and then stop. This works well as long as the load moment of inertia and friction levels are within design limits, and the step rate is within the capability of the motor and controller. The brushless DC motor has, by our earlier definition, no inherent capability of moving in discrete steps. However, with the addition of very few parts the brushless DC motor can perform better than step motors in incremental motion systems.

### 6.11.2. STEP MOTOR PERFORMANCE LIMITATIONS

With the ever-increasing technological demands on incremental motion devices, step motor systems are continually called upon to provide higher step rates and slew speeds, and to handle larger inertial loads. While the step motor industry is continually improving their products to handle increased demands, the problems inherent in the motor design principles come to the foreground as step rate demands increase. Following is a list of the major problems with currently available step motor systems:

1) Settling time. The open-loop step motor controller cannot cope well with the damped oscillatory settling behavior of the step motor shaft into a final position.
2) Lack of availability of a variety of step angles. Step motors have fixed step angles, which may not always fit given applications. A change in step angle involves re-tooling the rotor and stator laminations, a very expensive procedure.
3) Slew speed problems. Step motors are limited in high speed slew rates and have "forbidden zones" of operation, where the system can lose synchronism.
4) "Jerky" speed. In some applications the step motor's inherent reluctance cogging causes speed modulations which are inconsistent with acceptable performance.
5) Inability to handle large inertial loads. Because of the pulse excitation control, large inertial loads present difficulties for step motors.

The most common method of solving some of these problems in high performance systems is to provide an incremental shaft angle encoder to aid in the step switching, and the result is improved step rate and settling time. The user then has the following equipment: a power supply, a digital logic controller, a multi-phase switching controller, an ecoder and a motor. The parts are shown in block diagram form in Fig. 6.11.1.

### 6.11.3. BRUSHLESS DC MOTOR INCREMENTAL MOTION CONTROL

In order to get high performance "step motor" action with controllable damping


Fig. 6.11.1. High performance step motor system.
and with slew rates limited only by mechanical strength of the motor and counting rate limitations of the logic circuit, we can demonstrate the system as shown in Fig. 6.11.2. Included in the components, in addition to the parts shown in Fig. 6.11.1 (the closed-loop step motor system) are the commutation encoder, the DC brushless tachometer and the DC servo compensation network. The latter part is adjustable to accommodate varying load moments of inertia and other factors affecting system stability. There are, of course, other differences in signal processing and logic, but the purpose of this discussion is to demonstrate, in a general way, how this is accomplished. This system
has performance parameters exceeding the closed-loop step motor system in Fig. 6.11.1. It is more versatile in that by choice of proper position encoder one can have any desired number of steps per revolution. In addition, the system will have the high torque-to-inertia ratio of a DC motor, will behave as a DC motor at high speeds and therefore will have higher operational range than the equivalent step motor.

In Table 6.11.1 we see a brief summary of the main features of each system. A photograph of a system demonstrator is shown


Fig. 6.11.2. High performance BLM "electronic" step motor.

Step motors used in their "natural" operational range are well suited for step applications. Whenever high performance step
requirements are encountered it is worthwhile to investigate the advantages of brushless "electronic step" DC motor systems.

## TABLE 6.11.1

## BRIEF COMPARISON OF CHARACTERISTICS OF CLOSED-LOOP STEP MOTORS AND BRUSHLESS MOTORS FOR INCREMENTAL MOTION

## STEP MOTOR

Has an inherent number of discrete shaft stop positions.

Requires a high number of winding switching sequences.

Winding inductance is a dominant factor which may be limiting high speed performance.

Has non-linear torque-speed characteristics.

Requires incremental shaft angle encoder.

The number of encoder lines is dependent on motor step angle.

The incremental encoder and circuitry provide improved position damping over the open-loop motor system.

Position stiffness dependent on magnetic detent force and step angle.

BRUSHLESS DC MOTOR

Objective is to provide shaft rotation without any preferred positions due to motor construction.

Requires a low number of winding switching sequences.

Due to a lower number of switching sequences the winding inductance is a lesser problem at high shaft speeds.

Has linear torque-speed characteristics.

Requires incremental shaft angle encoder.

The number of encoder lines is independent of motor parameters.

Has optimum position damping response if analog tachometer is used.

Position stiffness dependent on encoder line-to-line angle, motor torque.


Fig. 6.11.3. Brushless DC motor and controller for demonstration of "electronic step motor" performance.

## Chapter 7

## Optical Encoders

### 7.1. INTRODUCTION

As referred to in this discussion, an encoder is an electromechanical device used to monitor the motion or position of an operating mechanism, and to translate that information into a useful output. The output can take the form of a simple system status indication, or it can provide feedback control information or in other ways interface with related devices.

There are several position sensing techniques available to the system designer. The most widely used types of sensors fall into one of the following types: capacitive, magnetic, contact or optical. This discussion is confined to the latter type: linear and rotary optical encoders. Since optical encoders offer significant improvements over earlier and less sophisticated methods of position sensing, it is hoped that the reader will find this information to be useful in applying optical encoding techniques to new equipment designs, as well as to the upgrading of existing equipment.

### 7.2 OPTICAL ENCODER COMPONENTS

Typically, an optical encoder consists of four components: (1) a light source, which can be an incandescent lamp or lightemitting diode (LED), depending on design considerations; (2) a pattern of alternating opaque and translucent segments appearing
on an assembly (disc or scale) that stands between the light source and its associated light sensor, and which usually tracks the movement of the device being monitored; (3) the light sensor, often a phototransistor, but possibly a photovoltaic component (solar cell); and (4) conditioning circuitry such as may be required to convert sensor output to properly formatted information for the desired readout or interface. The interrelationship of these components is shown in Figure 7.2.1, which is a functional sketch of the elements of a simple rotary optical encoder.


Figure 7.2.1. Components of an Optical Encoder
For increased resolution, the light source is collimated, and an additional element the mask - is added between the disc and sensor. In such applications, the mask and disc produce a shuttering effect so that only when the translucent segments of
both are in alignment light is permitted to pass through to the sensor. This configuration is shown, also using a rotary encoder example, in Figure 7.2.2.


Figure 7.2.2. High Resolution Optical Encoder

### 7.3. TYPES OF OPTICAL ENCODERS

Two basic types of optical encoders are described here: linear and rotary. The distinction is simple: with the linear encoder, direct digital information can be obtained regarding the position and/or velocity of an arm moving along a linear axis. Rotary encoders are those designed to sense the movement and/or position of devices which rotate about an axis. By far the more common of the two, rotary encoders are either incremental or absolute in encoding function, as described below; and are available both as self-contained discrete components and as modules for integration into related end products.

### 7.3.1. INCREMENTAL ROTARY ENCODERS

Incremental encoders provide efficient and economical means for obtaining position or velocity data in many applications. This
results from the inherent simplicity of the incremental encoder, as compared to the absolute. With only one or two output channels, the incremental has fewer components and less complex code discs. Hence, the cost of a standard incremental encoder is typically 60 percent less than that of an absolute with equivalent capabilities.

Functionally, the incremental encoder generates a serial pulsetrain output by reading the number of increments traversed on the code disc track as the encoder shaft rotates. In order to know the exact amount of travel, it is necessary to count the output pulses with an external digital counter. The counter then maintains a memory of the position information for the encoder.

Should a power failure occur, the counter can reset and lose track of the position. An index pulse, once per revolution, can be used to recalibrate the counter after a power failure. For a fail-safe system, a battery backup system should be provided for the counter and encoder; or else an absolute encoder should be used. The determining factor on whether to use an absolute versus an incremental encoder lies in the cost trade-offs of the system.

Incremental encoders are used exclusively where velocity is the desired parameter. Velocity sensing functions include tachometers and velocity-feedback systems, as utilized in the tape transport industry. In these cases, a modular rotary encoder is normally mounted directly on the motor shaft. Incremental encoders are also used
widely in many position-determining applications. A typical position-sensing application is provided by the machine tool industry, where the position of the tool with respect to the work piece must be constantly observed. The rotary encoder gives a number of cycles of output for each inch of tool travel. A typical 2500 cycle encoder with $4 X$ mulitiplication provides measurement tolerances in the order of .0001 inch per inch of travel.

There is a variety of standard incremental encoder configurations for applications requiring resolutions up to 5000 cycles per revolution. Standard encoders can be provided with internal electronics for interfacing with any logic type (CMOS, TTL, DTL, etc.). Certain configuration sizes also permit the inclusion of multiplication and direction-sensing electronics. Additionally, incremental encoders can be provided with either a single channel output or with any of the following output combinations:
a. Dual output from a single count, with each output phased by 90 degrees (phase quadrature).
b. Single output plus once per revolution pulse (zero reference or index).
c. Dual output plus index.
d. All of the above options taken from two different tracks. . .as with the English/Metric encoder, which provides outputs both in English and Metric (e.g. 1000/2540), either individually or simultaneously.

### 7.3.2. ABSOLUTE ROTARY ENCODERS


#### Abstract

Absolute or "whole word" encoders generate a unique digital parallel output for every shaft position. This absolute positional information is normally expressed in code formats such as binary, BCD, Gray, XS3, XS14, etc. Each bit in the digital word represents an independent track on the encoder disc, and an independent output channel with its associated electronics. Special codes for various purposes are routinely created.


Since it is not uncommon for an absolute encoder to have 13 or more output channels, as compared with the normal one or two in the incremental encoder, the higher cost of the absolute is apparent. For many applications, however, this higher cost is clearly justified. The positional information is read from the disc in an absolute encoder and from an external counter in an incremental encoder. Therefore, the information read from an absolute encoder is unaffected by power failure or noise.

Absolute encoders are normally used for generating positional feedback information. One example of usage is for monitoring the distribtuion of water through various canals and reservoirs in water conservation programs. Depsite higher cost, absolute encoders are warranted for such applications, not only because of their inherent fail-safe features, but also because they afford a savings in power consumption. Absolute encoders need only be powered when information is required.

### 7.3.3. LINEAR ENCODERS

With a linear encoder you can discern the position or velocity of an arm moving along a linear axis. The encoder eliminates the need for a mechanical conversion, and thereby allows resolutions and accuracies that are unobtainable with a rack and pinion in the system.

Linear encoders operate on the same basic principle as the rotary encoders: the "transmissive optical" principle. A glass scale with light and dark increments is affixed to the arm in motion; the scale moves within a yoke-shaped read head that contains a source of illumination on one side of the scale and a sensing assembly on the other. Light passing through the scale is sensed by photodetectors, which generate the signal output.

The arm in motion can have any conceivable shape, configuration, length, depending upon the function being performed. There are as many different mechanical configurations of linear encoders as there are applications.

Linear scales can be designed for absolute as well as incremental encoding functions, in lengths ranging from less than an inch up to several feet. Resolution in linear encoders is expressed by the displacement of lines per inch, where one "line" equals an opaque segment on the scale and its adjacent clear segment. Scales have been designed for precision inspection equipment with resolutions of 10,000 lines per inch that are accurate to 50 millionths of an inch. A summary of the most common applications for linear encoders is given in Table 7.3.1 below. Figures 7.3.1 through 7.3.4 depict some of these applications.

TABLE 7.3.1. LINEAR ENCODER APPLICATIONS

| APPLICATION | USAGE | TYPICAL RESOLUTION AND ACCURACY |
| :---: | :---: | :---: |
| Disc Memory System | Position feedback information for locating the read heads over the respective tracks on the memory disc. | Resolution: 100-200 lines per inch. Accuracy: 50 to 100 millionths. |
| Machine Tools | Indicates the position of the tool with respect to the work piece. | Resolution: 500-2500 lines per inch. Accuracy: 100 to 500 millionths. |
| Coordinatographs and Plotters | Indicates postion and bypasses the lead screw. | Resolution: 10,000 lines per inch. Accuracy: 50 to 100 millionths. |
| Inspection Equipment | Precision measurement with visual display as applied to height gauges, comparators, etc. | Resolution: 10,000 lines per inch. Accuracy: 50 to 100 millionths. |



Figure 7.3.1. Disc memory type of linear head offers prealigned features, and provides 100 or 200 tracks per inch, and indications of BOT (beginning of travel) and EOT (end of travel).


Figure 7.3.3. Another disc memory application shows a linear head and scale to be aligned by the user when the unit is integrated into an end product. This encoder offers 400 tracks per inch, plus BOT and EOT.


Figure 7.3.2. This example of a linear head used for disc memory applications is similar to the head shown in Figure 7.3.1, offering 200 tracks per inch.


Figure 7.3.4. The head and scale used on coordinatographs provide 1000 lines per inch. Such units can be butted together, thereby providing readout over several feet.

### 7.3.4. MODULAR ENCODERS

Modular encoders are offered to enable a manufacturer to build an encoder into his product, rather than couple an external encoder onto the shaft. Modular encoders are provided as a sub-assembly, with two basic elements: 1) code disc and hub, 2) housing, sensor, light source and electronics. The customer then assembles and integrates these elements directly onto the shaft within the casement of his end product. The modular elements are prealigned in order to minimize the time and expense required of the customer for the final integration.

Applications which lend themselves best to the modular concept generally fall into three basic categories: (1) where coupling one shaft to another might introduce torsional resonance, create errors or in other ways detract from system performance; (2) where lowest possible cost is essential; and (3) where encoders are an integral part of the motor housing or case.

Most modular encoders are incremental, but are also available as absolute.

### 7.4. SPECIAL DESIGNS \& OPTIONS

Encoders for unique applications fit basically into three categories of design: 1) altitude reporting encoders, 2) multiturn encoders, 3) high-resolution encoders.

### 7.4.1. ALTITUDE REPORTING ENCODERS

These encoders are designed to be incorporated into the final assembly of altim-
eters. They derive motion through gears which are driven by the aneroid or by the servo. As the disc rotates, the differential pressure resulting from variations in altitude is converted into a digital output. The code is ICAO, which relates directly to altitude in increments of 100 feet. Encoders are supplied to the altimeter manufacturer as a modular subassembly. Figure 7.4.1 shows an example of this type.


Figure 7.4.1. An Altitude Reporting Encoder Disc

### 7.4.2. MULTITURN ENCODERS

Where high-bit capacity is required for defining long distances of motion, multiturn encoders make use of two or more discs mounted on a gear train. The gear ratio between the input and output discs dictates the total number of revolutions of the input shaft required to generate the full capacity output. For example, assume the input disc is a 100 count unit, and that the output disc is a 1000 count unit; also, the reduction gear ratio between input and output is 1000 to 1 . The reduction ratio causes the output disc to advance one count every time the input shaft makes one complete revolution (100 counts). Therefore, 1000 revolutions of the input
are required for one revolution of the output. The total capacity is thus the product of both discs, which equals 100,000. By changing the disc resolutions and the gear ratios, multiturn encoders can provide a variety of count ranges.

### 7.4.3. HIGH RESOLUTION ENCODERS

For special requirements, high resolution encoders are available, such as Size 40 (four inch diameter) with resolutions up to 35999 in 8-4-2-1 BCD. This encoder can be modified to provide 16 bits in natural binary or Gray code.

### 7.5. ENCODER DISCS

The disc determines the resolution and accuracy of the encoder. The purpose of the disc is to operate as a rotating shutter, alternately blocking and transmitting light from its source to a light sensor. The disc is designed with an arrangement of opaque and transparent segments, and may be manufactured with any material that lends itself to this criterion. Figure 7.5.1 presents some examples of discs.


Figure 7.5.1. Typical Encoder Discs

Discs are made from plastic, mylar, glass or metal. The superior properties of glass, in the areas of stability, rigidity, hardness and flatness, afford it as the most practical material for disc production, although the other materials have a higher shock and vibration resistance than glass. The opaque segments can be photo emulsion or metalized. The metalized glass has a harder surface than photo emulsion, and is more resistant to harsh environments.

Figure 7.5.2 illustrates two code discs: incremental and an absolute. Note that the incremental code (at left in the figure) is a single track. Provisions must be made to count the number of transitions to determine the position or velocity information. The absolute code represents a 4 bit natural binary. The 4 bits permit 16 discreet positions. Each position has a unique binary word; i.e., position 7 is represented by the word 0111.


Figure 7.5.2. Incremental (left) and absolute code discs.

### 7.6. DIGITAL CODES

With the large number of applications requiring the interfacing of mechanical devices and electronic controllers, the need for converting mechanical position into an electronic signal is increasingly being solved by optical encoders. Depending on the applications, various methods of describing or coding an analog value into a digital pattern have been devised.

An absolute code makes use of one track on the disc for each bit; therefore, a 4 bit encoder in natural binary has 4 tracks, an 8 bit encoder disc has 8 tracks, and so on. The track having the greatest number of opaque and translucent signals is considered to be the LSD - least significant digit. The track having the least number of signals is known as the MSD - most significant digit. The tracks provide information which is weighted in an increasing importance from the LSD to the MSD. In the natural binary code pattern, when changing from the decimal number 7 to number 8 and from number 15 to number 0 , all of the bits change value from logic 1 to logic 0 (or vice versa).

The least number of channels or bits are required with the natural binary code. Each bit represents a power of 2 , such as $1,2,4$, 8,16 , etc. The analog value is represented by binary bits: " 1 " representing a transparent segment and " 0 " representing the opaque segment. When the values of each bit are added together, the value of the analog number is obtained. For example, as
shown below, the analog number " 178 " is represented by:

| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | (bit value) <br> (state of |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |  |
| each bit) |  |  |  |  |  |  |  |  |

Since values or weights can be assigned to each binary digit or "bit", and then added to convert the analog value they represent, the natural binary code is a weighted code. And, since several bits may change state at a transition between two numbers, the natural binary code is also a polystrophic code. Some example analog numbers are shown here with their binary equivalents. $1=1,2=10,3=11,4=100,5=101$, $6=110,7=111,8=1000$ and, as above, $178=10110010$.

Polystrophic codes are so known because two or more bits may change at a transition between adjacent words. Figure 8.6.1 shows a natural binary count of 16 code (zero through 15), and could be read with four in-line sensors. Due to mechanical tolerances, there will be a finite timing error, or ambiguity, among the four sensors. Techniques to compensate for this ambiguity include " $V$ " scan and " U " scan sensor arrangements. In Figure 7.6.1 the sensors form a " $U$ " shape; hence the term $U$ scan.


Figure 7.6.1. U Scan Arrangement

Figure 7.6.2 represents the wave form from the $U$ scan sensors. Note the advanced and retarded wave forms of the two sensors for track 2. When these two wave forms are gated by the least significant track, a wave form is generated whose transitions have zero time errors. The logic gating to generate this decoded track can be expressed at $(\bar{T} \cdot 2 A+1 \cdot 2 R)=$ track 2 decoded.

WAVEFORMS


Figure 7.6.2. U Scan Sensor Waveform
The same logic expression is used to decode tracks 4 and 8 . For encoders with higher resolution, the sensors may be arranged in a "V" pattern. With this V Scan technique each decoded track is used to decode the next significant track. The V Scan technique simplifies mechanical alignment of the sensors.

A conversion of digital to decimal form can be simpler if a binary number is used to represent each decimal digit. This is known as Binary Coded Decimal system (BCD) or the 8421 code. The digits $0-9$ are natural binary numbers. However, ten is represented as 0001-0000 rather than 1010 as above. Thus, " 178 " is represented as

0001-0111-1000 in BCD format. More bits or tracks are required if such a code is used to encode a shaft position; however, the output is in BCD.

Gray code, being monstrophic, has the advantage of requiring only a single bit to change from any one word to adjacent word. The conversion from Gray Code to natural Binary code is fairly simple, requiring one exclusive-or gate. Decimal, Gray Code and Binary equivalents are shown below:

| DECIMAL | GRAY CODE | BINARY |
| :---: | :---: | :---: |
| 0 | 0000 | 0000 |
| 1 | 0001 | 0001 |
| 2 | 0011 | 0010 |
| 3 | 0010 | 0011 |
| 4 | 0110 | 0100 |
| 5 | 0111 | 0101 |
| 6 | 0101 | 0110 |
| 7 | 0100 | 0111 |
| 8 | 1100 | 1000 |
| 9 | 1101 | 1001 |
| 10 | 1111 | 1010 |
| 11 | 1110 | 1011 |

Many trade offs must be made in deciding which of the codes (monostrophic or polostrophic) should be directly read from the code disc. In most cases, a monostrophic code pattern on the disc will be used and convert through digital Igoic to a polostrophic code if deemed necessary for the application.

The following codes are commonly used in building encoders.
MONOSTROPHIC $\quad$ POLOSTROPHIC

- Gray Code Natural Binary
- ICAO (altitude $\quad \bullet 8421$ BCD
report code)
- XS-BCD (i.e. XS3,
XS3/XS14, etc.)
- Biquinary (2 out of
5 code - Johnson code)

The Excess 3 (XS3) Code is sometimes used when simple arithmetic circuits are used to perform subtraction by the 9 's compliment. If all bits are complimented or inverted in the Excess 3 Code, the 9 's compliment is easily formed.

## DECIMAL/EXCESS 3

| 0 | 0011 | 5 | 1000 |
| :--- | :--- | :--- | :--- |
| 1 | 0100 | 6 | 1001 |
| 2 | 0101 | 7 | 1010 |
| 3 | 0110 | 8 | 1011 |
| 4 | 0111 | 9 | 1100 |

The Excess 3 Code can also be represented by a monostrophic code.

| GRAY CODE |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| EXCESS 3 BCD |  |  |  |  |  |
| TENS | UNITS |  | TENS | UNITS |  |
| DECIMAL | DCBA | DCBA | DECIMAL | DCBA | DCBA |
| 0 | 0010 | 0010 | 5 | 0010 | 1100 |
| 1 | 0010 | 0110 | 6 | 0010 | 1101 |
| 2 | 0010 | 0111 | 7 | 0010 | 11.11 |
| 3 | 0010 | 0101 | 8 | 0010 | 1110 |
| 4 | 0010 | 0100 | 9 | 0010 | 1010 |

EXCESS 3 BCD
TENS UNITS

Some devices require the direct decimal code.

| 0 | 0000000001 | 5 | 0000100000 |
| :--- | :--- | :--- | :--- |
| 1 | 0000000010 | 6 | 0001000000 |
| 2 | 0000000100 | 7 | 0010000000 |
| 3 | 0000001000 | 8 | 0100000000 |
| 4 | 0000010000 | 9 | 1000000000 |

Numerous other codes serve special purposes including error correcting (Hamming) codes, ICAO code used for automatic aircraft altitude reporting, and others.

Natural Binary and Gray Codes require the least number of tracks. The justification for the additional tracks needed for BCD is the ease of its application in interfacing with decimal display or BCD systems.

### 7.7. MECHANICAL CONSIDERATIONS

To the end user, the optical encoder is primarily a device designed to provide an electrical signal. The manufacturer views the optical encoder as an opto-mechanical assembly from which rotational or linear position, speed and/or acceleration can be determined. The optical encoder, as an opto-mechanical device, is only as good as the cumulative effects, or limits, of error that all the mechanical components of the assembly might exhibit. Such parameters as speed, slew rate, torque, inertia, size and weight, runout, shaft loading, eccentricity and end-play, are of major importance.

### 7.7.1 SIZE AND WEIGHT

Except for aerospace applications, encoder weight is rarely of any concern. However, size can significantly limit encoder resolution. Figure 7.7.1. depicts the relationship of counts/revolution to code pattern centerline diameter and window opening (each adjacent translucent and opaque section on the code provides one complete cycle.)


Resolution $\equiv N=\frac{\pi D}{2 X}$
Where:
D = Code Pattern Centerline $\mathrm{X}=$ Window Opening $\mathrm{N}=$ Counts/Revolution

Figure 7.7.1 Counts per revolution vs. centerline diameter and window opening.

### 7.7.2. SLEW SPEED AND SLEW RATE

Angular speed of the encoder disc, the code pattern diameter and segment size are related parameters defining the frequency of a generated output signal from an optical encoder. The linear speed of a point on a disc equals the angular velocity $\omega$ times the radius $R$ of the point in question. It is this linear speed, divided by the segment cycle, that generates the output frequency of the encoder, as shown in Figure 7.7.2.


Figure 7.7.2. Frequency Determination
In equation form, the frequency becomes:
$\mathrm{f}=$ (linear rate of speed of code pattern) - (number of complete window cycles per linear inch,

## or

$$
f=\frac{W N}{2 \pi}-\frac{n\left(R P M^{\prime} s\right)}{60}
$$

Thus, the frequency output of the optical encoder is generated by the angular speed of the disc.

Slew rate is the maximum velocity at which an encoder will be required to perform. A typical example is tape rewind in a tape transport system. Slew speed is an indicator of how fast the encoder may be rotated without introducing conditions detrimental to the bearings or other components of the mechanical assembly. As slew speed becomes an important factor, it can be enhanced by reducing the preload on the bearings and by using a high grade type of bearing in the assembly.

### 7.7.3. TORQUE, INERTIA

## AND ACCELERATION

Torque, inertia and angular acceleration are equated in rotational motion by $\mathrm{T}=\mathrm{J} \alpha$ where $\mathrm{T}=$ torque, $\mathrm{J}=$ inertia and $\alpha=$ acceleration. This represents the interdependency of available torque of the driving motor, the desired system acceleration (or deceleration) and the added system inertia of the optical encoder. Acceleration is usually not a major concern in optical encoders. However, in extremely high acceleration applications, bonding strength of disc to hub and material stresses must be adequate to withstand the higher torques.

Friction torque must be considered as either starting torque or running torque, which are analogous to static friction and dynamic friction. Starting and running torques are greatly affected by the encoder requirements, and are contributed by not only the code disc, but also by the quality of bearings, bearing preload and bearing seals. Where extremely small, low inertia DC motors are employed in servo systems, encoder torques can be greatly reduced.

However, this torque reduction has tradeoff penalties. Reducing bearing preload and/or bearing sealing requirements will reduce torque, but will also increase radial and axial endplay, and will permit greater particle contamination in the bearing.

Disc inertia may be of paramount importance in some systems. In high acceleration or velocity servos where fast access, over-
shoot or settling time are significant, low inertia discs are mandatory. As resolution requirements also increase, larger discs may be necessary. And, since inertia increases by the 4th power of the disc radius, many trade-offs have to be considered. Inertia varies in direct proportion to the mass of the disc. Therefore, certain disc material trade-offs must be considered. Mylar and photoplast discs, while lighter in weight than glass, may present other mechanical difficulties. the best results have been obtained with very thin glass discs.

### 7.8. MANUFACTURING AND HANDLING

Manufacturing techniques and assembly sequence will vary considerably, depending on the application the encoder is designed to fulfill. The variables are dictated by the options required to meet the expected encoder performance, environmental conditions, and reliability.

### 7.9. ENCODER RELIABILITY AND DESIGN CONSIDERATIONS

Encoder failure can be divided into four areas: (1) light source failure, (2) bearing failure, (3) electronics failure and (4) environmentally caused failure.

The lifetime of an encoder's light source depends on the type used, vibration level, power supply, and ambient temperature. Incandescent lamps can withstand higher temperatures, while LED's withstand
higher vibration levels and have a longer average life. Power supply regulation has a large effect on incandescent lamp life, as filament life is inversely proportional to the twelfth power of the applied voltage.

$$
\begin{aligned}
& t_{2}=t_{1}\left(\frac{1}{v+\Delta v}\right)^{12} \\
& t_{1}=\text { Lamp life at voltage } v \\
& t_{2}=\text { Lamp life at voltage }(v+\Delta v)
\end{aligned}
$$

Thus, a one percent increase in filament voltage will shorten lamp life by eleven percent. When incandescent lamps are used in encoders they are derated and burned in for approximately 300 hours, to maximize life and statize output. Thus, average filament lifetimes of 50,000 to 100,000 hours are possible.

To minimize error caused by axial and radial play in the encoder shaft, the bearings are usually class 7 , and are preloaded near the recommended limit. However, excessive preload is the usual cause of early bearing failure. Thus, the bearing housing used in encoders must be carefully designed to obtain maximum life without excessive wear.

The environment has a large impact on encoder design. Potential humidity, temperature or dust contamination may require special consideration in the specification of encoders.

Where seals are necessary, encoders employ an O-ring seal between the outer cover and
bearing assembly. Condensation or liquid splashing usually requires the use of sealed bearings. By incorporating O-ring shaft seals, low rpm encoders can be made submersible. Where both humidity and temperature must be tolerated, metalized discs and masks are usually used instead of photographic emulsions.

To achieve high resolution and accuracy, the mask and disc surfaces are usually spaced .001 to .006 inches apart. Any emulsion swelling due to high temperature, $120^{\circ} \mathrm{F}$ or above, or high humidity with possible condensation, or dust contamination may limit the life of the encoder.

The electronic circuits used in most encoders are generally very reliable and normally outlast the light source and bearings in encoders. Two exceptions are: In very high temperature environments above $125^{\circ} \mathrm{C}$, and with complex encoders using several integrated circuits. Early failure of integrated circuits or "infant mortality" can be as high as . 5 or $1 \%$ during the first one thousand hours if commercial non-stress-tested devices are used. Thus, if an encoder design uses eight IC's, as many as $4 \%$ of the encoders may fail during the first 1,000 hours due to "infant mortality." Simple burn-in or artificial aging procedures can reduce this rate from .5\% to .1\%.

Design considerations as related to encoder life are summarized in Table 7.9.1, following.

TABLE 7.9.1. COMPONENT TRADEOFFS

| ENCODER COMPONENT <br> OPTION | ADVANTAGE | DISADVANTAGE |
| :--- | :--- | :--- |
| Incandescent Lamp | High light output. Resistant <br> to high temperatures. | Filament sensitive to vibration. <br> Requires lens for collimation. |
| L.E.D. | Long life. Simple to install. <br> Requires less current | Less light output. Usually in- <br> frared cannot be visually <br> observed. Limited to about <br> $80^{\circ} \mathrm{C} . ~ H i g h e r ~ c o s t ~ t h a n ~ i n c a n-~$ <br> descent. |
| Silicon Photo <br> Voltaic Cell | Less expensive. Resistant to <br> temperatures to 150 | Low cost prevents hermetic <br> sealing. |
| Photo Transistor | Higher sensitivity; can be <br> hermetically sealed. Higher <br> output. | Limited to upper temperature <br> limit. Higher cost. |
| Photographic Disc <br> Emulsion | Less expensive | Only moderately hard. Not <br> resistant to high humidity. <br> Limited to $120^{\circ} \mathrm{C}$. |
| Metal on Glass | Resistant to high tempera- <br> ture, humidity \& scratching. | Expensive. |
| Mylar Disc | Low inertia. | Limited disc size. |
| Photoplast Disc | Low inertia. High <br> resistance to shock. | Less rigid disc. |
| Sealed Bearings | Less contamination <br>  <br> liquids. | More expensive. Higher <br> running torque. |

### 7.10. ACCURACY AND ERRORS

The total accuracy of an encoder-incremental or absolute-is affected by the combined error potential of the encoder/system configuration. This error can be summarized as follows:

## Total Error = encoder error + system interaction errors

### 7.10.1 ENCODER ERROR

Encoder error is the aggregate of quantization error, instrument error and signal processing errors, including generation and transmission errors.
7.10.1.1. Quantization Error - some quantization error is present in any digital device. In the case of optical encoders, it is determined by the intricacy of the pattern printed or deposited on the disc or scale, which in turn is dictated by desired accuracy, output requirements and other interfacing provisions. As shown in the figure below, quantization error will be a maximum of $\frac{1}{2 N}$, where $N$ is the number of lines in one disc revolution (or for linear types, the number of lines in one scale length). Thus, the accuracy of position readout is limited by this type of error.

7.10.1.2. Instrument Error - is caused by the following defects; the total is usually of the same order of magnitude as the quantization error.

1. Assembly error and manufacturing mechanical tolerance buildup
2. Mask or disc misalignment
3. Mechanical vibration
4. Temperature variation
5. Electrical noise
6. Faulty power supply regulation

The limiting errors for rotary encoders are usually eccentricity, disc pattern variation and runout on the disc face. These mechanical variations will produce "jitter" or "flutter". These are partly interdependent. Jitter will always be increased by disc eccentricity, but is also caused by inaccuracies in the photo reduction process.

Eccentricity is caused by radial bearing play, inaccurate dynamic centering of the disc pattern on the hub and by excessive clearance between the disc hub and shaft. Eccentricity produces several errors in the signal output, noted as follows.

Amplitude Modulation - if the encoder output is an analog or quasi sine wave, eccentricity will produce an amplitude variation such as shown below.


Frequency Modulation - usually appearing as jitter on the oscilloscope, is one component of observed jitter, depicted below.


$$
\frac{F_{2} \cdot F_{1}}{F_{1}}\left(\frac{\Delta R}{R}\right) \times 100 \%=\% \text { Flutter }
$$

Inter-channel Jitter - if the optical sensors for the two channels are separated by angle $\theta$, eccentricity will produce additional jitter of an amount equal to:

\% Inter-channel

$$
\text { Jitter }=\frac{\Delta R}{R} \frac{N}{2 \pi} \frac{\theta}{180^{\circ}} \times 100 \%
$$

Where
$N=$ number of lines on disc
$\theta=$ angular separation in degrees of sensors
$\frac{\Delta R}{R}=$ \% eccentricity

Oscilloscope traces of the output of Tracks $A$ and $B$ depicted above, would appear as shown:


QUADRATURE $=90^{\circ}\left( \pm 9^{\circ}\right)$

Angular Error - is the difference between the angle indicated by the encoder and the actual shaft angle, due to imperfect centering of the disc pattern with respect to the axis of rotation.

Angular error $=\arcsin \frac{R_{\max }-R_{\text {min }}}{R_{\min }}$
Where $\mathrm{R}_{\text {max }}=$ maximum distance from the code pattern to the axis of rotation
$R_{\text {min }}=$ the minimum distance

For example, encoder discs are usually centered by either optical or electronic means. Typical centering accuracy is $\pm 5 \%$ of one line width. Thus, for $R=1$ inch, $N=1000$, eccentricity error will be approximately $3 \times 10^{-4}$ inches.

If this is added to the eccentricity caused by shaft runout of .0002 in ., and a hub/ shaft clearance of .0002 in., total worst case eccentricity is .0007 in . Thus, a 1000 count, 2 -inch diameter encoder could have $.07 \%$ amplitude modulation, which would seldom be significant. Assuming $.07 \%$ frequency modulation, and angular error of $.04^{\circ}$; and if angular seperation between tracks $A$ and $B$ is $20^{\circ}$, then Jitter $=1.2 \%$.

### 7.10.1.3. Signal Processing Errors

Signal Generation - for most encoder applications, the electronic circuitry for signal generation senses the point at which the voltage output crosses through zero. Thus, any shift in the zero crossover can be an error. Besides eccentricty and disc runout discussed earlier, gap variation, excessive mask-disc gap, and temperature drift can cause crossover error. Zero crossover error appears as a constant, as a random variation or as a rapid periodic error (jitter).

As shown in the figures below, a single light sensor cannot produce a zero voltage output without an offset bias due to noncollimated light, scattering, finite maskdisc gaps, and mask misalignment.


Collimated Light


Sensor


Although a bias voltage can be used to cancel the DC offset of a single sensor, a small gap variation or temperature change would cause the DC offset to change; therefore, causing a change in the zero crossovers.


The use of two sensors per track output will nearly cancel this drift if the sensors are operated back-to-back or "push-pull". Thus, two masks and two sensors per track
are used with the masks arranged to produce signals with $180^{\circ}$ phase difference from the sensors.




Output with DC offset due to temperature or gap variation is shown:


Thus, the modulations and drift in the zero crossover are minimized by the use of two sensors per track output. This error cancellation requires closely matched sensors for output, temperature tracking, and high frequency rolloff. Other errors reduced by "back-to-back" sensors are changes in light excitation and errors due to poor voltage regulation.

Signal Transmission - problems encountered in trasmitting the encoder signal to the receiving electronics are usually those of electrical noise or signal distortion. These could result in gain or loss of counts, and hence incorrect data on the performance of the device being monitored.

Electrical noise can be minimized by the following techniques: (1) using shielded cable, (2) installing a low value pull-up resistor, typically 100 ohms, (3) use of a line driver and (4) use of a differential line driver. In the latter case, a differential line driver is used where long interconnections exist. A shielded twisted pair cable and mated line receiver are used.

### 7.10.2 SYSTEM INTERACTION ERRORS

System interaction errors are the result of the effects of gears, racks and pinions and drive belts used to couple the encoder with the system. Also, in the case of linear encoders, misalignment can cause significant cosine errors.

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## Appendix

## SI System and Conversion Factors

## A.1. SI SYSTEM OF UNITS (METRIC SYSTEM)

The International System of Units (in French: Système International d'Unités SI) was given its name in 1960 by the XI. Conférence Générale des Poids et Mesures. The system covers the whole science and technology. It was derived from the MKSA (metre-kilogram-second-ampere) system of units, which covers only mechanics, electricity, and magnetism.

The former CGS (centimetre-gram-second) system of units is still sometimes used (especially in magnetism), regardless of whether its use has been deprecated by law in metric nations. Therefore, the Conversion Factors Table in Appendix A.2. introduces some of the CGS magnetic units and their relation to SI and British units.

The SI is an absolute (independent of gravity), coherent system of metric units, which is based on seven base units of the following quantities:

| QUANTITY | UNIT | SYMBOL |
| :--- | :--- | :---: |
| length | metre | m |
| mass | kilogram | kg |
| time | second | s |
| electric <br> current | ampere | A |
| temperature | kelvin | K |
| luminous <br> intensity | candela | cd |
| amount <br> of <br> substance | mole | mol |

These base units are supported by two supplementary units:

| plane angle | radian | rad |
| :---: | :---: | :--- |
| solid angle | steradian | sr |

From the base and supplementary units is successively and systematically derived a series of derived units which, together with base and supplementary units form the SI. All these units are coherent; i.e., in their definition equations are no numerical coefficients at all. Therefore, also in practical equations are no numerical coefficients, if all quantities are expressed in base, supplementary or derived units.

In addition, there is a group of non-SI units, which are allowed to be used together with SI units. These units are related to corresponding SI units by means of certain numerical coefficients. The reasons for their use are either historical or practical, like the use of minute, hour and day for time, angular degree and revolution for plane angle, litre for volume, degree Celsius for temiperature, etc. In the following text, each kind of unit will be distinguished by:
(B) for base units,
(S) for supplementary units,
(D) for derived units,
(N) for non-Sl units.

In the SI system, the following prefixes are used to indicate multiples or submultiples of units:

| MULTIPLE | PREFIX | SYMBOL |
| :--- | :--- | :--- |
| $10^{18}$ | exa | E |
| $10^{15}$ | peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{2}$ | hecto | $\mathrm{h}^{*}$ |
| $10^{1}$ | deka | da |
| $10^{-1}$ | deci | $\mathrm{d}^{*}$ |
| $10^{-2}$ | centi | $\mathrm{c}^{*}$ |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

*to be avoided wherever possible

Examples:

| millimetre | $1 \mathrm{~mm}=10^{-3} \mathrm{~m}$ |
| :--- | :--- |
| kilovolt | $1 \mathrm{kV}=10^{3} \mathrm{~V}$ |
| microampere | $1 \mu \mathrm{~A}=10^{-6} \mathrm{~A}$ |
| nanofarad | $1 \mathrm{nF}=10^{-9} \mathrm{~F}$ |
| femtogram | $1 \mathrm{fg}=10^{-15} \mathrm{~g}$ |
| gigawatt | $1 \mathrm{GW}=10^{9} \mathrm{~W}$ |

DEFINITIONS OF BASE AND
SUPPLEMENTARY SI UNITS

## Length

| (B) $\quad$ metre | m |
| :--- | :---: |
| $1 \mathrm{~m}=1650763.73 \lambda$, where $\lambda$ is the wave- |  |
| length of radiation corresponding to the |  |
| transition between the levels $2 \mathrm{p}_{10}$ and |  |
| $5 \mathrm{~d}_{5}$ of the krypton- 86 atom in vacuum |  |

Originally, the metre was defined as $1 /\left(4 \times 10^{7}\right)$ of the Earth's meridian at sea level and realized by the length of the metallic international prototype.

Frequently used multiples:

| kilometre | $1 \mathrm{~km}=10^{3} \mathrm{~m}$ |
| :--- | :--- |
| centimetre | $1 \mathrm{~cm}=10^{-2} \mathrm{~m}$ |
| millimetre | $1 \mathrm{~mm}=10^{-3} \mathrm{~m}$ |
| micrometre | $1 \mu \mathrm{~m}=10^{-6} \mathrm{~m}$ |

## Mass

| (B) kilogram | kg |
| :--- | :--- |
| 1 kg is the mass of the international mass |  |
| prototype which is secured by the Inter- |  |
| national Bureau of Weights and Measures |  |
| at Sévres, France |  |

Very accurately, 1 kg is the mass of $1 \ell$ (litre) $=1 \mathrm{dm}^{3}=10^{-3} \mathrm{~m}^{3}$ of water at $4^{\circ} \mathrm{C}$ and at normal atmospherical pressure.

Frequently used multiples and non-SI units:
$(\mathrm{N})$ tonne (metric ton)
or megagram $1 \mathrm{t}=10^{3} \mathrm{~kg}=1 \mathrm{Mg}$
gram $\quad 1 \mathrm{~g}=10^{-3} \mathrm{~kg}$
milligram $\quad 1 \mathrm{mg}=10^{-6} \mathrm{~kg}=10^{-3} \mathrm{~g}$

## Time

| (B) second | s |
| :--- | :--- |
| $\mathbf{1} \mathrm{s}=9$ 192 631770 T , where T is the |  |
| period of radiation corresponding to the |  |
| transition between the two hyperfine |  |
| levels of the ground state of the cesium-133 |  |
| atom |  |

Frequently used multiples and non-SI units:
( N$)$ day (mean solar day)

$$
1 \mathrm{~d}=24 \mathrm{~h}=86400 \mathrm{~s}
$$

(N) hour

$$
1 \mathrm{~h}=3600 \mathrm{~s}
$$

(N) minute

$$
1 \mathrm{~min}=60 \mathrm{~s}
$$

millisecond

$$
1 \mathrm{~ms}=10^{-3} \mathrm{~s}
$$

microsecond

$$
1 \mu \mathrm{~s}=10^{-6} \mathrm{~s}
$$

nanosecond

$$
1 \mathrm{~ns}=10^{-9} \mathrm{~s}
$$

## Electric current

(B)
A

1 A is the DC current which, being maintained in two straight, parallel and infinitely long conductors with negligible circular cross-section, spaced in vacuum 1 m apart, causes the force of $2 \times 10^{-7} \mathrm{~N}$ per metre of length between them

Frequently used multiples:

$$
\begin{array}{ll}
\text { kiloampere } & 1 \mathrm{kA}=10^{3} \mathrm{~A} \\
\text { milliampere } & 1 \mathrm{~mA}=10^{-3} \mathrm{~A} \\
\text { microampere } & 1 \mu \mathrm{~A}=10^{-6} \mathrm{~A} \\
\text { nanoampere } & 1 \mathrm{nA}=10^{-9} \mathrm{~A} \\
\text { picoampere } & 1 \mathrm{pA}=10^{-12} \mathrm{~A}
\end{array}
$$

## Temperature (thermodynamic)

| (B) kelvin | K |
| :--- | :--- |
| $1 \mathrm{~K}=1 / 273.16$ of the thermodynamic <br> temperature of the triple point of water |  |

Temperature is measured and designated in two different scales:
a) absolute temperature

$$
\text { kelvin } \quad K
$$

b) normal temperature
$(\mathrm{N})$ degree Celsius ${ }^{\circ} \mathrm{C}$
where
$1 \mathrm{~K}=1^{\circ} \mathrm{C}$
(magnitude of degrees)
and
$0^{\circ} \mathrm{C}=273.15 \mathrm{~K}$, so that
$\Theta\left[{ }^{\circ} \mathrm{C}\right]=\Theta[\mathrm{K}]-273.15$
Reference points are:

1. Triple point of water (calibration point)

$$
273.16 \mathrm{~K}=0.01^{\circ} \mathrm{C}
$$

2. Freezing point of water

$$
273.15 \mathrm{~K}=0^{\circ} \mathrm{C}
$$

3. Absolute zero
(lowest possible temperature)

$$
0 \mathrm{~K}=-273.15^{\circ} \mathrm{C}
$$

Originally, the degree Celsius was defined as $1 / 100$ of the temperature difference between the boiling and freezing points of water at normal atmospherical pressure.

## Luminous intensity

| (B) candela | cd |
| :--- | :--- |
| 1 cd is the normal (perpendicular) luminous |  |
| intensity of $1 / 600000 \mathrm{~m}^{2}$ of the surface |  |
| of an absolutely black body at the tempera- |  |
| ture of solidification of platinum ( 2046.5 K ) |  |
| and at pressure of 101.325 kPa |  |

The candela is one of photometric units (like lumen, lux, etc.) which are related to optical properties of the human eye. Therefore, in physical radiation theory and measurement, only the radiometric units (watt for radiant flux, etc.) must be used.

## Amount of substance

| (B) mole | mol |
| :--- | :--- |
| $\mathbf{1}$ mol is the amount of substance of a sys- |  |
| tem which contains as many elementary |  |
| entities as there are atoms in 0.012 kg of |  |
| carbon 12 |  |

## Plane angle

| (S) radian | rad |
| :--- | :--- |
| $\mathbf{1}$ rad is the plane angle between two radii of |  |
| a circle which cut off on the circumference |  |
| an arc equal in length to the radius |  |

There are $2 \pi$ rad in a full circle.

## Solid angle

| (S) steradian | sr |
| :--- | :--- |
| $\mathbf{1}$ sr is the solid angle which, having its vertex |  |
| in the center of a sphere, cuts off an area |  |
| of the surface of the sphere equal to the |  |
| square of the radius of the sphere |  |

There are $4 \pi \mathrm{sr}$ in a full sphere.

## REVIEW OF DERIVED SI UNITS AND NON-SI UNITS

This review is limited to mechanics, thermodynamics, radiation, photometry, electricity and magnetism, and contains only the units related to the contents of this handbook.

The successive method of derivation from the base and supplementary units is described and the dimension of each unit, expressed by means of basic dimensions ( $\mathrm{m}, \mathrm{kg}, \mathrm{s}, \mathrm{A}, \mathrm{K}, \mathrm{cd}$ ), is introduced, e.g., $[V]=m^{2} \mathrm{~kg} \mathrm{~s}^{-3} A^{-1}$

## A. MECHANICAL UNITS

## Area

| (D) $\quad$ square metre | $\mathrm{m}^{2}$ |
| :--- | :--- |

Frequently used multiples and non-SI units:
square kilometre $\quad 1 \mathrm{~km}^{2}=10^{6} \mathrm{~m}^{2}$
$(\mathrm{N})$ hectare $\quad 1 \mathrm{ha}=10^{4} \mathrm{~m}^{2}$
(N) are $1 \mathrm{a}=10^{2} \mathrm{~m}^{2}$
square centimetre $1 \mathrm{~cm}^{2}=10^{-4} \mathrm{~m}^{2}$
square millimetre $1 \mathrm{~mm}^{2}=10^{-6} \mathrm{~m}^{2}$

## Volume

| (D) $\quad$ cubic metre | $\mathrm{m}^{3}$ |
| :--- | :--- |

Frequently used multiples and non-SI units:
(N) litre
$1 \ell=1 \mathrm{dm}^{3}=10^{-3} \mathrm{~m}^{3}$
(N) millilitre $1 \mathrm{~m} \ell=10^{-3} \ell=1 \mathrm{~cm}^{3}$
cubic
centimetre $1 \mathrm{~cm}^{3}=10^{-6} \mathrm{~m}^{3}$
cubic
millimetre $1 \mathrm{~mm}^{3}=10^{-9} \mathrm{~m}^{3}$

## Velocity (linear)

(D) metre per second $\quad \mathrm{m} \mathrm{s}^{-1}$

Frequently used multiples and non-SI unit:
( N ) kilometre per hour

$$
1 \mathrm{~km} / \mathrm{h}=(1 / 3.6) \mathrm{m} / \mathrm{s}
$$

centimetre per second

$$
1 \mathrm{~cm} / \mathrm{s}=10^{-2} \mathrm{~m} / \mathrm{s}
$$

millimetre per second

$$
1 \mathrm{~mm} / \mathrm{s}=10^{-3} \mathrm{~m} / \mathrm{s}
$$

The velocity of electromagnetic radiation in vacuum $c=2.99793 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

## Acceleration (linear)

(D) metre per square second $\mathrm{m} \mathrm{s}^{-2} \quad$| (D) |
| :--- |

The standard (average) gravitational acceleration is internationally stated by the value $\mathrm{g}=9.80665 \mathrm{~m} / \mathrm{s}^{2}$ (exactly).

## Angular displacement

| (S) $\quad$ radian | rad |
| :--- | :--- |
|  | $[\mathrm{rad}]=\mathrm{m} / \mathrm{m}=1$ |

Non-SI units:
(N) angular degree

$$
\begin{aligned}
1^{0} & =(2 \pi / 360) \mathrm{rad} \\
& =0.017453 \mathrm{rad}
\end{aligned}
$$

$(\mathrm{N})$ revolution

$$
1 \mathrm{r}=360^{\circ}=(2 \pi) \mathrm{rad}=6.2832 \mathrm{rad}
$$

## Angular velocity

| (D)radian <br> per second | $\mathrm{rad} \mathrm{s}^{-1}$ |
| :---: | :---: |
|  | $\left[\mathrm{rad} \mathrm{s}^{-1]}=\mathrm{s}^{-1}\right.$ |

Non-SI units:
( N ) angular degree per second

$$
1^{\circ} / \mathrm{s}=0.017453 \mathrm{rad} / \mathrm{s}
$$

$(\mathrm{N})$ revolution per second

$$
1 \mathrm{r} / \mathrm{s}=6.2832 \mathrm{rad} / \mathrm{s}
$$

$(\mathrm{N})$ revolution per minute

$$
1 \mathrm{r} / \mathrm{min}=0.10472 \mathrm{rad} / \mathrm{s}
$$

Note: Symbols rpm, krpm are common in technical literature.

## Angular acceleration

| (D)radian per <br> square second | $\left[\mathrm{rad} \mathrm{s}^{-2}\right.$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |

Non-SI unit:
$(\mathrm{N})$ angular degree per square second

$$
1^{\circ} / \mathrm{s}^{2}=0.017453 \mathrm{rad} / \mathrm{s}^{2}
$$

## Force

| (D) newton | N |
| :--- | :---: |
|  | $[\mathrm{N}]=\mathrm{m} \mathrm{kg} \mathrm{s}^{-2}$ |
| 1 N is the force which accelerates the <br> mass of 1 kg by $1 \mathrm{~m} / \mathrm{s}^{2}$ |  |

Technical unit of force (not approved for use in the SI), which is still sometimes used:

## kilopond <br> $1 \mathrm{kp}=9.80665 \mathrm{~N}$

1 kp is the force which accelerates the mass of 1 kg by $\mathrm{g}=9.80665 \mathrm{~m} / \mathrm{s}^{2}$ so that 1 kp is the approximate weight (heaviness) of the mass of 1 kg . The kilopond is also called kilogram-force (kgf).

## Torque

(D) newton metre

| Nm |
| :---: |
| $[\mathrm{Nm}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-2}$ |

1 Nm is the torque produced by the force of 1 N acting on the radius of 1 m

Technical unit of torque (not approved for use in the SI ), which is still sometimes used:
kilopond metre

$$
1 \text { kpm = } 9.80665 \mathrm{Nm}
$$

## Moment of inertia

(D) kilogram - square metre
kg m ${ }^{2}$

Moment of inertia of a body (with respect to the given axis of rotation) is given by the volume integral

$$
\mathrm{J}=\int \mathrm{r}^{2} \mathrm{dm}
$$

m
where dm is the mass element of the body and $r$ is its radius with respect to the axis of rotation.

## Energy, work, amount of heat

| (D) | joule | J |
| :---: | :---: | :---: |
|  |  | $[\mathrm{J}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-2}$ |
| 1 J is the work of the force of 1 N along the path of 1 m |  |  |

Other derived, non-SI (approved) and not approved units:
(D) wattsecond

$$
1 \mathrm{Ws}=1 \mathrm{~J}
$$

(N) kilowatthour

$$
1 \mathrm{kWh}=3.6 \times 10^{6} \mathrm{~J}
$$

( N ) electronvolt

$$
1 \mathrm{eV}=1.60206 \times 10^{-19} \mathrm{~J}
$$

Not approved units still sometimes used:
kilopond metre
$1 \mathrm{kpm}=9.80665 \mathrm{~J}$
kilocalorie (nutrition calorie)
$1 \mathrm{kcal}=4186.8 \mathrm{~J}$
1 kcal is amount of heat which increases the temperature of $1 \ell$ of water by $1^{\circ} \mathrm{C}$

## Power

| (D) watt | W |
| :--- | :---: |
|  | $[\mathrm{W}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |
| 1 W is the power performing the work of |  |
| 1 J in 1 s |  |

Not approved units still sometimes used:
kilopond metre per second
$1 \mathrm{kpm} / \mathrm{s}=9.80665 \mathrm{~W}$
horsepower (metric)

$$
\begin{aligned}
1 \mathrm{hp} & =75 \mathrm{kpm} / \mathrm{s} \\
& =735.5 \mathrm{~W}
\end{aligned}
$$

Pressure, stress
(D) pascal

| Pa |
| :---: |
| $[\mathrm{Pa}]=\mathrm{m}^{-1} \mathrm{~kg} \mathrm{~s}^{-2}$ |

1 Pa is the pressure developed by the force of 1 N over the area of $1 \mathrm{~m}^{2}$
$1 \mathrm{~Pa}=1 \mathrm{~N} / \mathrm{m}^{2}$
kilopascal
$1 \mathrm{kPa}=10^{3} \mathrm{~N} / \mathrm{m}^{2}$
megapascal

$$
\begin{aligned}
1 \mathrm{MPa} & =10^{6} \mathrm{~N} / \mathrm{m}^{2} \\
& =1 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

Not approved technical units still sometimes used:
kilopond per square centimetre

$$
1 \mathrm{kp} / \mathrm{cm}^{2}=98.0665 \mathrm{kPa}
$$

technical atmosphere

$$
1 \mathrm{at}=1 \mathrm{kp} / \mathrm{cm}^{2}
$$

Not approved units still sometimes used in meteorology:

The reference atmospherical pressure, so called "standard atmosphere" is stated by the definition

$$
\begin{aligned}
1 \mathrm{~atm} & =101.325 \mathrm{kPa} \\
& =1013.25 \mathrm{mbar}
\end{aligned}
$$

millibar
$1 \mathrm{mbar}=100 \mathrm{~Pa}$
torr
1 torr $=133.322 \mathrm{~Pa}$
1 torr is equal to hydrostatical pressure of 1 mm column of mercury at $0^{\circ} \mathrm{C}$ and $\mathrm{g}=9.80665 \mathrm{~m} / \mathrm{s}^{2}(1 \mathrm{~atm}=760 \mathrm{torr})$

## Frequency of oscillations

| (D) hertz | Hz |
| :--- | :--- |
|  | $[\mathrm{Hz}]=\mathrm{s}^{-1}$ |

frequency of oscillations is the number of complete cycles of a periodic repeated movement during the time interval of 1 s

Frequently used multiples:
millihertz

$$
1 \mathrm{mHz}=10^{-3} \mathrm{~Hz}
$$

kilohertz
$1 \mathrm{kHz}=10^{3} \mathrm{~Hz}$
megahertz

$$
1 \mathrm{MHz}=10^{6} \mathrm{~Hz}
$$

gigahertz

$$
1 \mathrm{GHz}=19^{9} \mathrm{~Hz}
$$

## Wave number

(D) reciprocal metre
$\mathrm{m}^{-1}$

## Dynamic viscosity

(D) newton-second per square metre

| $\mathrm{N}-\mathrm{s} / \mathrm{m}^{2}$ |
| :---: |
| $\left[\mathrm{~N}-\mathrm{s} / \mathrm{m}^{2}\right]=\mathrm{m}^{-1} \mathrm{~kg} \mathrm{~s}^{-1}$ |

## Kinematic viscosity

(D)

| square metre <br> per second | $\mathrm{m}^{2} / \mathrm{s}$ |
| :---: | :---: |

B. THERMODYNAMIC, RADIOMETRIC

AND PHOTOMETRIC UNITS
Heat flux

| (D) watt | $W$ |
| :---: | :---: |
|  | $[W]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |

## Heat flux density

| (D)watt per <br> square metre | $\left[\mathrm{W} / \mathrm{m}^{2}\right.$ |
| :---: | :---: |
|  |  |

## Specific heat capacity

(D) joule per kilogram-kelvin

Thermal conductivity

| (D)watt per <br> metre-kelvin | $[\mathrm{W} / \mathrm{m}-\mathrm{K}]=\mathrm{m} \mathrm{kg} \mathrm{s}^{-3} \mathrm{~K}^{-1}$ |
| :---: | :---: |

## Thermal resistivity

| (D) $\begin{array}{c}\text { metre-kelvin } \\ \text { per watt }\end{array}$ | $m-K / W$ |
| :---: | :---: |
|  | $[m-K / W]=m^{-1} \mathrm{~kg}^{-1} \mathrm{~s}^{3} K$ |

Thermal resistance (between two points of a given structure)

| (N) degree Celsius per watt | ${ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| :---: | :---: |
|  | $\left[{ }^{\circ} \mathrm{C} / \mathrm{W}\right]=\mathrm{m}^{-2} \mathrm{~kg}^{-1} \mathrm{~s}^{\mathbf{3}} \mathrm{K}$ |

## Radiant flux

| (D) watt | W |
| :---: | :---: |
|  | $[W]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |

## Radiant flux density (irradiance)

|  | watt per <br> (D) <br> square metre |
| :---: | :---: |
|  | $\left[\mathrm{W} / \mathrm{m}^{2}\right]=\mathrm{kg} \mathrm{s}^{-3}$ |

Radiant flux intensity

| (D)watt per <br> steradian | $\mathrm{W} / \mathrm{sr}$ |
| :--- | :---: |
|  | $[\mathrm{W} / \mathrm{sr}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |

Luminous flux

| (D) lumen | Im |
| :--- | :---: |
|  | $[\mathrm{Im}]=\mathbf{c d}$ |

$$
1 \mathrm{~lm}=1 \mathrm{~cd}-\mathrm{sr}
$$

Illuminance

| (D) lux | 1 x |
| :---: | :---: |
|  | $[1 \mathrm{x}]=\mathrm{cd} \mathrm{m}$ |

$$
1 \mathrm{~lx}=1 \mathrm{~lm} / \mathrm{m}^{2}
$$

(D)
candela per
square metre
$\mathrm{cd} / \mathrm{m}^{2}$

## C. ELECTRICAL AND MAGNETIC UNITS

## Electric potential, voltage

| (D) volt | V |
| :--- | :--- |
|  | $[\mathrm{V}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3} \mathrm{~A}^{-1}$ |
| 1 V is the voltage (or potential difference) <br> between two points of a conductor con- <br> ducting the current of 1 A if the power <br> expended in this part of the conductor is <br> 1 W |  |

Frequently used multiples:
kilovolt

$$
1 \mathrm{kV}=10^{3} \mathrm{~V}
$$

millivolt

$$
1 \mathrm{mV}=10^{-3} \mathrm{~V}
$$

microvolt

$$
1 \mu V=10^{-6} V
$$

## Electric charge

(D)

| coulomb | $C$ |
| :---: | :---: |
|  | $[C]=s A$ |

1 C is the charge delivered by constant current of $1 A$ in 1 s

The elementary electric charge (charge of proton)

$$
e=(1.60206 \pm 0.00003) \times 10^{-19} \mathrm{C}
$$

Electric flux density

| coulomb <br> per <br> square <br> metre | $\left[C m^{-2}\right]=m^{-2} \mathrm{~s} A$ |
| :---: | :---: |
|  |  |

## Electric field intensity (strength)

| volt <br> per <br> metre | $\left[\mathrm{V} \mathrm{m}^{-1}\right]=\mathrm{m} \mathrm{kg} \mathrm{s}^{-3} \mathrm{~A}^{-1}$ |
| :---: | :---: |
|  |  |

Electric field intensity ( $E$ ) is related to electric flux density (D) by
$\mathrm{D}=\epsilon_{\mathrm{r}} \epsilon_{\mathrm{o}} \mathrm{E}$
where
$\epsilon_{\mathrm{r}}$ is relative premittivity, $\left[\epsilon_{\mathrm{r}}\right]=1$

$$
\epsilon_{\mathrm{o}}=8.85416 \times 10^{-12} \mathrm{~F} / \mathrm{m}
$$

is the permittivity of vacuum

## Capacitance

| (D) farad | $F$ |
| :---: | :---: |
|  | $[F]=m^{-2} \mathrm{~kg}^{-1} \mathrm{~s}^{4} A^{2}$ |

1 F is the capacitance of a capacitor which, being charged by the charge of 1 C , has the terminal voltage of 1 V

Frequently used multiples:
microfarad

$$
1 \mu F=10^{-6} F
$$

nanofarad

$$
1 \mathrm{nF}=10^{-9} \mathrm{~F}
$$

picofarad

$$
1 \mathrm{pF}=10^{-12} \mathrm{~F}
$$

## Magnetic flux

| (D) weber | Wb |
| :--- | :--- |
|  | $[\mathrm{Wb}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-2} \mathrm{~A}^{-1}$ |

1 Wb is the magnetic flux which, being linearly attenuated to zero in 1 s , induces in surrounding turn an emf of 1 V
$1 \mathrm{~Wb}=1 \mathrm{Vs}$ (voltsecond)

## Magnetic flux density (magnetic induction)

| (D) $\quad$ tesla | $T$ |
| :---: | :---: |
|  | $[T]=\mathrm{kg} \mathrm{s}^{-2} A^{-1}$ |

1 T is the magnetic flux density of a homogenous magnetic field which acts by the force of 1 N per 1 m of a conductor conducting the current of 1 A , if the field direction is perpendicular to the axis of the conductor
$1 \mathrm{~T}=1 \mathrm{~Wb} / \mathrm{m}^{2}$
Magnetic field intensity (strength)
(D) ampere per metre $\quad \mathrm{Am}^{-1}$

Magnetic field intensity $(\mathrm{H})$ is defined according to the 1st Maxwell equation by the curve integral

$$
\oint_{\mathbf{C}}
$$

where $\Sigma \mathrm{I}$ is the sum of currents inside the closed curve, $\mathbf{C}$, and $\mathbf{d} L$ is the element of $\mathbf{C}$. It is related to the magnetic flux density (B) by
$B=\mu_{r} \mu_{o} H$
where
$\mu_{\mathrm{r}}$ is the relative permeability, $\left[\mu_{\mathrm{r}}\right]=1$

$$
\begin{aligned}
\mu_{\mathrm{O}}= & 1.25664 \times 10^{-6} \mathrm{H} / \mathrm{m} \\
& \text { is the permeability of vacuum }
\end{aligned}
$$

Permeability and permittivity of vacuum are natural constants related together by

$$
c=\frac{1}{\sqrt{\epsilon_{\mathrm{o}} \mu_{\mathrm{O}}}}=2.99793 \times 10^{8} \mathrm{~m} / \mathrm{s}
$$

which is the velocity of electromagnetic radiation in vacuum.

## Inductance

| (D) henry | H |
| :--- | :---: |
|  | $[H]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-2} A^{-2}$ |
| 1 H is the inductance of a conductor in |  |
| which is generated the counter emf of 1 V |  |
| if the current in the conductor is linearly |  |
| changed by 1 A in 1 s |  |

## Current density

(D) ampere per square metre $\mathrm{Am}^{-2}$

This unit is too small for practical use. Practical unit is ampere per square millimetre

$$
1 \mathrm{~A} / \mathrm{mm}^{2}=10^{6} \mathrm{~A} / \mathrm{m}^{2}
$$

## Resistance

| (D) ohm | $\Omega$ |
| :---: | :---: |
|  | $[\Omega]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3} \mathrm{~A}^{-2}$ |

$1 \Omega$ is the resistance of a conductor which, being connected to the voltage of 1 V , conducts the current of 1 A

$$
1 \Omega=\frac{1 \mathrm{~V}}{1 \mathrm{~A}}
$$

The same unit is used for reactance and impedance.

Frequently used multiples:
megohm

$$
1 \mathrm{M} \Omega=10^{6} \Omega
$$

kiloohm

$$
1 \mathrm{k} \Omega=10^{3} \Omega
$$

milliohm

$$
1 \mathrm{~m} \Omega=10^{-3} \Omega
$$

## Resistivity

| (D)ohm- <br> metre | $\Omega \mathrm{m}$ |
| :---: | :---: |
|  | $[\Omega \mathrm{m}]=\mathrm{m}^{3} \mathrm{~kg} \mathrm{~s}^{-3} \mathrm{~A}^{-2}$ |

This unit is too large for practical use. Usually, the resistivity is measured and stated for the sample of a conductor which is 1 m long and has the cross-section of $1 \mathrm{~mm}^{2}$. Therefore, the practical unit is

$$
1 \Omega \mathrm{~mm}^{2} \mathrm{~m}^{-1}=10^{-6} \Omega \mathrm{~m}=1 \mu \Omega \mathrm{~m}
$$

which is called microohm-metre. For example, resistivity of copper is

$$
\rho \mathrm{Cu}=0.0172 \mu \Omega \mathrm{~m}
$$

Resistance of a given conductor is then given by

$$
\mathrm{R}=\rho \frac{L}{\mathrm{~S}}\left[\Omega ; \mu \Omega \mathrm{m}, \mathrm{~m}, \mathrm{~mm}^{2}\right]
$$

## Conductance



1 S is the conductance of a conductor which has the resistance of $1 \Omega$

Conductance is reciprocal of resistance:

$$
1 \mathrm{~S}=\frac{1}{1 \Omega}=\frac{1 \mathrm{~A}}{1 \mathrm{~V}}
$$

Electric power, reactive power, apparent power
a) power (effective power)

| (D) $\quad$ watt | $W$ |
| :---: | :---: |
|  | $[W]=m^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |

b) reactive power

| (N) $\quad$ var | var |
| :--- | :---: |
|  | $[$ $v a r] ~$ | $\mathrm{~m}^{2} \mathrm{~kg} \mathrm{~s}^{-3} \mathrm{C}$

c) apparent power

| (D) $\quad$ voltampere | VA |
| :---: | :---: |
|  | $[\mathrm{VA}]=\mathrm{m}^{2} \mathrm{~kg} \mathrm{~s}^{-3}$ |

The coherency of SI units and great practical advantage of their use is best demonstrated in the following set of examples:
a) Power, transmitted by rotating shaft, given as the product of torque $\mathbf{T}$ and angular velocity $\omega$

$$
\mathrm{P}=\mathrm{T} \omega \quad[\mathrm{~W} ; \mathrm{Nm}, \mathrm{rad} / \mathrm{s}]
$$

b) Centrifugal force acting on a body with the mass $m$ which moves by the velocity $\mathbf{v}$ along a curved path with instantaneous radius $r$

$$
F=m \frac{v^{2}}{r} \quad[N ; k g, m / s, m]
$$

c) Acceleration torque necessary to accelerate a rotating body with the moment of inertia $\mathbf{J}$ by angular acceleration $a$

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{J} a \quad\left[\mathrm{Nm} ; \mathrm{kg} \mathrm{~m}^{2}, \mathrm{rad} / \mathrm{s}^{2}\right]
$$

d) Force acting on a current-carrying conductor in homogenous magnetic field perpendicular to conductor axis (I = current in conductor, $L=$ length of conductor, $\mathbf{B}=$ magnetic flux density)

$$
F=B L I \quad[N ; T, m, A]
$$

e) Voltage generated in conductor, moved in homogenous magnetic field perpendicular to conductor axis

$$
E=B L v \quad[V ; T, m, m / s]
$$

f) Voltage generated in a turn due to time change of magnetic flux

$$
\begin{equation*}
e=-\frac{d \Phi}{d t} \tag{V;Wb,s}
\end{equation*}
$$

g) Voltage generated in the coil with an inductance $L$ due to time change of the current in the coil

$$
\begin{equation*}
v_{L}=L \frac{d i}{d t} \tag{V;H,A,s}
\end{equation*}
$$

h) Motor power loss expressed in terms of torque and angular velocity for output and armature current and terminal DC voltage for input

$$
\begin{aligned}
P_{L}=P_{i}-P_{0}= & I_{a} V-T \omega \\
& {[W ; A, V, N m, r a d / s] }
\end{aligned}
$$

i) Amount of heat, developed in a resistor over a period of time

$$
W=R I^{2} t
$$

## A. 2 UNITS' CONVERSION FACTORS AND TABLES

CONVERSION FACTORS
Unless otherwise specified, the units $o z$ and $l b$ in the following tables are units of force.

|  | m metre <br> km kilometre <br> cm centimetre <br> mm millimetre <br> $\mu \mathrm{m}$ micrometre <br> nm naometre <br> $\AA$ angstrom | $\begin{gathered} 1 \mathrm{~m}=10^{-3} \mathrm{~km} \\ 10^{2} \mathrm{~cm} \\ 10^{3} \mathrm{~mm} \\ 10^{6} \mu \mathrm{~m} \\ 10^{9} \mathrm{~nm} \\ 10^{10} \AA \\ \hline \end{gathered}$ | $\begin{aligned} 1 \mathrm{~m}= & 39.370 \mathrm{in} \\ & 3.2808 \mathrm{ft} \\ & 1.0936 \mathrm{yd} \end{aligned}$ |
| :---: | :---: | :---: | :---: |
|  |  | $1 \mathrm{in}=25.4 \mathrm{~mm}$ (exactly) |  |
|  |  | $1 \mathrm{ft}=12 \mathrm{in}=0.3048 \mathrm{~m}$ |  |
|  |  | $1 \mathrm{yd}=3 \mathrm{ft}=0.9144 \mathrm{~m}$ |  |
|  |  | $1 \mathrm{mi}($ statute mile) $=1609.344 \mathrm{~m}=5280 \mathrm{ft}$ |  |
|  |  | 1 nautical mile $=1852 \mathrm{~m}$ |  |
|  |  | $1 \mathrm{mil}=10^{-3} \mathrm{in}=0.0254 \mathrm{~mm}$ |  |
|  | $\mathrm{m}^{2}$ square metre <br> $\mathrm{km}^{2}$ square kilometre <br> ha hectare <br> a are <br> $\mathrm{cm}^{2}$ square centimetre <br> $\mathrm{mm}^{2}$ square millimetre | $\begin{aligned} 1 \mathrm{~m}^{2}= & 10^{4} \mathrm{~cm}^{2} \\ & 10^{6} \mathrm{~mm}^{2} \end{aligned}$ | $\begin{array}{r} 1 \mathrm{~m}^{2}=1550.0 \mathrm{in}^{2} \\ 10.764 \mathrm{ft}^{2} \end{array}$ |
|  |  |  | $1 \mathrm{~cm}^{2}=0.155 \mathrm{in}^{2}$ |
|  |  | $\begin{aligned} 1 \mathrm{in}^{2}= & 645.16 \mathrm{~mm}^{2}=6.4516 \mathrm{~cm}^{2} \\ & 6.9444 \times 10^{-3} \mathrm{ft}^{2} \end{aligned}$ |  |
|  |  | $1 \mathrm{ft}^{2}=144 \mathrm{in}^{2}=0.0929 \mathrm{~m}^{2}$ |  |
|  |  | $\begin{aligned} 1 \mathrm{~km}^{2}= & 100 \mathrm{ha} \\ & 10^{4} \mathrm{a} \\ & 10^{6} \mathrm{~m}^{2} \\ & 0.3861 \mathrm{mi}^{2} \end{aligned}$ | $\begin{aligned} 1 \mathrm{ha}= & 10^{4} \mathrm{~m}^{2}=100 \mathrm{a} \\ & 2.471 \text { acres } \end{aligned}$ |
|  |  |  | $\begin{array}{r} 1 \mathrm{a}=100 \mathrm{~m}^{2} \\ 0.01 \mathrm{ha} \end{array}$ |
|  |  | $\begin{aligned} 1 \mathrm{mi}^{2}= & 640 \text { acres } \\ & 2.59 \mathrm{~km}^{2} \end{aligned}$ | $\begin{aligned} & \hline 1 \text { acre }= 0.40469 \mathrm{ha} \\ & 4046.9 \mathrm{~m}^{2} \end{aligned}$ |
|  | $\mathrm{m}^{3}$ cubic metre <br> $\mathrm{dm}^{3}$ cubic decimetre <br> $\ell$ litre $\left.(=\mathrm{dm})^{3}\right)$ <br> $\mathrm{m} \ell$ millilitre $\left(=\mathrm{cm}^{3}\right)$ <br> $\mathrm{cm}^{3}$ cubic centimetre <br> $\mathrm{mm}^{3}$ cubic millimetre | $\begin{aligned} & 1 \mathrm{~m}^{3}= 10^{3} \mathrm{dm}^{3} \\ & 10^{3} \ell \\ & 10^{6} \mathrm{~cm}^{3} \\ & 10^{6} \mathrm{ml} \\ & 10^{9} \mathrm{~mm}^{3} \\ & \hline \end{aligned}$ | $\begin{aligned} 1 \mathrm{~m}^{3}= & 6.1024 \times 10^{4} \mathrm{in}^{3} \\ & 35.315 \mathrm{ft}^{3} \end{aligned}$ |
|  |  | $1 \mathrm{in}^{3}=16.387 \mathrm{~cm}^{3}$ |  |
|  |  | $1 \mathrm{ft}^{3}=1728 \mathrm{in}^{3}=2.8317 \times 10^{-2} \mathrm{~m}^{3}$ |  |
|  |  | $1 \mathrm{~m} \ell=0.0338 \mathrm{fl} \mathrm{oz}$ $1 \mathrm{fl} \mathrm{oz}=29.5737 \mathrm{~m} \ell$  <br> $1 \ell$ $=33.8 \mathrm{fl} \mathrm{oz}$ $1.8047 \mathrm{in}^{3}$ |  |
|  |  |  |  |
|  |  | 1.0567 qt | $1 \mathrm{qt}=0.94636 \ell$ |
|  |  | 0.26417 gal | $1 \mathrm{gal}=3.7854 \ell$ |



| $\stackrel{\mathscr{H}}{\mathbb{C}}$ | kg kilogram <br> g gram <br> t tonne <br>  (metric ton) | $\begin{aligned} 1 \mathrm{~kg}= & 10^{3} \mathrm{~g}=10^{-3} \mathrm{t} \\ & 35.274 \mathrm{oz} \text { (mass) } \\ & 2.2046 \mathrm{lb} \text { (mass) } \\ & 6.8522 \times 10^{-2} \text { slug } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: |
|  |  | 1 oz (mass) $=28.3495 \mathrm{~g}$ |  |
|  |  | $\begin{aligned} 1 \mathrm{lb} \text { (mass) }= & 16 \mathrm{oz} \text { (mass) } \\ & 0.45359 \mathrm{~kg} \end{aligned}$ |  |
|  |  | 1 slug $=14.5939 \mathrm{~kg}$ |  |
|  |  | $\begin{gathered} 1 \mathrm{t}=10^{3} \mathrm{~kg} \\ 2204.6 \mathrm{lb} \\ \text { (mass) } \\ 1.1023 \text { shortton } \end{gathered}$ | $\begin{aligned} 1 \text { short } & \\ \quad \text { ton }= & 2000 \mathrm{lb} \text { (mass) } \\ & 907.185 \mathrm{~kg} \\ & 0.907185 \mathrm{t} \end{aligned}$ |
| $\begin{aligned} & \text { U } \\ & \text { ఝ } \\ & \text { O} \end{aligned}$ | N newton <br> kp kilopond <br> kgf (= kp) kilogram-force <br> p pond <br> gf $(=\mathrm{p})$ gram-force | $\begin{aligned} 1 \mathrm{~N}= & 0.10197 \mathrm{kp} \\ & 0.22481 \mathrm{lb} \\ & 3.5969 \mathrm{oz} \\ & 7.2330 \text { poundals } \end{aligned}$ |  |
|  |  | $\begin{aligned} \hline 1 \mathrm{kp}= & 9.80665 \mathrm{~N} \\ & 2.2046 \mathrm{lb} \\ & 35.274 \mathrm{oz} \end{aligned}$ |  |
|  |  | $\begin{aligned} 1 \mathrm{oz}= & 0.27801 \mathrm{~N} \\ & 2.83495 \times 10^{-2} \mathrm{kp} \\ & 28.3495 \mathrm{p} \end{aligned}$ |  |
|  |  | $\begin{aligned} & \hline 1 \mathrm{lb}= 16 \mathrm{oz} \\ & 4.4482 \mathrm{~N} \\ & 0.45359 \mathrm{kp} \\ & \hline \end{aligned}$ |  |
|  |  | 1 poundal $=0.138255 \mathrm{~N}$ |  |
|  | Pa pascal <br> kPa kilopascal <br> MPa megapascal <br> $\mathrm{kp} / \mathrm{cm}^{2}$ kilopond per square centimetre <br> at $\quad\left(=\mathrm{kp} / \mathrm{cm}^{2}\right)$ technical atmosphere <br> $\mathrm{kp} / \mathrm{mm}^{2}$ kilopond per square millimetre | $\begin{gathered} 1 \mathrm{~Pa}=1.0197 \times 10^{-5} \mathrm{at} \\ 1.45034 \times 10^{-4} \\ \mathrm{lb} / \mathrm{in}^{2} \end{gathered}$ | $\begin{array}{r} \hline 1 \mathrm{at}=1 \mathrm{kp} / \mathrm{cm}^{2} \\ 98.0665 \mathrm{kPa} \\ 14.223 \mathrm{lb} / \mathrm{in}^{2} \end{array}$ |
|  |  | $\begin{gathered} 1 \mathrm{kPa}=1.0197 \times 10^{-2} \mathrm{at} \\ 0.145034 \\ \mathrm{lb} / \mathrm{in}^{2} \\ \hline \end{gathered}$ | $\begin{array}{r} 1 \mathrm{kp} / \mathrm{mm}^{2}=100 \mathrm{at} \\ 9.80665 \mathrm{MPa} \\ 1422.3 \mathrm{lb} / \mathrm{in}^{2} \end{array}$ |
|  |  | $\begin{aligned} 1 \mathrm{MPa}= & 10.197 \mathrm{at} \\ & 145.034 \mathrm{lb} / \mathrm{in}^{2} \end{aligned}$ |  |
|  |  | $1 \mathrm{lb} / \mathrm{in}^{2}=6.895 \mathrm{kPa}=0.07031 \mathrm{at}$ |  |


|  | Nm newton metre kpm kilopond metre | $\begin{aligned} & 1 \mathrm{Nm}= 0.10197 \mathrm{kpm} \\ & 0.73756 \mathrm{lb}-\mathrm{ft} \\ & 8.85075 \mathrm{lb}-\mathrm{in} \\ & 141.612 \mathrm{oz}-\mathrm{in} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: |
|  |  | $\begin{aligned} & 1 \mathrm{kpm}= 9.80665 \mathrm{Nm} \\ & 1.3887 \times 10^{3} \mathrm{oz}-\mathrm{in} \end{aligned}$ |
|  |  | $\begin{aligned} & 1 \mathrm{oz}-\mathrm{in}= 7.0615 \times 10^{-3} \mathrm{Nm} \\ & 7.2008 \times 10^{-4} \mathrm{kpm} \end{aligned}$ |
|  |  | $\begin{aligned} 1 \mathrm{lb}-\mathrm{ft}= & 192 \mathrm{oz}-\mathrm{in}=12 \mathrm{lb}-\mathrm{in} \\ & 1.3558 \mathrm{Nm} \\ & 0.13825 \mathrm{kpm} \end{aligned}$ |
|  | $\mathrm{kg} \mathrm{m}^{2}$ kilogram-square metre $\mathrm{kg} \mathrm{cm}^{2}$ kilogram-square centimetre $\mathrm{kg} \mathrm{mm}^{2}$ kilogram-square millimetre <br> $\mathrm{g} \mathrm{cm}^{2}$ gram-square centimetre | $\begin{aligned} 1 \mathrm{~kg} \mathrm{~m}^{2}= & 10^{4} \mathrm{~kg} \mathrm{~cm}^{2}=10^{6} \mathrm{~kg} \mathrm{~mm}^{2}=10^{7} \mathrm{~g} \mathrm{~cm}^{2} \\ & 8.85075 \mathrm{Ib}-\mathrm{in}-\mathrm{s}^{2} \\ & 141.612 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2} \end{aligned}$ |
|  |  | $1 \mathrm{~kg} \mathrm{~cm}^{2}=0.01416 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}$ |
|  |  | $\begin{aligned} 1 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}= & 6.25 \times 10^{-2} \mathrm{lb}-\mathrm{in}-\mathrm{s}^{2} \\ & 7.06155 \times 10^{-3} \mathrm{~kg} \mathrm{~m}^{2} \\ & 70.6155 \mathrm{~kg} \mathrm{~cm}^{2} \end{aligned}$ |
|  | J joule <br> Ws wattsecond <br> kWh kilowatthour <br> kpm kilopond metre <br> kcal kilocalorie <br> (nutrition calorie) <br> cal calorie | $\begin{array}{ll} \hline 1 \mathrm{~J}= & 1 \mathrm{Ws} \\ & 2.7778 \times 10^{-7} \mathrm{kWh} \\ & 0.10197 \mathrm{kpm} \\ & 2.38846 \times 10^{-4} \mathrm{kcal} \\ & 9.4781 \times 10^{-4} \mathrm{Btu} \\ \hline \end{array}$ |
|  |  | $\begin{aligned} 1 \text { kcal }= & 10^{3} \mathrm{cal} \\ & 4186.8 \mathrm{~J} \\ & 1.1630 \times 10^{-3} \mathrm{kWh} \\ & 3.9683 \mathrm{Btu} \end{aligned}$ |
|  |  | $\begin{aligned} \hline 1 \mathrm{kWh}= & 3.6 \times 10^{6} \mathrm{~J} \\ & 859.845 \mathrm{kcal} \\ & 3.4121 \times 10^{3} \mathrm{Btu} \end{aligned}$ |
|  |  | $\begin{aligned} 1 \mathrm{Btu}= & 1055.06 \mathrm{~J} \\ & 2.9307 \times 10^{-4} \mathrm{kWh} \\ & 0.251997 \mathrm{kcal} \end{aligned}$ |


| $\begin{aligned} & \text { ~ } \\ & \text { 를 } \\ & \text { O} \end{aligned}$ | W watt <br> kW kilowatt <br> $\mathrm{kpm} / \mathrm{s}$ kilopond metre <br> per second <br> hp horse power (metric) |  | $\begin{aligned} 1 \mathrm{~W}= & 10^{-3} \mathrm{~kW} \\ & 0.10197 \mathrm{kpm} / \mathrm{s} \\ & 0.73756 \mathrm{lb}-\mathrm{ft} / \mathrm{s} \\ & 1.3596 \times 10^{-3} \mathrm{hp} \text { (metric) } \\ & 1.3410 \times 10^{-3} \mathrm{hp} \text { (British) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
|  |  |  | $\begin{aligned} 1 \mathrm{hp}(\text { British })= & 550 \mathrm{lb}-\mathrm{ft} / \mathrm{s} \\ & 745.7 \mathrm{~W} \\ & 1.0139 \mathrm{hp} \text { (metric) } \end{aligned}$ |
|  |  |  | $\begin{aligned} 1 \mathrm{hp} \text { (metric) }= & 75 \mathrm{kpm} / \mathrm{s} \\ & 735.5 \mathrm{~W} \\ & 0.98632 \mathrm{hp} \text { (British) } \end{aligned}$ |
|  | expressed as product of torque and angular velocity (speed of rotation) |  | $\mathrm{P}=\mathrm{T} \omega \quad$ [W; Nm, rad/s] |
|  |  |  | $\mathrm{P}=0.10472 \mathrm{~T} \mathrm{n} \mathrm{[W;} \mathrm{Nm}, \mathrm{rpm]}$ |
|  |  |  | $\mathrm{P}=7.3948 \times 10^{-4} \mathrm{~T}$ n [W; oz-in, rpm] |
|  |  |  | $\mathrm{P} \approx 10^{-6} \mathrm{~T} \mathrm{n} \quad$ (hp; oz-in, rpm] |
|  | $\begin{aligned} & \mathrm{K} \\ & { }^{\mathrm{o}} \mathrm{C} \end{aligned}$ | kelvin degree Celsius | magnitude of degrees $1 \mathrm{~K}=1^{\circ} \mathrm{C}=1.8^{\circ} \mathrm{F}$ |
|  |  |  | $0^{\circ} \mathrm{C}=273.15 \mathrm{~K}=32{ }^{\circ} \mathrm{F}$ |
|  |  |  | $\left.\Theta\left[{ }^{\circ} \mathrm{C}\right]=\left(\Theta{ }^{\circ} \mathrm{F}\right]-32\right) / 1.8$ |
|  |  |  | $\Theta\left[{ }^{\circ} \mathrm{F}\right]=1.8 \Theta\left[{ }^{\mathrm{O}} \mathrm{C}\right]+32$ |
|  |  |  | $\Theta[\mathrm{K}]=\Theta\left[{ }^{\circ} \mathrm{C}\right]+273.15$ |
|  | $\begin{array}{\|l} \mathrm{Wb} \\ \mathrm{Vs} \\ \mathrm{Mx} \end{array}$ | weber <br> voltsecond maxwell | $\begin{aligned} & \hline 1 \mathrm{~Wb}= 1 \mathrm{Vs} \\ & 10^{8} \mathrm{Mx} \\ & 10^{5} \text { kilolines } \\ & 10^{8} \text { lines } \\ & \hline \end{aligned}$ |
|  |  |  | 1 line $=1 \mathrm{Mx}=10^{-8} \mathrm{~Wb}$ |
|  |  | tesla gauss | $\begin{aligned} 1 \mathrm{~T}= & 1 \mathrm{~Wb} / \mathrm{m}^{2} \\ & 10^{4} \mathrm{G} \\ & 10^{4} \text { lines } / \mathrm{cm}^{2} \\ & 6.4516 \times 10^{4} \text { lines } / \mathrm{in}^{2} \end{aligned}$ |
|  |  |  | $\begin{aligned} 1 \text { line } / \mathrm{in}^{2}= & 0.1550 \mathrm{G} \\ & 1.550 \times 10^{-5} \mathrm{~T} \end{aligned}$ |
|  | $\begin{array}{\|l} \mathrm{A} / \mathrm{m} \\ \mathrm{Oe} \end{array}$ | ampere per metre oersted | $\begin{aligned} \hline 1 \mathrm{~A} / \mathrm{m}= & 10^{-2} \mathrm{~A} / \mathrm{cm} \\ & 1.2566 \times 10^{-2} \mathrm{Oe} \\ & 2.54 \times 10^{-2} \mathrm{~A} \text {-turn } / \mathrm{in} \end{aligned}$ |
|  |  |  | $\begin{aligned} 1 \mathrm{Oe}= & \text { 79.577 A/m } \\ & 2.0213 \mathrm{~A}-\mathrm{turn} / \mathrm{in} \end{aligned}$ |
|  |  |  | $\begin{array}{r} 1 \mathrm{~A} \text {-turn } / \mathrm{in}=39.370 \mathrm{~A} / \mathrm{m} \\ 0.49474 \mathrm{Oe} \end{array}$ |


| $\begin{aligned} & \infty \\ & \frac{\infty}{2} \\ & 6 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | TORQUE CONSTANT | $\begin{aligned} \hline 1 \mathrm{Nm} / \mathrm{A}= & 0.10197 \mathrm{kpm} / \mathrm{A} \\ & 141.612 \mathrm{oz}-\mathrm{in} / \mathrm{A} \end{aligned}$ |
| :---: | :---: | :---: |
|  |  | $\begin{aligned} 1 \mathrm{kpm} / \mathrm{A}= & 9.80665 \mathrm{Nm} / \mathrm{A} \\ & 1.3887 \times 10^{3} \mathrm{oz}-\mathrm{in} / \mathrm{A} \end{aligned}$ |
|  |  | $\begin{aligned} 1 \mathrm{oz}-\mathrm{in} / \mathrm{A}= & 7.06155 \times 10^{-3} \mathrm{Nm} / \mathrm{A} \\ & 7.2008 \times 10^{-4} \mathrm{kpm} / \mathrm{A} \end{aligned}$ |
|  | VOLTAGE | $1 \mathrm{~V} / \mathrm{krpm}=9.5493 \times 10^{-3} \mathrm{~V} / \mathrm{rad} \mathrm{s}^{-1}$ |
|  | CONSTANT | $1 \mathrm{~V} / \mathrm{rad} \mathrm{s}^{-1}=104.72 \mathrm{~V} / \mathrm{krpm}$ |
|  |  | $\begin{aligned} 1 \mathrm{Nm} / \mathrm{rad} \mathrm{~s}^{-1}= & 141.612 \mathrm{oz}-\mathrm{in} / \mathrm{rad} \mathrm{~s}^{-1} \\ & 104.72 \mathrm{Nm} / \mathrm{krpm} \end{aligned}$ |
|  | $\begin{gathered} \text { DAMPING } \\ \text { CONSTANT } \end{gathered}$ | $\begin{aligned} 1 \mathrm{Nm} / \mathrm{krpm}= & 141.612 \mathrm{oz} \text {-in } / \mathrm{krpm} \\ & 9.5493 \times 10^{-3} \mathrm{Nm} / \mathrm{rad} \mathrm{~s}^{-1} \end{aligned}$ |
|  | viscous DAMPING | $\begin{aligned} 1 \mathrm{oz}-\mathrm{in} / \mathrm{rad} \mathrm{~s}^{-1}= & 7.0615 \times 10^{-3} \mathrm{Nm} / \mathrm{rad} \mathrm{~s}^{-1} \\ & 104.72 \mathrm{oz}-\mathrm{in} / \mathrm{krpm} \end{aligned}$ |
|  | FACTOR | $\begin{aligned} 1 \mathrm{oz}-\mathrm{in} / \mathrm{krpm}= & 7.0615 \times 10^{-3} \mathrm{Nm} / \mathrm{krpm} \\ & 9.5493 \times 10^{-3} \mathrm{oz}-\mathrm{in} / \mathrm{rad} \mathrm{~s}^{-1} \end{aligned}$ |
|  | SPEED REGULATION CONSTANT | $\begin{array}{ll} 1 \mathrm{rpm} / \mathrm{oz}-\mathrm{in} & =0.14161 \mathrm{krpm} / \mathrm{Nm} \\ 1 \mathrm{krpm} / \mathrm{Nm} & =7.0615 \mathrm{rpm} / \mathrm{oz}-\mathrm{in} \end{array}$ |



Example shown: $100 \mathrm{~W}=1 \mathrm{Nm} \times 100 \mathrm{rad} / \mathrm{s}$

|  | kg m ${ }^{2}$ | $\mathrm{g} \mathrm{cm}{ }^{2}$ | oz-in-s ${ }^{2}$ | $\mathrm{lb}-\mathrm{in}-\mathrm{s}^{2}$ | oz*in ${ }^{\text {2 }}$ | $1 b^{*}$-in ${ }^{2}$ | $1 \mathrm{lb}^{*}-\mathrm{ft}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{kg} \mathrm{m}{ }^{2}$ | 1 | $10^{7}$ | 141.612 | 8.85075 | $\begin{aligned} & 5.46745 \\ & \times 10^{4} \end{aligned}$ | $\begin{aligned} & 3.41716 \\ & \times 10^{3} \end{aligned}$ | 23.7303 |
| $\mathrm{g} \mathrm{cm}{ }^{2}$ | $10^{-7}$ | 1 | $\begin{aligned} & 1.41612 \\ & \times 10^{-5} \end{aligned}$ | $\begin{aligned} & 8.85075 \\ & \times 10^{-7} \end{aligned}$ | $\begin{gathered} 5.46745 \\ \times 10^{-3} \end{gathered}$ | $\begin{aligned} & 3.41716 \\ & \times 10^{-4} \end{aligned}$ | $\begin{gathered} 2.37303 \\ \times 10^{-6} \end{gathered}$ |
| oz-in-s ${ }^{2}$ | $\begin{gathered} 7.06155 \\ \times 10^{-3} \end{gathered}$ | $\begin{gathered} 7.06155 \\ \times 10^{4} \end{gathered}$ | 1 | $\begin{aligned} & 6.25 \\ & \times 10^{-2} \end{aligned}$ | 386.088 | 24.1305 | 0.167573 |
| $\mathrm{lb}-\mathrm{in}-\mathrm{s}^{2}$ | 0.112985 | $\begin{aligned} & 1.12985 \\ & \times 10^{6} \end{aligned}$ | 16 | 1 | $\begin{aligned} & 6.17741 \\ & \times 10^{3} \end{aligned}$ | 386.088 | 2.68117 |
| oz**in ${ }^{2}$ | $\begin{aligned} & 1.82901 \\ & \times 10^{-5} \end{aligned}$ | 182.901 | $\begin{aligned} & 2.59008 \\ & \times 10^{-3} \end{aligned}$ | $\begin{aligned} & 1.61880 \\ & \times 10^{-4} \end{aligned}$ | 1 | $\begin{aligned} & 6.25 \\ & \times 10^{-2} \end{aligned}$ | $\begin{aligned} & 4.34028 \\ & \times 10^{-4} \end{aligned}$ |
| $1 b^{*}-\mathrm{in}^{2}$ | $\begin{gathered} 2.92641 \\ \times 10^{-4} \end{gathered}$ | $\begin{aligned} & 2.92641 \\ & \times 10^{3} \end{aligned}$ | $\begin{gathered} 4.14413 \\ \times 10^{-2} \end{gathered}$ | $\begin{aligned} & 2.59008 \\ & \times 10^{-3} \end{aligned}$ | 16 | 1 | $\begin{gathered} 6.94444 \\ \times 10^{-3} \end{gathered}$ |
| $1 b^{*}-\mathrm{ft}^{2}$ | $\begin{gathered} 4.21403 \\ \times 10^{-2} \end{gathered}$ | $\begin{aligned} & 4.21403 \\ & \times 10^{5} \end{aligned}$ | 5.96755 | 0.372972 | 2304 | 144 | 1 |

*units of mass
Tab. A.1. Moment of inertia conversion factors
To convert from $A$ to $B$, multiply by entry in table.
Example: $1 \mathrm{~kg} \mathrm{~m}^{2}=141.612 \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}$

|  | Nm | kpm (kg*-m) | $\mathrm{g}^{*}-\mathrm{cm}$ | oz-in | lb-in | $\mathrm{lb}-\mathrm{ft}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nm | 1 | 0.101972 | $\begin{aligned} & 1.01972 \\ & \times 10^{4} \\ & \hline \end{aligned}$ | 141.612 | 8.85075 | 0.737562 |
| $\begin{aligned} & \mathrm{kpm} \\ & \left(\mathrm{~kg}^{*}-\mathrm{m}\right) \end{aligned}$ | 9.80665 | 1 | $10^{5}$ | $\begin{aligned} & 1.38874 \\ & \times 10^{3} \end{aligned}$ | 86.7962 | 7.23301 |
| $\mathrm{g}^{*}-\mathrm{cm}$ | $\begin{aligned} & 9.80665 \\ & \times 10^{-5} \end{aligned}$ | $10^{-5}$ | 1 | $\begin{aligned} & 1.38874 \\ & \times 10^{-2} \end{aligned}$ | $\begin{aligned} & 8.67962 \\ & \times 10^{-4} \end{aligned}$ | $\begin{aligned} & 7.23301 \\ & \times 10^{-5} \end{aligned}$ |
| oz-in | $\begin{aligned} & 7.06155 \\ & \times 10^{-3} \end{aligned}$ | $\begin{aligned} & 7.20077 \\ & \times 10^{-4} \end{aligned}$ | 72.0077 | 1 | $6.25 \times 10^{-2}$ | $\begin{aligned} & 5.20833 \\ & \times 10^{-3} \end{aligned}$ |
| lb -in | 0.112985 | $\begin{aligned} & 1.15212 \\ & \times 10^{-2} \end{aligned}$ | $\begin{aligned} & 1.15212 \\ & \times 10^{3} \end{aligned}$ | 16 | 1 | $\begin{aligned} & 8.33333 \\ & \times 10^{-2} \end{aligned}$ |
| lb -ft | 1.35582 | 0.138255 | $\begin{aligned} & 1.38255 \\ & \times 10^{4} \end{aligned}$ | 192 | 12 | 1 |

*units of force
Tab. A.2. Torque conversion factors
To convert from $A$ to $B$, multiply by entry in table.
Example: $1 \mathrm{oz}-\mathrm{in}=7.06155 \times 10^{-3} \mathrm{Nm}$

EXPLANATION TO TAB. A. 1 AND A. 2

The British system of units has two forms:
a) Technical (gravitational) system
which is more common in engineering (and is also used in this book) and which is based on the following base units:

1 s - for time
1 ft - for length
1 lb (force) - for force
The derived unit of mass is then 1 s/ug, defined as a mass which is accelerated by $1 \mathrm{ft} / \mathrm{s}^{2}$ upon application of a force of 1 lb .
b) Physical (absolute) system
which is based on the following base units:

1 s - for time
1 ft - for length
1 lb - for mass

The derived unit of force is then 1 poundal, a force which accelerates the mass of 1 lb by $1 \mathrm{ft} / \mathrm{s}^{2}$.

The basic equation which relates acceleration torque $\mathbf{T}_{\mathbf{a}}$, moment of inertia $\mathbf{J}$ and angular acceleration $a$ has therefore two forms, depending on the system of units used:
a) in the British technical system

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{J} a \quad\left[\mathrm{oz}-\mathrm{in} ; \mathrm{oz}-\mathrm{in}-\mathrm{s}^{2}, \mathrm{rad} / \mathrm{s}^{2}\right]
$$

where $o z$ is the unit of force, and
b) in the British physical system

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{J} a \quad\left[\text { poundal-ft; } \mathrm{lb}-\mathrm{ft}^{2}, \mathrm{rad} / \mathrm{s}^{2}\right]
$$

where $l b$ is the unit of mass.

The unit of torque, poundal-ft, is not common and is not used in engineering practice but the moment of inertia, $\mathbf{J}$, is sometimes given in units of $l b-f t^{2}$ (or, in related units, as $l b-i n^{2}$ or $\left.o z-i n^{2}\right)$ - even in technical literature. Therefore, in Tab. A. 1 all necessary conversion factors are introduced.

In British units of torque, which are introduced in Tab. A.2, the $o z$ and $l b$ are units of force, according to technical form of the British system.

As for the SI (metric) units in both tables, see the Appendix A.1.

|  |  | OUTPUT POWER [W] |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10 | 20 | 30 | 40 | 50 | 60 | 80 | 100 | 120 | 140 | 160 | 200 | 250 | 300 | 400 | 500 | 600 | 800 | 1000 | 1200 |
|  | 200 | 477 | 955 | 1.43 | 1.91 | 2.39 | 2.86 | 3.82 | 4.77 | 5.73 | 6.68 | 7.64 | 9.55 | 11.9 | 14.3 | 19.1 | 23.9 | 28.6 | 38.2 | 47.7 | 57.3 |
|  | 400 | 239 | 477 | 716 | 955 | 1.19 | 1.43 | 1.91 | 2.39 | 2.86 | 3.34 | 3.82 | 4.77 | 5.97 | 7.16 | 9.55 | 11.9 | 14.3 | 19.1 | 23.9 | 28.6 |
|  | 600 | 159 | 318 | 477 | 637 | 796 | 955 | 1.27 | 1.59 | 1.91 | 2.23 | 2.55 | 3.18 | 3.98 | 4.77 | 6.37 | 7.96 | 9.55 | 12.7 | 15.9 | 19.1 |
|  | 800 | 119 | 239 | 358 | 477 | 597 | 716 | 955 | 1.19 | 1.43 | 1.67 | 1.91 | 2.39 | 2.98 | 3.58 | 4.77 | 5.97 | 7.16 | 9.55 | 11.9 | 14.3 |
|  | 1000 | 95.5 | 191 | 286 | 382 | 477 | 573 | 764 | 955 | 1.15 | 1.34 | 1.53 | 1.91 | 2.39 | 2.86 | 3.82 | 4.77 | 5.73 | 7.64 | 9.55 | 11.5 |
|  | 1200 | 79.6 | 159 | 239 | 318 | 398 | 477 | 637 | 796 | 955 | 1.11 | 1.27 | 1.59 | 1.99 | 2.39 | 3.18 | 3.98 | 4.77 | 6.37 | 7.96 | 9.55 |
|  | 1400 | 68.2 | 136 | 205 | 273 | 341 | 409 | 546 | 682 | 819 | 955 | 1.09 | 1.36 | 1.71 | 2.05 | 2.73 | 3.41 | 4.09 | 5.46 | 6.82 | 8.19 |
|  | 1600 | 59.7 | 119 | 179 | 239 | 298 | 358 | 477 | 597 | 716 | 836 | 955 | 1.19 | 1.49 | 1.79 | 2.39 | 2.98 | 3.58 | 4.77 | 5.97 | 7.16 |
|  | 1800 | 53.1 | 106 | 159 | 212 | 265 | 318 | 424 | 531 | 637 | 743 | 849 | 1.06 | 1.33 | 1.59 | 2.12 | 2.65 | 3.18 | 4.24 | 5.31 | 6.37 |
|  | 2000 | 47.7 | 95.5 | 143 | 191 | 239 | 286 | 382 | 477 | 573 | 668 | 764 | 955 | 1.19 | 1.43 | 1.91 | 2.39 | 2.86 | 3.82 | 4.77 | 5.73 |
| $\bar{E}$ | 2200 | 43.4 | 86.8 | 130 | 174 | 217 | 260 | 347 | 434 | 521 | 608 | 694 | 868 | 1.09 | 1.30 | 1.74 | 2.17 | 2.60 | 3.47 | 4.34 | 5.21 |
| $\stackrel{\square}{2}$ | 2400 | 39.8 | 79.6 | 119 | 159 | 199 | 239 | 318 | 398 | 477 | 557 | 637 | 796 | 995 | 1.19 | 1.59 | 1.99 | 2.39 | 3.18 | 3.98 | 4.77 |
| 岂 | 2700 | 35.4 | 70.7 | 106 | 141 | 177 | 212 | 283 | 354 | 424 | 495 | 566 | 707 | 884 | 1.06 | 1.41 | 1.77 | 2.12 | 2.83 | 3.54 | 4.24 |
| ¢ | 3000 | 31.8 | 63.7 | 95.5 | 127 | 159 | 191 | 255 | 318 | 382 | 446 | 509 | 637 | 796 | 955 | 1.27 | 1.59 | 1.91 | 2.55 | 3.18 | 3.82 |
|  | 3500 | 27.3 | 54.6 | 81.9 | 109 | 136 | 164 | 218 | 273 | 327 | 382 | 437 | 546 | 682 | 819 | 1.09 | 1.36 | 1.64 | 2.18 | 2.73 | 3.27 |
|  | 4000 | 23.9 | 47.7 | 71.6 | 95.5 | 119 | 143 | 191 | 239 | 286 | 334 | 382 | 477 | 597 | 716 | 955 | 1.19 | 1.43 | 1.91 | 2.39 | 2.86 |
|  | 4500 | 21.2 | 42.4 | 63.7 | 84.9 | 106 | 127 | 170 | 212 | 255 | 297 | 340 | 424 | 531 | 637 | 849 | 1.06 | 1.27 | 1.70 | 2.12 | 2.55 |
|  | 5000 | 19.1 | 38.2 | 57.3 | 76.4 | 95.5 | 115 | 153 | 191 | 229 | 267 | 306 | 382 | 477 | 573 | 764 | 955 | 1.15 | 1.53 | 1.91 | 2.29 |
|  | 5500 | 17.4 | 34.7 | 52.1 | 69.4 | 86.8 | 104 | 139 | 174 | 208 | 243 | 278 | 347 | 434 | 521 | 694 | 868 | 1.04 | 1.39 | 1.74 | 2.08 |
|  | 6000 | 15.9 | 31.8 | 47.7 | 63.7 | 79.6 | 95.5 | 127 | 159 | 191 | 223 | 255 | 318 | 398 | 477 | 637 | 796 | 955 | 1.27 | 1.59 | 1.91 |
|  | 6500 | 14.7 | 29.4 | 44.1 | 58.8 | 73.5 | 88.1 | 118 | 147 | 176 | 206 | 235 | 294 | 367 | 441 | 588 | 735 | 881 | 1.18 | 1.47 | 1.76 |
|  | 7000 | 13.6 | 27.3 | 40.9 | 54.6 | 68.2 | 81.9 | 109 | 136 | 164 | 191 | 218 | 273 | 341 | 409 | 546 | 682 | 819 | 1.09 | 1.36 | 1.64 |
|  | 7500 | 12.7 | 25.5 | 38.2 | 50.9 | 63.7 | 76.4 | 102 | 127 | 153 | 178 | 204 | 255 | 318 | 382 | 509 | 637 | 764 | 1.02 | 1.27 | 1.53 |
|  | 8000 | 11.9 | 23.9 | 35.8 | 47.7 | 59.7 | 71.6 | 95.5 | 119 | 143 | 167 | 191 | 239 | 298 | 358 | 477 | 597 | 716 | 955 | 1.19 | 1.43 |
|  |  |  |  |  |  |  |  |  | TOR | UE $\uparrow$ |  |  |  |  |  |  |  | [mNm |  | [ Nm |  |

Tab. A.3. Relationship of the speed, torque and output power of a motor (metric units)

EASY REMINDER: 100W correspond approximately to 1 Nm at 1000 rpm

|  |  | OUTPUT POWER [hp] |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1/75 | 1/60 | 1/50 | 1/40 | 1/35 | 1/30 | 1/25 | 1/20 | 1/15 | 1/12 | 1/10 | 1/8 | 1/6 | 1/4 | 1/3 | 1/2 | 5/8 | 3/4 | 1 | 3/2 |
|  | $\begin{array}{r} 850 \\ 1140 \\ 1200 \\ \hline \end{array}$ | 11.2 | 14 | 16.8 | 21 | 24 | 28 | 33.7 | 42.1 | 56.1 | 70.1 | 84.1 | $\begin{aligned} & 148 \\ & 110 \\ & 105 \end{aligned}$ | $\begin{aligned} & 198 \\ & 149 \\ & 140 \end{aligned}$ | $\begin{aligned} & 296 \\ & 220 \\ & 210 \end{aligned}$ | $\begin{aligned} & 396 \\ & 294 \\ & 269 \end{aligned}$ | $\begin{aligned} & 592 \\ & 440 \\ & 420 \end{aligned}$ | 740 550 525 | $\begin{aligned} & 883 \\ & 660 \\ & 630 \end{aligned}$ | 1184 880 840 | $\begin{aligned} & 1776 \\ & 1320 \\ & 1260 \end{aligned}$ |
|  | 1425 | 9.4 | 11.8 | 14.2 | 17.7 | 20.2 | 23.6 | 28.4 | 35.5 | 47.3 | 59 | 70.8 | 90 | 115 | 180 | 235 | 360 | 450 | 540 | 720 | 1080 |
|  | 1600 | 8.4 | 10.5 | 12.6 | 15.8 | 18.0 | 21 | 25.2 | 31.5 | 42.1 | 52.6 | 63.1 | 78.8 | 105 | 158 | 210 | 316 | 395 | 474 | 632 | 943 |
|  | 1725 | 7.8 | 9.8 | 11.7 | 14.6 | 16.7 | 19.5 | 23.4 | 29.3 | 39 | 48.8 | 58.5 | 72.8 | 97 | 146 | 194 | 291 | 364 | 437 | 582 | 874 |
|  | 1800 | 7.5 | 9.4 | 11.2 | 14 | 16 | 18.7 | 22.4 | 28 | 37.4 | 46.8 | 56.1 | 69.9 | 91 | 140 | 182 | 280 | 350 | 418 | 559 | 836 |
|  | 2000 | 6.7 | 8.4 | 10.1 | 12.6 | 14.4 | 16.8 | 20.2 | 25.3 | 33.6 | 42.1 | 50.5 | 63.2 | 84.2 | 126 | 168 | 252 | 315 | 378 | 504 | 756 |
|  | 2200 | 6.1 | 7.6 | 9.2 | 11.5 | 13.i | 15.3 | 18.4 | 23 | 30.6 | 38.2 | 45.9 | 57.3 | 76.4 | 115 | 153 | 230 | 287 | 335 | 460 | 670 |
|  | 2400 | 5.6 | 7.0 | 8.4 | 10.5 | 12.0 | 14.0 | 16.8 | 21.0 | 28.0 | 36.0 | 42.1 | 52.6 | 70.1 | 105 | 140 | 210 | 263 | 315 | 420 | 630 |
| E | 2600 | 5.2 | 6.5 | 7.8 | 9.7 | 11.1 | 12.9 | 15.5 | 19.4 | 25.9 | 32.4 | 38.8 | 48.6 | 64.7 | 97.2 | 129 | 194 | 243 | 291 | 389 | 582 |
| 은 | 2800 | 4.8 | 6.0 | 7.2 | 9.0 | 10.8 | 12.0 | 14.4 | 18.0 | 24.0 | 30.0 | 36.0 | 45.0 | 60 | 90 | 120 | 180 | 225 | 270 | 360 | 540 |
| 号 | 3000 | 4.5 | 5.6 | 6.7 | 8.4 | 9.6 | 11.2 | 13.5 | 16.8 | 22.4 | 28.1 | 33.6 | 42.1 | 56.1 | 84.1 | 112 | 168 | 210 | 252 | 336 | 504 |
| ¢ | 3200 | 4.2 | 5.3 | 6.3 | 7.9 | 9.0 | 10.5 | 12.6 | 15.8 | 21.0 | 26.3 | 31.5 | 39.4 | 52.6 | 78.8 | 105 | 158 | 197 | 236 | 315 | 472 |
|  | 3450 | 3.9 | 4.9 | 5.8 | 7.3 | 8.4 | 9.8 | 11.7 | 14.6 | 19.5 | 24.4 | 29.3 | 36.4 | 48.8 | 72.8 | 97.7 | 146 | 182 | 218 | 292 | 436 |
|  | 3600 | 3.7 | 4.7 | 5.6 | 7.0 | 8.0 | 9.4 | 11.2 | 14.0 | 18.7 | 23.4 | 28.1 | 35.1 | 46.8 | 70.1 | 93.6 | 140 | 175 | 210 | 280 | 420 |
|  | 3800 | 3.5 | 4.4 | 5.3 | 6.6 | 7.6 | 8.8 | 10.6 | 13.3 | 17.7 | 22 | 26.6 | 33.2 | 44.3 | 66.4 | 88.6 | 133 | 166 | 199 | 265 | 399 |
|  | 4000 | 3.4 | 4.2 | 5.0 | 6.3 | 7.2 | 8.4 | 10.1 | 12.6 | 16.8 | 21 | 25.3 | 31.5 | 42.1 | 63.2 | 84.2 | 126 | 158 | 189 | 252 | 378 |
|  | 4500 | 3 | 3.7 | 4.5 | 5.6 | 6.4 | 7.5 | 9 | 11.2 | 15 | 18.7 | 22.4 | 28 | 37.4 | 56 | 74.8 | 112 | 140 | 168 | 224 | 336 |
|  | 5000 | 2.7 | 3.4 | 4 | 5 | 5.8 | 6.7 | 8.1 | 10.1 | 13.5 | 16.8 | 20.2 | 25.2 | 33.7 | 50.5 | 67.3 | 101 | 126 | 152 | 202 | 303 |
|  | 5500 | 2.45 | 3.1 | 3.7 | 4.6 | 5.2 | 6.1 | 7.3 | 9.2 | 12.2 | 15.3 | 18.4 | 22.9 | 30.6 | 45.9 | 61.2 | 91 | 114 | 137 | 182 | 273 |
|  | 6000 | 2.2 | 2.8 | 3.4 | 4.2 | 4.8 | 5.6 | 6.7 | 8.4 | 11.2 | 14 | 16.8 | 21 | 28 | 42 | 56.1 | 84 | 105 | 126 | 168 | 252 |
|  | 6500 | 2.1 | 2.6 | 3.1 | 3.9 | 4.4 | 5.2 | 6.2 | 7.8 | 10.4 | 13.0 | 15.6 | 19.4 | 25.9 | 38.8 | 51.8 | 77 | 96 | 116 | 154 | 232 |
|  | 7000 | 1.9 | 2.4 | 2.9 | 3.6 | 4.1 | 4.8 | 5.8 | 7.2 | 9.6 | 12 | 14.4 | 18 | 24 | 36 | 48 | 72 | 90 | 108 | 144 | 216 |
|  |  |  |  |  |  |  |  |  |  | RQUE | [oz-in |  |  |  |  |  |  |  |  |  |  |

Tab. A.4. Relationship of the speed, torque and output power of a motor (British units)

## Electro-Craft Corporation Products

Electro-Craft began the development of high quality DC motors, speed controls and servo systems in 1960. Today, we manufacture a broad range of speed, position and motion control devices for all types of industry applications.

In the following pages, you will find a catalog of the wide variety of products designed and manufactured by ElectroCraft. The products detailed in this section are standard Electro-Craft designs and are manufactured regularly in one or more of our four factories.

They are the products of over 20 years of precision motion control experience. Experience not simply in motor design, but in total motion, position and speed control devices. Unlike some suppliers, all Electro-Craft products are developed by
our engineering staff and manufactured, under .rigid quality control, in our own facilities. That means that we have an in-depth understanding of both the component parts and of complete systems. All products which may be combined are cross-referenced accordingly and the individual products are detailed with charts demonstrating their performance both in system and alone.

We have found that the particular speed, position and motion control criteria of our customers can require performance and hardware specifications not available on our standard products. If, in attempting to match your design criteria to a standard Electro-Craft design, you cannot find precisely what you are looking for, all of the products contained in this section may be modified to suit your needs.

## DC Servomotors

Automation and the need for precisely controlled speed and torque have brought about a renewed interest in the use of DC motors. Over the years, Electro-Craft has developed a broad line of sophisticated servomotors and motor-tachometers each incorporating the most advanced design technology available - for a wide range of applications.

In the following section, you will find detailed specifications and performance data on the complete Electro-Craft motor line. In the main, these products may be ordered for immediate application simply by specifying the model number and quantity needed.

Should you require a motor whose characteristics vary slightly from an Electro-Craft standard design, our Applications Engineering Department can help you with a
modified design. All standard products both moving coil and iron core motors may be equipped with modified windings, filters, custom casting and other minor changes you need to fit our product to yours. In some cases, your application requirements may require a unique product, a custom design. Electro-Craft engineers draw upon two decades of design experience and can equip you with precisely the motor application demands.

Electro-Craft has a complete line of standard DC servomotors and motor-tachometers. They are available from stock at Electro-Craft or any of our eight stocking distributors throughout Europe and Japan.

## E-400

The E-400 is a high performance permanent magnet servomotor which combines many of the characteristics of a "torque" motor with those of a low-inertia moving coil type. The motor has excellent regulation and low inherent ripple torque.


## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color


|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 6.07 | 4.49 | . 64 | 1.60 | 40 |  |
| B | 7.59 | 5.61 | 1.02 | 2.32 | 31 | Peak torque (oz. in.) 240 |
| C | 9.62 | 7.11 | 1.63 | 3.86 | 25 | Max. rated continuous torque at stall (oz. in.) 32.0 Armature inertia (oz. in. sec. ${ }^{2} .0033$ |
| D | 12.14 | 8.98 | 2.58 | 5.28 | 20 | Damping factor (oz. in./KRPM) 1.0 |
| E | 15.17 | 11.22 | 4.10 | 8.17 | 16 | Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.0 (Armature-to-ambient) |
| F | 19.22 | 14.21 | 6.45 | 12.64 | 12.5 | Electrical time constant (msec) 1.90 |
| G | 24.03 | 17.77 | 10.30 | 19.95 | 10 | Mechanical time constant (msec) 8.2 Motor weight 3 lb .15 oz . |
| H | 30.35 | 22.44 | 15.43 | 31.60 | 8 | Static Friction Torque 3.0 oz. in. |



## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Rated brush life 5,000 hours @ 1 KRPM Maximum friction torque 7 oz . in. Maximum shaft radial load 30 lb .,

1 in. from front bearing continuous

## CONSTRUCTION FEATURES

## Winding insulation Class $F$

ABEC Class 1 ball bearings
PVC lead insulation
Totally enclosed housing
Available option:
Integrally mounted optical encoder (see pp. 40-45) NEMA 42CZ mounting face

$\mathrm{V}=$ Terminal Voltage
The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $10^{\prime \prime} \times 10^{\prime \prime} \times 1 / 4^{\prime \prime}$ Heat Sink.

## E-508/E-512

The E-508/E-512 series is a moderate cost of FHP D.C. permanent magnet motors designed for a broad range of industrial applications, such as tape transport reel drives or position control systems. Reliability and long life have been designed into these motors by the use of corrosionresistant shaft and housing materials and permanently lubricated ball bearings. Pictured at the right is the E-508 and the longer E-512.


## HOW TO ORDER

To order, you must supply sales model number.
Order codes are shown below in color


|  | $\begin{aligned} & \text { 읃 } \\ & \vdots \\ & \frac{E}{3} \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 508 | A | 2.60 | 1.92 | . 85 | 1.13 | 24 |
|  | B | 3.28 | 2.42 | 1.34 | 1.57 | 19 |
|  | C | 4.13 | 3.05 | 2.12 | 2.28 | 15 |
|  | D | 5.20 | 3.84 | 3.36 | 3.40 | 12 |
| 510 | A | 5.88 | 4.35 | 1.24 | 3.39 | 24 |
|  | B | 7.41 | 5.48 | 1.99 | 4.92 | 19 |
|  | C | 9.34 | 6.91 | 3.15 | 7.31 | 15 |
|  | D | 11.76 | 8.70 | 4.99 | 11.10 | 12 |
| 512 | A | 8.68 | 6.42 | 1.64 | 5.65 | 24 |
|  | B | 10.94 | 8.09 | 2.64 | 8.41 | 19 |
|  | C | 13.78 | 10.19 | 4.18 | 12.68 | 15 |
|  | D | 17.36 | 12.84 | 6.63 | 19.47 | 12 |

MODEL 508
Peak torque (oz. in.) 62
Max. rated continuous torque at stall (oz. in.) 10
Armature inertia (oz. in. sec. ${ }^{2}$ ) . 0015
Damping factor (oz. in./KRPM) . 1
Thermal resistance ( ${ }^{\circ} \mathrm{C}$./watt) 6.84
(Armature-to-ambient)
Electrical time constant (msec) . 90
Mechanical time constant (msec) 29
Motor weight 1 lb .2 oz.
Static friction torque 3 oz . in.
MODEL 510
Peak torque 9(oz. in.) 140
Max. rated continuous torque at stall (oz. in.) 19
Armature inertia (oz. in. sec. ${ }^{2}$ ) . 0038
Damping factor (oz. in./KRPM) . 1
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 5.0
(Armature-to-ambient)
Electrical time constant (msec) 2.06
Mechanical time constant (msec) 24.7
Motor weight 1 lb .8 oz .
Static friction torque 3 oz . in.

## MODEL 512

Peak torque (oz. in.) 208
Max. rated continuous torque at stall (oz. in.) 25
Armature inertia (oz. in. $\mathrm{sec}^{2}$ ) . 0062
Damping factor (oz. in./KRPM) . 2
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.18
(Armature-to-ambient)
Electrical time constant (msec) 2.77
Mechanical time constant (msec) 24
Motor weight 2 lb .8 oz .
Static friction torque 3 oz . in.


SHAFT RUNOUT. 001 IN./IN.

## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$
Maximum no load speed 6 KRPM
Rated brush life 5,000 hours @ 1 KRPM
Maximum friction torque 3 oz . in.
Maximum shaft radial load 10 lbs .,
0.5 in . from front bearing continuous
for a minimum of 1,000 hours

## CONSTRUCTION FEATURES

Winding insulation Class F
ABEC Class 1 ball bearings
PVC lead insulation
Totally enclosed housing
Available options:
Integrally mounted tachometer (see pp. 60-61)
Integrally mounted optical encoder (see pp. 40-45)

SPEED TORQUE CURVES
MODEL 510


TORQUE (OZ. IN.) $\quad V T=$ Terminal Voltage

The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $10^{\prime \prime} \times 10^{\prime \prime} \times 1 / 4^{\prime \prime}$ heat sink.

## E-530/E-532

The E-530/E-532 series incorporates the inherent low ripple, low inertia, and long life characteristics of a moving coil tachometer in an economical motor-tach package.

## HOW TO ORDER

To order you must supply sales model number.
Order codes are shown below in color.


|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 530 | A | 10.02 | 7.41 | 1.24 | 3.39 | 24 |
|  | B | 12.63 | 9.34 | 1.99 | 4.92 | 19 |
|  | C | 15.91 | 11.76 | 3.15 | 7.31 | 15 |
|  | D | 20.04 | 14.82 | 4.99 | 11.10 | 12 |
| 531 | A | 11.77 | 8.70 | 1.44 | 4.53 | 24 |
|  | B | 14.83 | 10.96 | 2.31 | 6.67 | 19 |
|  | C | 18.69 | 13.81 | 3.66 | 9.99 | 15 |
|  | D | 23.54 | 17.40 | 5.81 | 15.28 | 12 |
| 532 | A | 14.81 | 10.95 | 1.64 | 5.65 | 24 |
|  | B | 18.66 | 13.80 | 2.64 | 8.41 | 19 |
|  | C | 23.51 | 17.38 | 4.18 | 12.68 | 15 |
|  | D | 29.63 | 21.90 | 6.63 | 19.47 | 12 |

## MODEL E-530

Peak torque (oz. in.) 240
Max. rated continunus torque at stall (oz. in.) 29
Armature intertia (including tach) (oz. in. $\mathrm{sec}^{2}$ ) . 0039
Damping factor (oz. in./KRPM) . 1
Thermal resistance ( ${ }^{\circ} \mathrm{C} /$ watt) 5.0
(Armature-to-ambient)
Electrical time constant (msec) 2.06
Mechanical time constant (msec) 8.8
Motor weight 2 lb .1 oz .
Static Friction Torque 3 oz . in.

## MODEL E-531

Peak torque (oz. in.) 282
Max. rated continuous torque at stall (oz. in.) 38
Armature inertia (including tach) (oz. in. sec. ${ }^{2}$ ) . 0051
Damping factor (oz. in./KRPM) . 15
Thermal resistance ( ${ }^{\circ} \mathrm{C}$./watt) 4.60
(Armature-to-ambient)
Electrical time constant (msec) 2.46
Mechanical time constant (msec) 9.4
Motor weight 2 lb .8 oz .
Static Friction Torque 3 oz . in.

## MODEL E-532

Peak torque (oz. in.) 355
Max. rated continuous torque at stall (oz. in.) 50
Armature inertia (including tach) (oz. in. $\mathrm{sec}^{2}$ ) . 0063
Damping factor (oz. in./KRPM) . 2
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt $) 4.18$
(Armature-to-ambient)
Electrical time constant (msec) 2.77
Motor weight 3 lb .0 oz .
Static Friction Torque 3 oz . in.


## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Maximum no load speed 6 KRPM
Rated brush life 5,000 hours @ 1 KRPM Maximum friction torque 3 oz . in. Maximum shaft radial load 10 lbs.
0.5 in . from front bearing continuous for
a minimum of 1,000 hours

## CONSTRUCTION FEATURES

Winding insulation Class F ABEC Class 1 ball bearings PVC lead insulation
Totally enclosed housing Available options:
Integrally mounted tachometer (see pp. 58-59) Integrally mounted optical encoder (see pp. 40-45)

TACHOMETER WINDING VARIATIONS

Winding Constants

| Output Voltage Gradient <br> (Volts/KRPM) | 3.0 | 2.5 |
| :--- | :--- | :--- |
| Armature Resistance (ohms) | 160 | 160 |
| Armature Inductance (mhy) 4 <br> Optimum Load Impedance  | 10,000 | 4 |
| 10,000 |  |  | (ohms)

(See pp. 54-61 for more detailed tachometer information)

SPEED TORQUE CURVES

MODEL E-530


MODEL E-531


RMS TORQUE (OZ-IN)

The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $10^{\prime \prime} \times 10^{\prime \prime} \times 1 /{ }^{\prime \prime}$ heat sink.

## E-540/E-542

The E-540/E-542 series of FHP D.C. permanent magnet motors is derived from the E-508/E-512 series with increased output performance resulting from an enhanced magnetic circuit. This increased performance has substantially added to the cost effectiveness and versatility of these motors used in a variety of applications in professional and industrial products.


## HOW TO ORDER

To order, you must supply sales model number.
Order codes are shown below in color


|  | 읗 <br> $\vdots$ <br> 3 |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{c} \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 540 | A | 10.02 | 7.41 | 1.24 | 3.39 | 24 |
|  | B | 12.63 | 9.34 | 1.99 | 4.92 | 19 |
|  | C | 15.91 | 11.76 | 3.15 | 7.31 | 15 |
|  | D | 20.04 | 14.82 | 4.99 | 11.10 | 12 |
| 541 | A | 11.77 | 8.70 | 1.44 | 4.53 | 24 |
|  | B | 14.83 | 10.96 | 2.31 | 6.67 | 19 |
|  | C | 18.69 | 13.81 | 3.66 | 9.99 | 15 |
|  | D | 23.54 | 17.40 | 5.81 | 15.28 | 12 |
| 542 | A | 14.81 | 10.95 | 1.64 | 5.65 | 24 |
|  | B | 18.66 | 13.80 | 2.64 | 8.41 | 19 |
|  | C | 23.51 | 17.38 | 4.18 | 12.68 | 15 |
|  | D | 29.63 | 21.90 | 6.03 | 19.47 | 12 |

## MODEL 540

Peak torque (oz. in.) 240
Max. rated continuous torque at stall (oz. in.) 29
Armature inertia (oz. in. sec.2) . 0038
Damping factor (oz.in./KRPM) . 1
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 5.0
(Armature-to-ambient)
Electrical time constant (msec) 2.06
Mechanical time constant (msec) 8.8
Motor weight 2 lb .1 oz .
Static Friction Torque 3 oz . in.

## MODEL 541

Peak torque (oz. in.) 282
Max. rated continuous torque at stall (oz. in.) 38
Armature inertia (oz. in. sec.2) . 005
Damping factor (oz. in./KRPM) . 15
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.60
(Armature-to-ambient)
Electrical time constant (msec) 2.46
Mechanical time constant (msec) 9.4
Motor weight 2 lb .8 oz .
Static Friction Torque 3 oz . in.
MODEL 542
Peak torque (oz. in.) 355
Max. rated continuous torque at stall (oz. in.) 50
Armature inertia (oz. in. $\mathrm{sec}^{2}$ ) . 0062
Damping factor (oz. in./KPRM) . 2
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.18 (Armature-to-ambient)
Electrical time constant (msec) 2.77
Mechanical time constant ( msec ) 8.2
Motor weight 3 lb .0 oz .
Static Friction Torque 3 oz . in.


SHAFT RUNOUT . 001 IN./IN.

## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$
Maximum no load speed 6 KRPM
Rated brush life 5,000 hours @ 1 KRPM
Maximum friction torque 3 oz . in.
Maximum shaft radial load 10 lbs .,
0.5 in . from front bearing continuous
for a minimum of 1,000 hours

## CONSTRUCTION FEATURES

Winding insulation Class F ABEC Class 1 ball bearings PVC lead insulation
Totally enclosed housing
Available options:
Integrally mounted tachometer (pp. 60-61)
Integrally mounted optical encoder (see pp. 40-45)

## SPEED TORQUE CURVES

MODEL E-540


MODEL E-541

$\mathrm{V} T=$ Terminal Voltage

MODEL E-542


The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $10^{\prime \prime} \times 10^{\prime \prime} \times 1 / 4^{\prime \prime}$ heat sink.

## E-586/E-588

The E-586/E-588 series offers the performance and cost advantages of an integral motor/tachometer. This unit is used in a wide range of velocity and position controls.

## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color.

$$
E-\overline{\text { motor-tach type }}-\overline{\text { motor winding }}-\frac{S_{\text {tach winding }}}{\text { for Special }}
$$

|  |  |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{c} \\ & 0.0 \\ & 0 \\ & \hline \end{aligned} \cdot \bar{o}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 586 | A | 10.02 | 7.41 | 1.24 | 3.39 | 24 |
|  | B | 12.63 | 9.34 | 1.99 | 4.92 | 19 |
|  | C | 15.91 | 11.76 | 3.15 | 7.31 | 15 |
|  | D | 20.04 | 14.82 | 4.99 | 11.10 | 12 |
| 587 | A | 11.77 | 8.70 | 1.44 | 4.53 | 24 |
|  | B | 14.83 | 10.96 | 2.31 | 6.67 | 19 |
|  | C | 18.69 | 13.81 | 3.66 | 9.99 | 15 |
|  | D | 23.54 | 17.40 | 5.81 | 15.28 | 12 |
| 588 | A | 14.81 | 10.95 | 1.64 | 5.65 | 24 |
|  | B | 18.66 | 13.80 | 2.64 | 8.41 | 19 |
|  | C | 23.51 | 17.38 | 4.18 | 12.68 | 15 |
|  | D | 29.63 | 21.90 | 6.63 | 19.47 | 12 |

## MODEL E-586

Peak torque (oz. in.) 240
Max. rated continuous torque at stall (oz. in.) 29
Armature inertia (including tach) (oz. in. $\mathrm{sec}^{2}$ ) .0052
Damping factor (oz.in./KRPM) 1
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 5.0
(Armature-to-ambient)
Electrical time constant (msec) 2.06
Mechanical time constant (msec) 8.8
Motor weight 2 lb .1 oz .
Static Friction Torque 3 oz. in.
MODEL E-587
Peak torque (oz. in.) 282
Max. rated continuous torque at stall (oz. in.) 38
Armature inertia (including tach) (oz. in. sec.2) . 0064
Damping factor (oz. in./KRPM) . 15
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.60 (Armature-to-ambient)
Electrical time constant (msec) 2.46
Mechanical time constant (msec) 9.4
Motor weight 2 lb .8 oz .
Static Friction Torque 3 oz. in.
MODEL E-588
Peak torque (oz. in.) 355
Max. rated continuous torque at stall (oz. in.) 50
Armature inertia (including tach) (oz. in. $\mathrm{sec}^{2}$ ) . 0078
Damping factor (oz. in/KRPM) . 2
Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 4.18
(Armature-to-ambient)
Electrical time constant (msec) 2.77
Mechanical time constant (msec) 8.2
Motor weight 3 lb .0 oz .
Static Friction Torque 3 oz. in.


## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$
Maximum no load speed 6 KRPM
Rated brush life 5,000 hours @ 1 KRPM
Maximum friction torque 3 oz . in.
Maximum shaft radial load 10 lbs.
0.5 in . from front bearing continuous for
a minimum of 1,000 hours

## CONSTRUCTION FEATURES

Winding insulation Class F $A B E C$ Class 1 ball bearings PVC lead insulation
Totally enclosed housing
Integrally mounted optical encoder (see pp. 40-45)

TACHOMETER WINDING VARIATIONS

| Winding Constants | A | B | c | D |
| :---: | :---: | :---: | :---: | :---: |
| Output Voltage Gradient (Volts/KRPM) | 3.0 | 7.0 | 14.2 | 21 |
| Armature Resistance (ohms) (includes temperature compensation network) | 570 | 570 | 720 | 950 |
| Armature Inductance (mhy) | 6.2 | 33 | 138 | 255 |
| Optimum Load Impedance | 5,000 | 5,000 | 5,000 | 10,000 |

(ohms)
(See pp. 60-61 for more detailed tachometer information)

## SPEED TORQUE CURVES



The S.O.A.C. Iine represents the safe operating area, continuous with unit mounted on $10^{\prime \prime} \times 10^{\prime \prime} \times 1_{4}{ }^{\prime \prime}$ heat sink.

## E-660/E-670

The Model E-660/E-670 series permanent magnet DC motors are designed to provide high cost effectiveness in a variety of professional and industrial equipment. Applications include direct drive for reel motors in tape transports, web handling systems and material handling systems.


## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color

*Available tachometer: E-596, M-110. For tachometer specifications see pp. 58-61.

|  | $\begin{aligned} & \text { 들 } \\ & \stackrel{1}{0} \\ & \stackrel{C}{3} \end{aligned}$ |  |  |  |  |  | MODEL 660 <br> Peak torque (oz. in.) 679 <br> Max. rated continuous torque at stall (oz. in.) <br> Armature inertia (oz. in. sec. ${ }^{2}$ ) . 032 <br> Damping factor (oz. in./KRPM) . 68 <br> Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 2.8 <br> (Armature-to-ambient) | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Electrical time constant (msec) 3.7 |  |
| 660 | A | $\begin{aligned} & 12.82 \\ & 17.09 \end{aligned}$ | 9.48 12.64 | . 40 | 2.22 2.74 | 53 40 | Mechanical time constant (msec) 11.6 |  |
|  | C | 21.53 | 15.93 | . 85 | 3.87 | 32 | Motor weight 6 lb .2 oz . |  |
|  | D | 27.13 | 20.07 | 1.34 | 5.71 | 25 |  |  |
|  | E | 34.19 | 25.28 | 2.15 | 8.70 | 20 |  |  |
|  | F | 42.74 | 31.60 | 3.40 | 13.32 | 16 |  |  |
|  | G | 53.42 | 39.50 | 5.36 | 20.57 | 13 | MODEL 670 |  |
|  | H | 67.31 | 49.77 | 8.56 | 32.41 | 10 | Peak torque (oz. in.) 903 |  |
|  | J | 84.40 | 62.41 | 13.46 | 50.54 | 8 | Max. rated continuous torque at stall (oz, in.) | 165 |
| 670 | A | 18.64 | 13.78 | . 52 | 3.29 | 48 | Armature inertia (oz. in. sec. ${ }^{2}$ ) . 05 |  |
|  | B | 22.94 | 16.96 | . 63 | 3.84 | 39 |  |  |
|  | C | 28.90 | 21.37 | 1.00 | 5.52 | 31 | Damping factor (oz. in./KRPM) 1.0 |  |
|  | D | 36.42 | 26.93 | 1.59 | 8.21 | 25 | Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 2.1 |  |
|  | E | 45.89 | 33.93 | 2.54 | 12.61 | 20 | (Armature-to-ambient) |  |
|  | F | 57.34 | 42.40 | 4.02 | 19.40 | 16 | Electrical time constant (msec) 4.6 |  |
|  | G | 71.68 | 53.00 | 6.33 | 30.06 | 13 | Mechanical time constant (msec) 11.2 |  |
|  | H | 90.31 | 66.78 | 10.11 | 47.44 | 10 | Mechanical time constant (msec) 11.2 |  |
|  | J | 113.24 | 83.74 | 15.90 | 74.07 | 8 | Motor weight 8 lb .6 oz . |  |



## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Rated brush life 5,000 hours @ 1 KRPM Maximum friction torque 7 oz . in. Maximum shaft radial load 30 lb .,

1 in. from front bearing continuous

## CONSTRUCTION FEATURES

Winding insulation Class F
ABEC Class 1 ball bearings
PVC lead insulation
Totally enclosed housing
Available option:
Integrally mounted optical encoder (see pp. 40-45) NEMA 42CZ mounting face


The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $12^{\prime \prime} \times 12^{\prime \prime} \times 1 / 2^{\prime \prime}$ Heat Sink.

## E-701/E-703

The E-701/E-703 series permanent magnet D.C. motors are designed to be used in a broad range of industrial servo applications where high torque output at low speeds, or high pulse torque output is particularly useful. Applications include direct drive for reel motors in tape transports and web handling systems, material handing systems and machine tool drives. Shown in the photos are the standard square flange (above) end cap and
 the optional "in line" round end cap.

## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color
$E-\overline{\text { motor model }}-$ - $-\frac{S_{\text {tach. model }} \text { winding }}{\text { for special }}$
*Available tachometer: E-596, E-796, M-110. For tachometer specifications see pp. 58-63.

|  | $\begin{aligned} & \text { O} \\ & \stackrel{5}{0} \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ |  |  |  |  |  | MODEL 701 <br> Peak torque (oz. in.) 750 <br> Max. rated continuous torque at stall (oz. in.) 150 <br> Armature inertia (oz. in. sec. ${ }^{2}$ ) . 10 <br> Damping factor (oz.in./KRPM) 5 <br> Thermal resistance ( ${ }^{\circ} \mathrm{C} . /$ watt) 2.2 <br> (Armature-to-ambient) <br> Electrical time constant (msec) 2.6 <br> Mechanical time constant ( msec ) 36.3 <br> Motor weight 6 lb .10 oz . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 701 | A | 10.43 | 7.71 | . 13 | . 73 | 72 | MODEL 702 |
|  | B | 20.39 | 15.08 | . 68 | 2.17 | 37 | Peak torque (oz. in.) 1350 <br> Max. rated continuous torque at stall (oz. in.) 300 |
|  | C | 32.62 | 24.12 | 1.24 | 3.91 | 23 | Armature inertia (oz. in. sec.2) . 15 |
|  | D | 40.77 | 30.15 | 2.60 | 7.48 | 18 | Thermal resistance ( ${ }^{\circ} \mathrm{C}$./watt) 1.6 (Armature-to-ambient) |
| 702 | A | 18.97 | 14.03 | . 17 | 1.46 | 71 | Electrical time constant (msec) 4.5 Mechanical time constant (msec) 19.1 |
|  | B | 37.12 | 27.45 | . 68 | 3.75 | 36 | Motor weight 8 lb .5 oz . |
|  | C | 59.40 | 42.92 | 1.65 | 8.61 | 23 | MODEL 703 |
| 703 | D | 74.25 | 54.90 | 2.58 | 12.83 | 18 | Peak torque (oz. in.) 2000 <br> Max. rated continuous torque at stall (oz. in.) 410 Armature inertia (oz. in. $\mathrm{sec}^{2}$ ) . 20 |
|  | A | 27.99 | 20.70 | . 22 | 2.20 | 31 | Thermal resistance ( ${ }^{\circ} \mathrm{C}$./watt) 1.3(Armature-to-ambient) |
|  | B | 54.77 | 40.50 | . 85 | 6.00 | 37 |  |
|  | C | 87.64 | 64.80 | 2.06 | 13.90 | 23 | Electrical time constant ( msec ) 6.0 Mechanical time constant (msec) 13.3 Motor weight 11 lb .0 oz. |
|  | D | 109.50 | 81.00 | 3.22 | 20.80 | 18 |  |



## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Rated brush life 5,000 hours @ 1 KRPM
Maximum friction torque 15 oz . in.
Maximum shaft radial load 30 lb .,
1 in. from front bearing continuous

## CONSTRUCTION FEATURES

Winding insulation Class F
ABEC Class 1 ball bearings
PVC lead insulation
Totally enclosed housing
Available option:
Integrally mounted optical encoder (see pp. 40-45) NEMA 46C mounting face

## SPEED TORQUE CURVES

MODEL E-701


MODEL E-702


MODEL E-703


RMS OUTPUT TORQUE (OZ-IN)

## $\mathrm{V}=$ Terminal Voltage

The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $12^{\prime \prime} \times 12^{\prime \prime} \times 1 / 2$ Heat Sink.

## E-5260

The E-5260 motor represents a new concept in DC servomotors. In applications requiring low torque at high speed and high torque at low speed the savings in amplifier costs can be very large. The back emf, or $K_{E}$, can be varied by over 3:1 by use of a small bias field winding power supply.

## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color

*Available tachometers: E-596, E-796, M-110. For tachometer specifications see pp. 58-63.

E-5260


RELATIONSHIP BETWEEN FLUX BIAS CURRENT AND TORQUE CONSTANT



MODEL E-5260 THERMAL DATA


NOMINAL SPECIFICATIONS AT $25^{\circ} \mathrm{C}$

| Parameter | Value | Units | Parameter | Value | Units |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Torque Constant $\mathrm{K}_{\mathrm{T}}$ |  |  | Armature Inductance | 4 | mH |
| Low Speed Drive | 50 | oz. in./A | Armature Moment of Inertia | 0.38 | oz. in. $\mathrm{s}^{2}$ |
| High Speed Slew | 14.4 | oz. in/A | Flux Bias Field Voltage | $\pm 32$ | $V$ - |
| Voltage Constant $K_{E}$ <br> Low Speed Drive | 37 | V/krpm | Flux Bias Field Current | $\pm 0.8$ | A |
| High Speed Slew | 10.6 | V/krpm | Full Load Torque (Uncooled) | 500 | oz. in. |
| Armature Resistance | . 8 | $\Omega$ | Maximum Armature Temperature | 155 | ${ }^{\circ} \mathrm{C}$ |

## E30-1/E30-2

The E30-1/E30-2 series permanent magnet DC motors are designed to be used in a broad range of industrial servo applications where high torque output at low speeds, or high pulse torque is particularly useful. Applications include axis drives for lathes, mills, machining centers and high speed winders. Shown in the photo is the standard square NEMA 56C flange.

## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color

*Available tachometer: E-796. For tachometer specifications see pp. 62-63.



## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Rated brush life 5,000 hours @ 1 KRPM Maximum friction torque 2.5 in . lbs. Maximum shaft radial load 45 lb .,

1 in. from front bearing continuous

## CONSTRUCTION FEATURES

Winding insulation Class $F$
ABEC Class 1, sealed ball bearings
PVC lead insulation
Totally enclosed housing
Available option:

## Terminal box

Integrally mounted optical encoder (see pp. 40-45)

SPEED TORQUE CURVES

MODEL E30-1

MOTOR SPEED (KRPM)

MODEL E30-2


The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $12^{\prime \prime} \times 12^{\prime \prime} \times 1 / 2^{\prime \prime}$ Heat Sink.

## E40-1/E40-2

The E40-1/E40-2 series permanent magnet DC motors are designed to be used in a broad range of industrial servo applications where torque output at low speeds, or high pulse torque is particularly useful. Applications include axis drive for lathes, mills, machining centers and pump drives. Shown in the photo is the standard square NEMA 56C flange.


## HOW TO ORDER

To order, you must supply sales model number. Order codes are shown below in color

*Available tachometer: E-796. For tachometer specifications see pp. 62-63.

|  | $\begin{aligned} & \text { 음 } \\ & \text { C } \\ & \frac{c}{3} \end{aligned}$ |  |  |  |  |  |  | Peak torque (in. lbs.) 300 <br> Max. rated continuous torque at stall (in. Ibs.) 65 <br> Armature inertia (oz. in. sec. ${ }^{2}$ ) 0.9 <br> Damping factor (in. Ibs./KRPM) 1.0 <br> Thermal resistance ( ${ }^{\circ} \mathrm{C} /$ watt) 1.0 <br> (Armature-to-ambient) <br> Electrical time constant (msec) 5 <br> Mechanical time constant (msec) 6 <br> Motor-tach weight 40 lbs . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E40-1 | A | 3.07 | 36.5 | . 12 | . 6 | 100 | 3000 | MODEL E40-2 |
|  | B | 4.64 | 55 | . 40 | 1.3 | 80 | 2000 | Peak torque (in. Ibs.) 300 |
|  | C | 6.07 | 72 | . 78 | 2.8 | 50 | 1500 | Max. rated continuous torque at stall (in. Ibs.) 85 Armature inertia (oz. in. $\mathrm{sec}^{2}$ ) 1.4 Damping factor (in. Ibs./KRPM) 1.2 |
| E40-2 | A | 3.12 | 37 | . 08 | . 4 | 100 | 3000 | Thermal resistance ( ${ }^{\mathrm{C}} \mathrm{C} /$ watt) .75 (Armature-to-ambient) |
|  | B | 4.13 | 49 | . 14 | . 6 | 80 | 2400 | Electrical time constant ( msec ) 4 Mechanical time constant (msec) 5 |
|  | C | 6.24 | 74 | . 32 | 1.3 | 50 | 1500 | Motor-tach weight 48 lbs . |



## SERIES MOTOR RATINGS

Maximum rated armature temperature $155^{\circ} \mathrm{C}$ Rated brush life 5,000 hours @ 1 KRPM Maximum friction torque 3.0 in . lbs. Maximum shaft radial load 45 lb ., 1 in. from front bearing continuous

## CONSTURCTION FEATURES

Winding insulation Class $F$
ABEC Class 1 sealed ball bearings.
PVC lead insulation
Totally enclosed housing
Available option:
Terminal box
Integrally mounted optical encoder (see pp. 40-45)

## SPEED TORQUE CURVES



RMS OUTPUT TORQUE (IN. LBS.)
$\mathrm{V}=$ Terminal Voltage
The S.O.A.C. line represents the safe operating area, continuous with unit mounted on $12^{\prime \prime} \times 12^{\prime \prime} \times 1 / 2^{\prime \prime}$ Heat Sink.

## M-1020

The M-1020 is the first step up to the high performance of moving coil motors. This model offers compact size, reasonable economy, and fast response with low inertia loads.

## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color

NOTE: This motor can be supplied with an integrally mounted incremental optical encoder, see pp. 40-45.

|  | Nower Rate (peak) <br> Naximum Continuous |
| :--- | :--- |
| Morque |  |

Let our application specialist examine your requirements.


M 1020
$\square$ Area of safe continuous operation without air cooling.

Area beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent basis provided armature temperature rating is not exceeded.


## M-1030

The Electro-Craft Model M-1030 is an intermediate performance DC moving coil servomotor designed for maximum economy over a broad range of applications. A significant range extension can be achieved by using forced air cooling due to the very efficient air baffle arrangement on this motor. The unit shown on the right has an Electro-Craft M-110 moving coil tachometer mounted on the rear end of the shaft.


## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color


NOTE: This motor can be supplied with an integrally mounted incremental optical encoder, see pp. 40-45.


## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to this question as numerous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle, direction reversals, load inertia and method of heat removal.

For the M-1030, operated under normal conditions, a life expectancy of $1.5 \times 10^{9}$ stop-start cycles at 100 cycles per second or $2 \times 10^{8}$ revolutions at 1500 RPM with 20 oz . in. output can be achieved before maintenance is required. A life prediction can be made for your application after all pertinent factors are known and a simulation test has been made.

Let our application specialist examine your requirements.



Area of safe continuous operation wihtout air cooling.
$\square$ Area of safe continuous operation with adequate air cooling.
Area beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent basis provided armature temperature rating is not exceeded.


MODEL M-1030 THERMAL DATA


## M-1040

The Electro-Craft Model M-1040 is a medium performance DC moving coil servomotor designed to fill the gap between low cost iron core motors and the higher cost moving coil motors. A significant range extension can be achieved by using forced air cooling due to the very efficient air baffle arrangement on this motor.


## HOW TO ORDER

To order, you must supply sales model number. Order Codes are shown below in color

Note: This motor can be supplied with an integrally mounted incremental optical encoder, see pp. 40-45.


Uncooled (no load)
Maximum Speed at Rating Load
Rated Torque (continuous at stall)
Rated Power Output
Power Rate (continuous)
Power Rate (peak)
No Load Acceleration at Maximum Continuous Torque
No Load Acceleration at Maximum Pulse Torque

Maximum Rated
Maximum Static
Friction Torque
Electrical Time Constant
Mechanical Time Constant Damping Factor
7 krpm

4 krpm
35 oz . in. 80 W $14 \mathrm{KW} / \mathrm{sec}$ $1425 \mathrm{KW} / \mathrm{sec}$
$58000 \mathrm{rad} / \mathrm{sec}^{2}$
$155^{\circ} \mathrm{C}$
2.5 oz . in.
0.1 msec
2.3 msec
1.0 oz . in./krpm
$580,000 \mathrm{rad} / \mathrm{sec}^{2}$

Armature Moment of Inertia (including bearings)
$1 / 2 \mathrm{in}$. Dia. Shaft, Mtr. Only 0.00058 oz. in. s2
$1 / 2 \mathrm{in}$. Dia. Shaft, Mtr. Tach 0.00065 oz . in. $\mathrm{s}^{2}$
$3 / 8 \mathrm{in}$. Dia. Shaft. Mtr. Only $0.00048 \mathrm{oz} . \mathrm{in} . \mathrm{s}^{2}$
$3 / 8 \mathrm{in}$. Dia. Shaft, Mtr. Tach $0.00055 \mathrm{oz} . \mathrm{in} . \mathrm{s}^{2}$

W/Air Coolinc ( $30 \mathrm{in} . \mathrm{H}_{2} \mathrm{O}$ )
12 krpm
4 krpm
75 oz . in. 200 W $58 \mathrm{KW} / \mathrm{sec}$ $1570 \mathrm{KW} / \mathrm{sec}$
$124000 \mathrm{rad} / \mathrm{s}$
$640000 \mathrm{rad} / \mathrm{s}$
$155^{\circ} \mathrm{C}$
2.5 oz . in.
0.1 msec
2.3 msec
1.0 oz . in. $/ \mathrm{krp}$

| 5.8 | 4.3 | 0.70 | 0.09 | 60 |
| ---: | ---: | ---: | ---: | ---: |
| 8.3 | 6.1 | 1.95 | 0.21 | 42 |
| 11.6 | 8.6 | 2.70 | 0.35 | 30 |

## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to thi question as numberous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle, direction reversals, load inertia and method of heat removal.
For the M-1040 operated under normal conditions a life expectancy of $1.5 \times 10^{9}$ stop-start cycles at 100 cycles per second or $1 \times 10^{8}$ revolutions at 1500 RPM with 20 oz . in. Output can be achieved before maintenance is required. A life prediction can be made for you application after all pertinent factors are known and a simulation test has been made.
Let our application specialists examine your requirements.

$\square$ Area of safe continuous operation without air cooling.

Area of safe continuous operation with adequate air cooling.

Area beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent basis provided armature temperature rating is not exceeded.


MODEL M-1040 THERMAL DATA


## $M-1125 / M-1140$

The M-1125/1140 series provides significantly improved performance over previous four-inch diameter moving coil motors.


## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color

| $\mathrm{M}-\frac{}{\text { motor model }}-\frac{}{\text { motor winding }}$ | $\begin{array}{ll} \text { for analog } & \text { tach windir } \\ \text { tachometer } & \text { see pp. } 58-5 \\ \text { if required } & \end{array}$ | for special |  |  |
| :---: | :---: | :---: | :---: | :---: |
| MOTOR RATING | Uncooled M-1125 | W/Air Cooling ( $30 \mathrm{in} . \mathrm{H}_{2} \mathrm{O}$ ) | Uncooled M-1140 | W/Air Cooling ( 30 in. $\mathrm{H}_{2} \mathrm{O}$ ) |
| Maximum Safe Speed (no load) | 6.7 krpm | 12 krpm | 5.3 krpm | 11 krpm |
| Maximum Speed at Rated Load | 3 krpm | 3 krpm | 3 krpm | 3 krpm |
| Rated Torque (continuous at stall) | 50 oz . in. | $110 \mathrm{oz}. \mathrm{in}$. | $60 \mathrm{oz}. \mathrm{in}$. | 120 oz . in. |
| Rated Power Output | 84 W | 184 W | 100 W | 200 W |
| Power Rate (continuous) | $39 \mathrm{KW} /$ sec | $188 \mathrm{KW} / \mathrm{sec}$ | $47 \mathrm{KW} / \mathrm{sec}$ | $188 \mathrm{KW} / \mathrm{sec}$ |
| Power Rate (peak) | $2088 \mathrm{KW} / \mathrm{sec}$ | 2088 KW/sec | $3520 \mathrm{KW} / \mathrm{sec}$ | $3520 \mathrm{KW} / \mathrm{sec}$ |
| No Load Acceleration at Maximum Continuous Torque | $111000 \mathrm{rad} / \mathrm{sec}^{2}$ | $244000 \mathrm{rad} / \mathrm{sec}^{2}$ | $111000 \mathrm{rad} / \mathrm{sec}^{2}$ | $222000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| No Load Acceleration at Maximum Pulse Torque | $810000 \mathrm{rad} / \mathrm{sec}^{2}$ | $810000 \mathrm{rad} / \mathrm{sec}^{2}$ | $960000 \mathrm{rad} / \mathrm{sec}^{2}$ | $960000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| Maximum Rated Armature Temp. | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ |
| Maximum Static Friction Torque | 4 oz . in. | 4 oz . in. | 4 oz . in. | 4 oz . in. |
| Electrical Time Constant | . 14 msec. | .14 msec . | . 1 msec . | . 1 msec . |
| Mechanical Time Constant | 1.1 msec. | 1.1 msec . | .83 msec | . 83 msec . |
| Damping Factor | 1.5 oz . in./krpm | 1.5 oz . in./krpm | 2.5 oz . in./krpm | $2.5 \mathrm{oz} . \mathrm{in} . / \mathrm{krpm}$ |
| Armature Moment of Inertia (including bearings) |  |  |  |  |
|  | M-1125 | M-1140 |  |  |
| Motor only | .00038 oz. in. sec ${ }^{2}$ | .00047 oz. in. $\sec ^{2}$ |  |  |
| Motor M-110 Tach | . 00045 oz. in. sec ${ }^{2}$ | .00054 oz. in. $\mathrm{sec}^{2}$ |  |  |

## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to this question as numerous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle, direction reversals, load inertia and method of heat removal.
For the M-1125/M-1140 operated under normal conditions a life expectancy of $1.5 \times 10^{9}$ stop-start cycles at 150 cycles per second or $3 \times 10^{8}$ revolutions at 1750 RPM with 35 oz . in. output can be achieved before maintenance is required. A life prediction can be made fol your application after all pertinent factors are known and a simulation test has been made.

Let our application specialist examine your requirements.

M. 1140


Area of safe continuous op eration without air cooling

Area of safe continuous op eration with adequate air cooling.

Areas beyond capacity of motor for continuous oper ation. Much higher output torque can be realized on an intermittent basis pro vided armature tempera ture rating is not exceeded


1125 1140

## M-1438/M-1440

The Electro-Craft Model M-1438/1440 highperformance DC moving coil servomotors have a theoretical acceleration from stall exceeding $10^{6} \mathrm{rad} / \mathrm{s}^{2}$. They are commonly used where the highest performance is required in computer peripheral and other capital equipment.


## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color

$\mathrm{M}-\overline{\text { motor model }}-\overline{\text { motor winding }}$| for analog <br> tachometer <br> if required |
| :--- |
| 110 |


|  | M-1438 |  | M-1440 |  |
| :---: | :---: | :---: | :---: | :---: |
| MOTOR RATING | Uncooled | W/Air Cooling ( $30 \mathrm{in} . \mathrm{H}_{2} \mathrm{O}$ ) | Uncooled | W/Air Cooling (30 in. $\mathrm{H}_{2} \mathrm{O}$ ) |
| Maximum Safe Speed (no load) | 4.5 krpm | 10 krpm | 4.5 krpm | 10 krpm |
| Maximum Speed at Rated Load | 2.5 krpm | 2.5 krpm | 2.5 krpm | 2.5 krpm |
| Rated Torque (continuous at stall) | 84 oz. in. | 170 oz. in. | 120 oz . in. | 229 oz. in. |
| Rated Power Output | 130 W | 280 W | 160 W | 320 W |
| Power Rate (continuous) | $76 \mathrm{KW} / \mathrm{sec}$ | $230 \mathrm{KW} / \mathrm{sec}$ | $59 \mathrm{KW} / \mathrm{sec}$ | $217 \mathrm{KW} / \mathrm{sec}$ |
| Power Rate (peak) | $4400 \mathrm{KW} / \mathrm{sec}$ | $4400 \mathrm{KW} / \mathrm{sec}$ | $1837 \mathrm{KW} / \mathrm{sec}$ | $1837 \mathrm{KW} / \mathrm{sec}$ |
| No Load Acceleration at Maximum Continuous Torque | $120,000 \mathrm{rad} / \mathrm{sec}^{2}$ | $215000 \mathrm{rad} / \mathrm{sec}^{2}$ | $70600 \mathrm{rad} / \mathrm{sec}^{2}$ | $135000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| No Load Acceleration at |  |  |  |  |
| Maximum Pulse Torque | $950000 \mathrm{rad} / \mathrm{sec}^{2}$ | $950000 \mathrm{rad} / \mathrm{sec}^{2}$ | $390000 \mathrm{rad} / \mathrm{sec}^{2}$ | $390,000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| Maximum Rated Armature Temp. | $155{ }^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ | $1550^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ |
| Maximum Static Friction Torque | 5 oz . in. | 5 oz . in. | 5. oz. in. | 5 oz . in. |
| Electrical Time Constant | . 16 msec . | . 16 msec | . 27 msec | .27 msec . |
| Mechanical Time Constant | . 69 msec | . 69 msec . | 1 msec | 1 msec |
| Damping Factor | 4 oz . in./krpm | 4 oz . in./krpm | 4 oz . in./krpm | 4 oz . in./krpm |
| Armature Moment of Inertia (ref. only) (including bearings) |  |  |  |  |
|  | M-1438 2 | M-1440 |  |  |
| Motor only Motor M-110 Tach | . 00070 oz. in.sec ${ }^{2}$ | . 0016 oz. in. $\sec ^{2}$ |  |  |
| Motor M-110 Tach Motor/Optical Tach | . 00077 oz. in. sec ${ }^{2}$ | . 0017 oz. in. sec ${ }^{2}$ |  |  |
| Motor/Optical Tach | . 00078 oz. in. $\mathrm{sec}^{2}$ |  |  |  |

## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to this question as numerous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle, direction reversals, load inertia and method of heat removal.
For the $\mathrm{M}-1438 / 1440$ operated under normal conditions a life expectancy of $1.5 \times 10^{9}$ stop-start cycles at 150 cycles per second or $3 \times 10^{8}$ revolutions at 1750 RPM with 60 oz , in. output can be achieved before maintenance is required. A life prediction can be made for your application after all pertinent factors are known and a simulation test has been made.


Area of safe continuous op eration without air cooling.

Area of safe continuous op eration with adequate air cooling.

Areas beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent basis pro vided armature tempera ture rating is not exceeded.




## M-1450/M-1460


#### Abstract

The Electro-Craft Model M-1450/M-1460 is a particularly powerful DC high-performance servomotor designed to drive moderately high inertial loads with the fast response characteristic of moving coil motors. Typical applications include high speed line printer drives, paper feed drives in high speed Optical Character recognition equipment and high speed material handling systems.




## HOW TO ORDER

To order you must supply sales model number. Order codes are shown below in color


|  | M-1450 |  | M-1460 |  |
| :---: | :---: | :---: | :---: | :---: |
| MOTOR RATINGS | Uncooled | W/Air Cooling ( 30 in. $\mathrm{H}_{2} \mathrm{O}$ ) | Uncooled | W/Air Cooling (30 in. $\mathrm{H}_{2} \mathrm{O}$ ) |
| Maximum Safe Speed (no load) | 4 krpm | 7.5 krpm | 4.5 krpm | 7.5 krpm |
| Maximum Speed at Rated Load | 2 krpm | 2 krpm | 2 krpm | 2 krpm |
| Rated Torque (continuous at stall) | 180 oz. in | 356 oz . in. | 220 oz. in. | 375 oz. in. |
| Rated Power Output | 267 W | 490 W | 300 W | 504 W |
| Power Rate (continuous) | $22 \mathrm{KW} / \mathrm{sec}$ | $76 \mathrm{KW} / \mathrm{sec}$ | $40 \mathrm{KW} / \mathrm{sec}$ | $140 \mathrm{KW} / \mathrm{sec}$ |
| Power Rate (peak) | $1480 \mathrm{KW} / \mathrm{sec}$ | 1300 KW/sec | $2400 \mathrm{KW} / \mathrm{sec}$ | $2400 \mathrm{KW} / \mathrm{sec}$ |
| No Load Acceleration at Maximum Continuous Torque | $17900 \mathrm{rad} / \mathrm{sec}^{2}$ | $32900 \mathrm{rad} / \mathrm{sec}^{2}$ | $33000 \mathrm{rad} / \mathrm{sec}^{2}$ | $61000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| No Load Acceleration at Maximum Pulse Torque | $139000 \mathrm{rad} / \mathrm{sec}^{2}$ | $139000 \mathrm{rad} / \mathrm{sec}^{2}$ | $240000 \mathrm{rad} / \mathrm{sec}^{2}$ | $240000 \mathrm{rad} / \mathrm{sec}^{2}$ |
| Maximun Rated Armature Temp. | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ | $155^{\circ} \mathrm{C}$ |
| Maximum Static Friction Torque | 9 oz . in. | 9 oz in. | $9 \mathrm{oz}. \mathrm{in}$. | 9 oz in. |
| Electrical Time Constant | .18 msec | . 18 msec | . 18 msec | .18 msec |
| Mechanical Time Constant | 3 msec | 3 msec | 1.6 msec | 1.6 msec |
| Damping Factor | 7.6 oz. in./krpm | 7.6 oz. in. $/ \mathrm{krpm}$ | 7.6 oz . in./krpm | 7.6 oz . in./krpm |
| Armature Moment of Inertia | . 01 oz. in. $\mathrm{sec}^{2}$ | . 01 oz. in. $\mathrm{sec}^{2}$ | . 0054 oz. in. $\mathrm{sec}^{2}$ | . 0054 oz. in. $\mathrm{sec}^{2}$ |

## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to this ques tion as numerous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle direction reversals, load inertia and method of heat removal.
For the $\mathrm{M}-1450 / \mathrm{M}-1460$ operated under normal conditions a life expectancy of $10^{9}$ stop-start cycles at 150 cycles per second o $2 \times 10^{8}$ revolutions at 1000 RPM with 150 oz . in. output can be achieved before maintenance is required. A life prediction can be mad for your application after all pertinent factors are known and a simulation test has been made.
Let our application specialist examine your requirements.


Area of safe continuous op eration without air cooling

Area of safe continuous op eration with adequate air cooling.

Areas beyond capacity of motor for continuous oper ation. Much higher output torque can be realized on an intermittent basis pro vided armature tempera ture rating is not exceeded

M-1450

M. 1460


## M-1600

The Electro-Craft Model M-1600 is a high-performance DC moving coil servomotor designed to meet the requirements of 250 IPS capstan drives. This motor has a theoretical acceleration from stall exceeding $10^{6} \mathrm{rad} / \mathrm{s}^{2}$, very high resonent frequencies, and various air cooling systems to suit customer requirements.


## HOW TO ORDER

To order, contact your local Electro-Craft sales engineering as this high performance model requires significant application engineering.
Note: This motor can be supplied with an incremental optical encoder, see pp. 40-45.

## Winding

A



MOTOR RATINGS
Maximum Safe Speed
(no load)
Maximum Speed at
Rated Load
Maximum Continuous
Stall Torque
Rated Power Output
Power Rate (continuous)
Power Rate (peak)
No Load Acceleration at
Maximum Continuous
Torque
No Load Acceleration at
Maximum Pulse Torque
Maximum Rated Armature
Temperature
Maximum Static Friction
Torque
Electrical Time Constant
Mechanical Time Constant
Damping Factor
Armature Moment of
Inertia
Thermal Resistance

W/Air Cooling
Uncooled
$7 \mathrm{krpm} \quad 7 \mathrm{krpm}$
4.5 krpm

60 oz. in. 120 oz . in 150 Watts 375 Watts $72 \mathrm{KW} / \mathrm{sec}$ 5920 KW/sec $280 \mathrm{KW} / \mathrm{sec}$ $6630 \mathrm{KW} / \mathrm{sec}$
$170,000 \mathrm{rad} / \mathrm{sec}^{2} \quad 360,000 \mathrm{rad} / \mathrm{sec}^{2}$

## MOTOR LIFE

Many users naturally ask what operational and brush life can be expected from a moving coil motor. There is no pat answer to the question as numerous application factors have a direct impact on motor life. Among these are peak current during acceleration, duty cycle, direction reversals, load moment of inertia and method of heat removal.

For the M-1600 operated under normal conditions a life expectance of $1.5 \times 10^{9}$ stop-start cycles at 180 cycles per second or $3 \times 10^{8}$ revolutions at 1750 rpm with 50 oz . in. output torque can be achieved before maintenance is required. A life prediciton can be made for your application after all pertinent factors are known and a simulation test has been made.

$\square$ Area of safe continuous operation without air cooling.

Area of safe continuous operation with adequate air cool. ing.
Area beyond capacity of motor for continuous operation. Much higher output torque can be realized on an intermittent basis provided armature temperature rating is not exceeded.


MODEL 1600 THERMAL DATA


## Encoders and Tachometers

The complete line of Electro-Craft DC motors is available with either integrally or separately mounted tachometers and encoders. As with the standard motor selection, these speed and position sensors are available simply by specifying the product number and the quantity desired. However, the applications for these devices are so broad and varied that we find a heavy demand for custom design units. Please contact the Electro-Craft Applications Engineering Department for assistance in designing a tachometer or encoder for your specific application.

## R-50

The R-50 Series encoder is a low cost modular, designed for ease of installation onto DC Servomotors. This encoder features solid state components including a reliable LED light source. The encoder outputs are available in either sine or $T^{2} \mathrm{~L}$ compatible square wave, single or quadrature, at a variety of pulse rates. The R-50 series is ideal for those low cost computer peripherial and process control applications.

## HOW TO ORDER:



## SPECIFICATIONS:

## Electrical Specifications:

Type 1 Output:
Input voltage: $5 \mathrm{VDC} \pm 5 \%$ @ 75 ma Output waveform $30 \mu \mathrm{amp}$ P-P min.


QUADRATURE, SINE WAVE OUTPUT ZERO CROSSING

Type 3 Output:
Input voltage +5 VDC $\pm 10 \%$ @ 100 ma Output waveform $T^{2} \mathrm{~L}$ compatible


QUADRATURE, SQUARE WAVE, TTL COMPATIBLE OUTPUT
Logic '0' . . 0.5 VDC max. @ 10 ma max. sink current Logic ' 1 ' . . .VCC @ 1 K ohm (TTL compatible)


Outline


Operating frequency. . $50 \mathrm{KH}_{\mathrm{z}}$ max standard Output format . . . . A leads B for CCW rotation as viewed looking at mounting surface
Operating temp. . . . . 0 to $55^{\circ} \mathrm{C}$
Flutter . . . . . . . . . .Less than 1.0\% once around Moment of inertia . . $8.5 \times 10^{-4} \mathrm{in} . \mathrm{oz} . \mathrm{sec}^{2}$

## Standard Available Resolutions:

192, 250, 400, 500, 1000

Other line counts within this range can be produced for a one time tooling charge.

## Motor Mounting Requirements:

1. Shaft runout: must be less than .005 TIR $1 / 2$ inch from mounting surface (disc hub mounting position)
2. Shaft to be perpendicular to mounting surface within 0.005 TIR
3. Shaft length $5 / 8$ inch min. from mounting surface
4. Shaft end play not to exceed .005 inch.

## Encoder Cover:

The standard cover mounts on two standoffs with the mounting screws supplied.

## R-1500

The R-1500 is the state-of-art modular encoder; designed to the highest standards for stability, and long term performance.

This modular encoder was solid state electronics and a new patented radial-line sensor array which minimizes sensitivity to motor shaf runout. This design provides for minimal installation time on motor. Models are available for either positive or negative motor shaft end play.


## HOW TO ORDER

To order, you must supply the sales model number. Codes are shown in color.


## SPECIFICATIONS:

## Electricl Specifications:

Type 1 Output:
Input voltage: 5 VDC $\pm 5 \%$ @ 70 ma .
Output waveform


Quadrature $90^{\circ} \pm 22.5^{\circ}$ at 5.0 VDC at $75 \mathrm{KH}_{z}$ Output format - differential sine wave suitable for input to comparator; A leads B for CCW rotation as viewed looking at mounting surface.

Type 3 Output:
Input voltage 5 VDC $\pm 5 \%$ @ 125 ma .
Output waveform: TTL compatible
Frequency response to $75 \mathrm{KH}_{\mathrm{Z}}$
Quadrature $90^{\circ} \pm 22.5^{\circ}$
Output format - A leads B for CCW rotation as viewed looking at mounting surface


QUADRATURE, SQUARE WAVE, TTL COMPATIBLE OUTPUT Output load impedance $470 \Omega$ typical


## SPECIFICATIONS:

## Electrical Specifications:

Type 4 Output
Input voltage: 5 VDC $\pm 5 \%$ @ 75 ma .


Output P/P 100 MV Min at 1 KH with 10 K load
Quadrature: $90^{\circ} \pm 36^{\circ}$ at 5 VDC and 50 KH
DC Offset: $10 \%$ of $\mathrm{P} / \mathrm{P}$
Output format - sine wave A leads B for CCW rotation as viewed looking at mounting surface.
Mechanical Specifications
Shaft end play specifications:

1. minus - preload permitting minus shaft movement

positive - preload permitting positive shaft movement


Mounting shaft end play: . 001 maximum
Nominal gap between disc and reticle: . 006 for resolutions up to 750, . 003 for 751 to 1000.
Shaft runout: must be less than . 0005 T.I.R. $1 / 2$ inch from mounting surface

## Resolution:

The following line counts are available as standard:
180, 200, 192, 250, 360, 400, 471, 500, 647, 720,
800, 1000
Other line counts within this range can be produced for a one time tooling change.

Discs:
Discs are metalized on glass pattern.
Disc inertia: $3 \times 10^{-5} \mathrm{oz}$. in. ${ }^{2}$
Low inertia disc available $1.5 \times 10^{-5} \mathrm{oz}$. in. ${ }^{2}$
Disc assemblies are available to fit standard shaft diameters:
$\begin{array}{cc}\text { Shaft }+.0000 & \text { Specify } \\ .2498 & 1 / 4 \\ .3123 & 5 / 16 \\ .3748 & 3 / 8 \\ .4373 & 7 / 16 \\ .4998 & 1 / 2 \\ .2360 & 6 . \mathrm{MM} \\ .3148 & 8 . \mathrm{MM}\end{array}$
Disc assemblies for other shaft sizes are available on request. Specify S on your order and detail requirements.

Mounting hole pattern


Encoder cover. The standard cover mounts on to the encoder base. If a special cover is required, i.e., a split cover or one with a hole in it for shaft exit specify S on the order and detail.

## R-2000

The R-2000 builds on the new technologies developed in the R-1500. In addition to high stability, ease of installation and multiple configurations, this unit is available with an index pulse and goes on to higher resolutions.
This modular encoder uses a L.E.D. light source
 and phototransistors for sensors and the new patented radial-line sensor array.

## HOW TO ORDER

To order, you must supply the sales model number. Codes are shown in color.
$\qquad$

Shaft End Play:
Minus
2. Positive


## SPECIFICATIONS:

Electrical Specifications:
Type 1 Output:
Input voltage: $5 \mathrm{VDC} \pm 5 \%$ @ 70 ma .
Output waveform Channel A $0^{\circ}, 180^{\circ}$



Index output amplitude dependant on resolution

Quadrature $90^{\circ} \pm 22.5^{\circ}$ at 5.0 VDC at $75 \mathrm{KH}_{\mathrm{z}}$. Tighter tolerance available as a special.
Output format - differential sine wave suitable for input to comparator: (on $A$ and $B$ channel) $A$ leads $B$
for CCW rotation as viewed looking at mounting surface. Output load impedance: (A and B channel) $470 \Omega$ typical

Type 2 Output:
Input voltage + and $\pm$ VDC \& 5.0 VDC @ 125 ma .
Output waveform: Sine wave zero-crossing for channels $A$ and $B$, single ended for index


* Other amplitudes available as special to be specified when ordering.


Quadrature $90^{\circ} \pm 22.5^{\circ}$ at 15 VDC at $75 \mathrm{KH}_{\mathrm{z}}$
Offset $\pm 5 \%$ of peak-to-peak
Frequency response to $75 \mathrm{KHz}_{\mathrm{Z}}$
Output load; greater than $1 \mathrm{k} \Omega$
Type 3 Output:
Input voltage 5 VDC $\pm 5 \%$ @ 125 ma .
Output waveform: TTL compatible
Frequency response to $75 \mathrm{KHz}_{\mathrm{Z}}$
Quadrature $90^{\circ} \pm 36^{\circ}$
Output format - A leads B for CCW rotation as viewed
looking at mounting surface
Mechanical Specifications
Shaft end play specifications:

1. minus - preload permitting 2. positive - preload permitting


Mounting shaft end play: . 001 maximum
Nominal gap between disc and reticle: . 006 for resolutions up to $750, .002$ for 751 to 1000 .
Shaft runout: must be less than . 005 T.I.R. $1 / 2$ inch from mounting surface
Resolution:
The following line counts are available as standard:

$$
\begin{aligned}
& 50,100,125,192,200,250,256,288,330,360,400 \\
& 460,471,476,500,525,556,600,640,647,688,720 \\
& 800,960,1000,1217,1250,1500
\end{aligned}
$$

Other line counts within this range can be produced for a one time tooling change.

Discs:
Discs are available with either metalized or photo emulsion pattern.
Disc inertia: Standard disc $-1.3 \times 10^{-4}$ oz. in. sec. ${ }^{2}$ Disc assemblies are available to fit standard shaft diameters:

| Shaft +.0000 | Specify |
| :--- | :---: |
| -.0002 | $1 / 4$ |
| .2498 | $5 / 16$ |
| .3123 | $3 / 8$ |
| .3748 | $7 / 16$ |
| .4373 | $1 / 2$ |
| .4998 | 6 MM |

Disc assemblies for other shaft sizes are available on request. Specify S on your order and detail requirements.

Mounting hole pattern


Encoder cover. The standard cover mounts on to the encoder base. The necessary mounting screws are supplied. If a special cover is required, i.e., a split cover or one with a hole in it for shaft exit specify $S$ on the order and detail.

## R-2520

The R-2520 Serives is a low cost, high performance encoder featuring solid state electronics and a new patented radial-line sensor array which minimizes sensitivity to disc runout and shaft loading.
This size 25 servo housed encoder is shielded against oil, moisture and dust. It is available in sine, amplified sine or $T^{2} L$ and CMOS square wave, signle or quadrature outputs. A once per revolution index is an available option on most line counts from 100 to 1500.
The R-2520 Series is ideal for applications requiring precision positioning or speed control such as in machine tool positioning or computer peripherals like printers and plotters.


HOW TO ORDER

*Order mating connector separately.

General Specifications:
Operation temperature range $0-60^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ}$ to $90^{\circ} \mathrm{C}$
Humidity $95 \%$ relative, no condensation
Bearing life $10^{9}$ revolutions
Torque . 06 oz . in.
Starting moment of inertia $5 \times 10^{-4} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{sec}^{2}$
Maximum speed 5000 rpm
Accuracy $\pm 2 \mathrm{~min}$ of arc std.
Direction of rotation - A leads B for CCW rotation when viewed from end of shaft.
LED light source
CODE. . . incremental
Electrical Specifications:
Type 1 Output
Output voltage 5 VDC $\pm 5 \%$ @ 70 ma


Quadrature $90^{\circ} \pm 36^{\circ}$
Output format - differential sine wave suitable for input to comparator.
Output load required $470 \Omega$ Typical
Frequency response
Data. . . 0 to $60 \mathrm{KH}_{\mathrm{Z}}$
Index. . 0 to $30 \mathrm{KH}_{\mathrm{z}}$ (Data)

## MECHANICAL OUTLINE


$L=2.45^{\prime \prime}$ with Cable - Standard
$L=3.0^{\prime \prime}$ with Connector
$X=.19^{\prime \prime}$ DA15P Connector
$X=.43^{\prime \prime}$ MS3102 Connector $\}$ Optional
SHAFT $1 / 4^{\prime \prime}$ OD Standard $3 / 8^{\prime \prime}$ OD Optional

## MOUNTING OPTIONS



| OUTPUT TERMINATION |  |  |  |
| :--- | :--- | :---: | :---: |
| FUNCTION | COLOR | DA15P | MS3102 |
| CH A | White | 13 | A |
| CH B | Green | 14 | B |
| Index | Orange | 15 | C |
| VCC | Red | 6 | D |
| GND | Black | 1 | E |
| +15VDC* | Yellow | 3 | F |
| -15VDC* | Blue | 4 | G |
| ${ }^{*}$ Type 2 Output only |  |  |  |

-ype 2 Output:
Input voltage + and $\pm 15$ VDC \& 5 VDC
Output waveform: Sine wave zero-crossing for channels $A$ and $B$, single ended for index
Output Load: greater than 1 K ohm


Offset $\pm 5 \%$ of peak-to-peak $\pm .16$ volts over specified range

TYPE 3 OUTPUT: $T^{2}$ L Compatible TYPE 4 OUTPUT: CMOS Compatible


Available Resolutions:
50, 100, 125, 192, 200, 250, 256, 288, 330, 360, $400,460,471,476,500,525,556,600,640,647$, $688,720,800,960,1000,1250,1500$

With Index:
50, 100, 125, 192, 200, 250, 256, 288, 330, 400, $471,500,525,600,640,647,688,720,800,960$, $1000,1250,1257,1500$

## R-2530

The R-2530 Series Incremental shaft encoder is a universal incremental encoder designed for most applications where shaft coupling is required.
This size 25 servo housed encoder is available with either sine or $T^{2} L$ compatible square wave output and optional once per revolution gated index.

The R-2530 Series plug in modular design concept allows for selection of options to suite different customers unique requirements including three shaft and increased resolution with $\times 4$ multiply.


## HOW TO ORDER

To order, you must supply the sales model number. Codes are shown in color.

R253

## Type Output:

 Differnetial sine wave Zerocrossing $\mathrm{T}^{2} \mathrm{~L}$compatible
$\qquad$ Mechanical construction: Specify T, S, L. - $\qquad$
$\qquad$ - $\qquad$ $-$ $\qquad$ -S Resolution: Signal conditioning electronics: Defines number of cycles in one revolution of code disc

## Number of

 Outputs A. Single B. Dual $90^{\circ}$ quadrature C. Single + indexD. Dual

+ index

Direction sensing Option:
Direction sensing with 2 times count multiplication Direction sensing with 4 times count multiplication Complimentary outputs, available with any of the above or with type 3 output.

| Mounting | Termination: |
| :---: | :---: |
| Option: | $\begin{aligned} & 18 \text {-inch } \\ & \text { pigtail } \end{aligned}$ |
| F2 | C1 DA 15P |
| F3 | C2 MS3102 |

General Specifications:
Operation temperature range $0-55^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ}$ to $90^{\circ} \mathrm{C}$
Humidity $95 \%$ relative, no condensation
Bearing life $19^{9}$ revolutions
Torque . 06 oz . in.
Starting moment of inertia $34 \times 10^{-4} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{sec}^{2}$
Maximum speed 5000 rpm
Accuracy $\pm$ min of arc
Direction of rotation - A leads B for CCW rotation when viewed from end of shaft.

Electrical Specifications:
Type 1 Output
Input voltage 5 VDC $\pm 5$ \%
Quadrature $90^{\circ} \mathrm{b}^{45}$ o


Quadrature $90^{\circ} \pm 45^{\circ}$
Output format - differential sine wave suitable for input to comparator.
Output load required $470 \Omega$ typical
Frequency response
Data . . . 0 to 50 KHz
Index . . 0 to 30 KHz (Data)

## MECHANICAL OUTLINE


$L=1.37$ for $T$
$L=2.06$ for $S$
$L=3.00$ for $L$
$\left.\begin{array}{l}X=.19^{\prime \prime} \text { DA15P Connector } \\ X=.43^{\prime \prime} \text { MS3102 Connection }\end{array}\right\}$ Optional
SHAFT $1 / 4^{\prime \prime}$ OD Standard
3/8" OD Optional

MOUNTING OPTIONS


3 PLCS on 2.000B.C.


4 PLCS on $2.000 \mathrm{~B} . \mathrm{C}$.
-Type 2 Output:
Input voltage 4 VDC $\pm 5 \%$ VDC @ 100 ma .
Output waveform


10\% MAX
Quadrature $90^{\circ} \pm 45^{\circ}$ at 5 VDC at $10 \mathrm{KH}_{z}$ Output load impedance $10 \mathrm{k} \Omega \mathrm{min}$ for rated output voltage Frequency response Data. . 0 to $50 \mathrm{KH}_{\mathrm{z}}$ Index. . 0 to $30 \mathrm{KH}_{\mathrm{Z}}$ (data)
Type 3 Output
Input voltage 5 VDC $\pm 5 \%$
Output waveform : $T^{2} L$


QUADRATURE, SQUARE WAVE, TTL COMPATIBLE OUTPUT


Quadrature $90^{\circ} \pm 45^{\circ} \mathrm{VDC}$ to $10 \mathrm{KHz}_{z}$ Frequency response

Data. . . 0 to $50 \mathrm{KHz}_{\mathrm{z}}$
Index. . 0 to $30 \mathrm{KH}_{\mathrm{z}}$
Direction Sensing:
General direction sensing is used to provide a logic indication of the direction of rotation as well as provide a means of increasing encoder resolution. DS1 specifies a unit with direction sensing. DS2 specifies direction sensing with 2 times count multiplication and DS4 indicates 4 times count mulitplication.


Resolution:
The following line counts are available as standard:
$1,2,8,48,50,100,120,128,150,200,250,256,264$,
$360,400,402,500,508,512,600,625,635,750,850$, $900,1000,1024,1035,1250,1500,1683,1714,1300$, 2000, 2500
Other line counts within this range can be produced for a one-time tooling charge.

## R-2530HD

The R-2530HD Series is a Heavy Duty version of the R-2530 series. It has been designed to be extremely rugged and reliable for those heavy duty machine tool and process control applications. This model is available with all the electrical options of the R-2530 series. It is provided with a $3 / 8$ inch diameter shaft which will take loads of 40 lbs . axial and 35 lbs . radial, also a series of seals that protect the electronics from the harsh operating environment.

## HOW TO ORDER

To order you must supply the sales model number. Codes are shown in color.


General Specifications:
Operation temperature range $0-70^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ}$ to $+90^{\circ} \mathrm{C}$
Humidity 95\% relative, no condensation
Shock 50 g for 11 msec
Vibration to $2000 \mathrm{H}_{\mathrm{z}}$ at 15 g 's
Sand and dust - not detrimental
Bearing life $10^{9}$ revolutions
Torque .06 oz . in.
Starting moment of inertia $5 \times 10^{-4} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{sec}^{2}$
Maximum speed 5000 rpm
Accuracy $\pm 2 \mathrm{~min}$. of arc
Direction of rotation - A leads B for CCW rotation when viewed from end of shaft.

## Electrical Specifications:

Type 1 Output
Output voltage 5 VDC $\pm 5 \%$


Quadrature $90^{\circ} \pm 450$
Output format - differential sine wave suitable for input to comparator.
Output load required $470 \Omega$ Typical
Frequency response
Data. . . 0 to $50 \mathrm{KHz}_{\mathrm{z}}$
Index. . . 0 to $30 \mathrm{KH}_{\mathrm{Z}}$ (Data)


Type 2 Output:
Input voltage 5 VDC $\pm 5 \%$ VDC @ 100 ma Output waveform


Quadrature $90^{\circ} \pm 45^{\circ}$ at 5 VDC at 10 KH
Output load impedance $10 \mathrm{k} \Omega \mathrm{m}$ in for rated output voltage Frequency response

Data. . . . 0 to $50 \mathrm{KH}_{\mathrm{z}}$
Index. . . 0 to $30 \mathrm{KH}_{\mathrm{Z}}$ (Data)
Type 3 Output
Input voltage 5 VDC $\pm 5 \%$ VDC
Output waveform: $T^{2} L$ compatible


ZUADRATURE, SQUARE WAVE, TTL COMPATIBLE OUTPUT


Quadrature $90^{\circ} \pm 45^{\circ} \mathrm{VDC}$ to $10 \mathrm{KH}_{\mathrm{z}}$
Frequency response
Data. . . . 0 to $50 \mathrm{KHz}_{\mathrm{z}}$
Index. . . 0 to $30 \mathrm{KH}_{\mathrm{Z}}$ (Data)
Direction Sensing:
General direction sensing is used to provide a logic indication of the direction of rotation as well as provide a means of increasing encoder resolution. DS1 specifies a unit with direction sensing, DS2 specifies direction sensing with 2 times count multiplication and DS4 indicates 4 times count multiplication.


CW
Rotation


Resolution:
The following line counts are available as standard:
$1,2,8,48,50,100,120,128,150,200,250,256,264,360$, $400,402,500,508,512,600,625,635,750,850,900,1000$, $1024,1035,1250,1280,1500,1683,1714,1800,2000,2500$ Other line counts within this range can be produced for a onetime tooling charge.

## R-2540

The R-2540 Series Dual Count Incremental shaft encoder offers the high reliability and versatility of the R-2530 with the advantage of providing output resolutions in both English and metric counts. The two outputs are generated from two separate code tracks on the same disc. Each track provides dual outputs in phasequadrature for direction sensing. The user can select either code track at any one time by means of input select line. (logic $1^{1}=$ english logic $\delta^{1}=$ metric)

## HOW TO ORDER

To order, you must supply the sales model number. Codes are shown in color.

R-254

| Type Output: Differential sine wave <br> 2. Zerocrossing <br> 3. TTL compatible |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |

Differential sine wave crossin

TTL compatible


General Specifications:
Operation temperature range $0-55^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ}$ to $90^{\circ} \mathrm{C}$
Humidity $95 \%$ relative, no condensation
Bearing life $19^{9}$ revolutions
Torque .06 oz . in.
Starting moment of inertia $34 \times 10^{-5} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{sec}^{2}$
Maximum speed 5000 rpm
Accuracy $\pm 2$ mins of arc
Direction of rotation - A leads B for CCW rotation when viewed from end of shaft.

## Electrical Specifications:

Type 1 Output
Input voltage $5 \mathrm{VDC} \pm 5^{\circ}$


Quadrature $90^{\circ} \pm 45^{\circ}$
Output format - differential sine wave suitable for input to comparator.

## MECHANICAL OUTLINE


$\mathrm{L}=3.0^{\prime \prime}$ with Cable - Standard
$\mathrm{L}=3.53^{\prime \prime}$ with Connector
$X=.19^{\prime \prime}$ DA15P Connector
$X=.43^{\prime \prime}$ MS3102 Connector
SHAFT $1 / 4^{\prime \prime}$ OD Standard 3/8" OD Optional

Dutput load impedence $470 \Omega$
=requency response
Data . . . 0 to 50 KHz
Index . . 0 to 20 KHz
Type 2 Output
Input voltage $5 \mathrm{VDC} \pm 5 \%$
Output waveform


Quadrature $90^{\circ} \pm 45^{\circ}$ at 5 VDC at $10 \mathrm{KH}_{z}$ Output load impedance $10 \mathrm{k} \Omega \mathrm{min}$ for rated output voltage
Frequency response $50 \mathrm{KHz}_{z}$
Data. . . 0 to $50 \mathrm{KH}_{\mathrm{Z}}$
Index. . 0 to $20 \mathrm{KHz}_{\mathrm{z}}$

## Type 3 Output

Input voltage $5 \mathrm{VDC} \pm 5 \%$
Output waveform: TTL compatible
Quadrature $90^{\circ} \pm 45^{\circ}$ VDC at $10 \mathrm{KH}_{\mathrm{Z}}$


## Resolution:

Dual count encoders permit selection of one of two different line counts by means of a logic " 1 " or " 0 " on an input line. The following dual line counts are available as standard:
1000/254, 1270/500, 1270/1000, 2000/508, 2000/1969, 2540/1000, 2540/2000, 2540/2500, 763/25

Other line counts within the range of 1 to 2500 can be produced for a one-time tooling change.

## R-2540XD

The R-2540XH Series is a Heavy Duty version of the R-2540. It has been designed to be extremely rugged and reliable for those heavy duty machine tool and process control applications. Like the R-2540 this encoder is available with switch selectible english/metric outputs. Also additional signal conditioning is available such as $\times 2$, or $\times 5$ interpolation providing standard output as high as 12,700 counts per revolution.

## HOW TO ORDER

To order you must supply the sales model number. Codes are shown in color.


## General Specifications:

Operation temperature range $0-55^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ}$ to $+95^{\circ} \mathrm{C}$
Humidity 95\% relative, no condensation
Shock 50 g for 11 msec
Vibration to $2000 \mathrm{H}_{\mathrm{z}}$ at 15 g 's
Sand and dust - not detrimental
Bearing life $10^{9}$ revolutions
Torque .06 oz . in.
Starting moment of inertia $5 \times 10^{-4} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{sec}^{2}$
Maximum speed 5000 rpm
Accuracy $\pm 2 \mathrm{~min}$. of arc
Direction of rotation - A leads B for CCW rotation when viewed from end of shaft.

## Electrical Specifications:

Type 1 Output
Output voltage 5 VDC $\pm 5 \%$

Signal conditioning electronics:
Interpolation
General interpolation is an electronic detection method used to increase the resolution of an encoder by a factor of 2 INT2 or by 5 INT5

The output characteristics of an interpolate encoder are considerably different than a typical direct unit. The usual terminology of quadrature accuracy and symmetry ratio have little meaning and should not be used. (For example, it is possible to have a given transition out of its correct position by as much as one cycle $-360^{\circ}$ - and the encoder will still count correctly).

A more useful term is transition accuracy (T.A.). This describes how far any given transition is from its correc location. It is expressed in percent of a quantum (Q), where a quantum is the interval between adjacent transitions.


When the encoder is operated at constant speed the outputs as displayed on a scope are shown at the right.

Specifications:

$$
\begin{array}{cl}
\text { Conditions: } & \begin{array}{l}
\text { Supply voltage } \\
\text { Temperature }
\end{array} \\
\hline 0 \text { to } 55^{\circ} \mathrm{C} \mathrm{CDC}
\end{array}
$$

Disc Resolution: (cycles per turn)


For INT 2 Units At 20 KC At 100 KC For INT 5 Units At 50 KC At 250 KC

For INT 2 Units At 20 KC At 100 KC For INT 5 Units At 50 KC At 250 KC
1270 and Below

| Transition | Quanta |
| :--- | :---: |
| Accuracy | Minimum |
| $\pm 0.5 \mathrm{Q}_{\mathrm{N}}$ | $0.5 \mathrm{Q}_{\mathrm{N}}$ |
| $\pm 1 \mathrm{Q}_{\mathrm{N}}$ | $0.2 \mathrm{Q}_{\mathrm{N}}$ |

Above 1270

| Transition | Quanta |
| :---: | :---: |
| Accuracy | Minimum |
| $\pm 1 \mathrm{Q}_{\mathrm{N}}$ | $0.25 \mathrm{Q}_{\mathrm{N}}$ |
| $\pm 1.5 \mathrm{Q}_{\mathrm{N}}$ | $0.1 \mathrm{Q}_{\mathrm{N}}$ |
|  |  |
| $\pm 2 \mathrm{Q}_{\mathrm{N}}$ | $0.25 \mathrm{Q}_{\mathrm{N}}$ |
| $\pm 3 \mathrm{Q}_{\mathrm{N}}$ | $0.1 \mathrm{Q}_{\mathrm{N}}$ |

## R-2553

The R-2533 Series absolute position encoder is available in either natural binary, binary coded decimal or grey code with up to 12 bits parrallel as standard resolution.
This absolute encoder provides a "whole word" output that assures an exact reading of shaft position in those applications where power to the encoder or system has been interrupted and then reinstated.
Low resolution absolute encoder ( 8 bits \& under)
 offer an optional latch output (quick look). Modular kit absolute encoders are also available for mounting to DC servomotors.

## HOW TO ORDER

To order, you must supply sales model number. Codes are shown in color


## General Specifications

Operating temperature range $0-50^{\circ} \mathrm{C}$
Storage temperature range $-30^{\circ} \mathrm{C}$ to $+90^{\circ} \mathrm{C}$
Humidity $95 \%$ relative, no condensation

Bearing life - $10^{9}$ revolutions
Torque .05 oz . in.
Starting moment of inertia $34 \times 10^{-5} \mathrm{oz}$. in. $\mathrm{sec}^{2}$
Maximum acceleration $50,000 \mathrm{rad} / \mathrm{scc}^{2}$
Maximum speed 5000 rpm (non operative)
Direction of rotation, CCW Standard, CW optional
Output waveform $-T^{2} \mathrm{~L}$ compatible
Input voltage 5 VDC $\pm 5 \%$
Frequency Response $50 \mathrm{KH}_{\mathrm{Z}}$
Accuracy $\pm 1 / 2$ bit

- S Special order options

The following list of special order options can be supplied Please specify with order.

- Shaft seal
- Low torque (. 01 oz. in.)
- Special output (cmos, line drives, etc.)
- Alternate face mounting hole pattern
- Special terminations, cables with connectors, etc.
- Latch (Quick look)



## LATCH OUTPUT OPTION DESCRIPTION

The latch output option provides up to 8 bits of storage, allowing for the holding (latch) of a shaft angle position code while the shaft continues to rotate. The position code is refreshed upon demand by the operator and will provide the exact shaft angle code as of the time of last inquiry. The latch option may be disabled by the operator at which time the output signals will continue to change with shaft rotation. The engaging and disengaging of the latch option is accomplished thru the input line.
"
*Length depends on exact combination of code, resolutions and termination. For 8 bit CG and NB length is 1.98 min , to 2.73 max. For all other lengths is 4.75 min . to 6.0 max.

## Code

BDC
XS3
XS14

Grey Code
Natural Binary
(8421)
(8421)
(8421)

|  | Cod |
| :--- | ---: |
| GC | Grey Co |
| NB | Natural |
|  |  |
|  |  |
| BDC | (8421) |
| XS3 | (8421) |
| XS14 | (8421) |

Complimentary outputs are also available, add a C at the end of the resolution code. For example if you want an encoder with 8 bits of natural binary plus compliments you would specify 8 NBC as the output and resolution model number.

## Resolution

8 bit $=256$ unique words per turn
9 bit $=512$ unique words per turn
10 bit $=1024$ unique words per turn
11 bit $=2048$ unique words per turn
12 bit $=4096$ unique words per turn
$100 \quad(0-99)$ unique words per turn
$360(0-359)$ unique words per turn
$1000(0-999)$ unique words per turn
$2000(0-1999)$ unique words per turn
$3600(0-3599)$ unique words per turn

## M-100/M-110

The model $\mathrm{M}-100$ is a separate moving coil tachometer designed for use in applications requiring high quality velocity feedback is with minimum system inertia load. Commutators are manufactured from coil silver and are diamond cut after assembly to a surface finish better than .000020 . Very long life-exceeding 10,000 hours at 3,000 RPM-may be obtained even in systems where high frequency shaft reversal and start-stop operation is a normal operating mode.
The model M-110 is a modular version of the same tachometer integrally mounted to any Electro-Craft
 servomotor. Shaft coupling problems are totally eliminated and due to the very low inertia of the tachometer armature, the lowest shaft torsional resonant frequency exceeds 3.5 KHz .

## HOW TO ORDER

To order, you must supply sales model number. For complete tachometer order

M-100 -
 -
(A winding supplied unless otherwise specified)

For modular tachometer order
motor model - winding model $/ 110-\frac{}{\text { tach winding }}-S_{\text {for special }}$
The M-110 tachometer can be supplied with any Electro-Craft motor.

Typical oscilloscope picture of $\mathrm{M}-100 / \mathrm{M}-110$ tachometer output showing AC ripple component

STANDARD RIPPLE FILTER SCHEMATIC



## PERFORMANCE SPECIFICATIONS

## Parameter Linearity

Ripple
M-100
M-110
Ripple Frequency
Speed Range
Temperature Coefficent
Armature Inertia

$$
\begin{aligned}
& \mathrm{M}-100 \\
& \mathrm{M}-110
\end{aligned}
$$

Insulation Resistance
Friction Torque
Torsional Resonant Frequency When Coupled to an Electro-Craft MCM Servomotor Rated Operating Life
Operating Temperature Range
$\frac{\text { Value }}{.2}$
1.5

2
19
1-6,000
$-.01$
$9 \times 10^{-5}$
$7 \times 10^{-5}$
10
.25

## 3.5

10,000
0-155
$\frac{\text { Units }}{\% \text { Max. }}$ Deviation from Perfect Linearity
Max Percentage Peak to Peak AC
Component in DC output at 167 to 6,000 RPM
Cycles/Revolution
RPM
$\% /{ }^{\circ} \mathrm{C}$
oz. in. $\mathrm{sec}^{2}$
oz. in. sec ${ }^{2}$
Megohm
oz. in., max.
$\mathrm{KHz}, \min$.
Hours at 3,000 RPM
${ }^{\circ} \mathrm{C}$

WINDING VARIATIONS

## Winding Constants <br> Output Voltage Gradient (Volts/KRPM) <br> Armature Resistance (ohms) <br> Armature Inducance (mhy) <br> Recommended Minimum Load Impedance (ohms)

| Winding |  |
| :--- | :--- |
| $\frac{A}{3.0}$ | $\frac{B}{2.5}$ |
| 160 | 160 |
| 4 | 4 |
| 10,000 | 10,000 |

## E-591/E-596

The Model E-591 separate permanent-magnet tachometer is a precision DC rate generator suitable for use in a wide range of industrial velocity and position control systems where extremely low inertia or ripple is not required. The E-591 is supplied with standard base and face mounting.

The Model E-596 is a modular version of the same tachometer integrally coupled to an Electro-Craft servomotor.

## HOW TO ORDER

To order, you must supply sales model number. For complete tachometer order
E-591 - $\frac{\text { tach winding }}{-}$ for special
For modular tachometer order
$\overline{\text { motor model }}-\frac{}{\text { motor winding }}-/ 596-\frac{S_{\text {for special }}}{}$
The E-596 tachometer can be supplied with all E-500, E-600, and E-700 series motors.

Typcial oscilloscope picture of E-591/596 tachometer output showing A.C. ripple component


STANDARD RIPPLE FILTER SCHEMATIC
$12 \mathrm{k} \Omega$



## PERFORMANCE SPECIFICATIONS

| $\frac{\text { Parameter }}{\text { Linearity }}$ | $\frac{\text { Value }}{.2}$ |
| :--- | :--- |
| Ripple | 5 |
|  |  |
| Ripple Frequency | 11 |
| Speed Range | $1-8,000$ |
| Temperature Coefficient | -.05 |
| Armature Inertia | $1.4 \times 10^{-3}$ |
| Insulation Resistance | 10 |
| Friction Torque | 3 |
| Rated Life | 5,000 |
| Operating Temperature Range | $0-75$ |

$\frac{\text { Value }}{.2}$ 5

11
1-8,000
$-.05$
10
3
0.75

Units
\% Max. Deviation from Perfect Linearity
Max. Percentage Peak to Peak AC Component in DC output at 500 RPM
Cycles/Revolution
RPM
$\% /{ }^{\circ} \mathrm{C}$
oz. in. $\mathrm{sec}^{2}$
Megohm
oz. in., max.
Hours at 3 KPRM
${ }^{\circ} \mathrm{C}$

WINDING VARIATIONS
Winding

## Winding Constants

Output Voltage Gradient (Volts/KPRM)
Armature Resistance (ohms)
(includes temperature compensation network)

Armature Inductance (mhy)
Optimum Load Impedance (ohms)

| $\frac{A}{3.0}$ | $\frac{B}{C .0}$ | $\frac{C}{14.2}$ | $\frac{D}{21}$ |
| :--- | :--- | :--- | :--- |
| 570 | 570 | 720 | 950 |
| 6.2 | 33 | 138 | 255 |
| 5,000 | 5,000 | 5,000 | 10,000 |

## E-796

The E-796 modular tachometer is used with larger servomotors in applications requiring a rugged mechanical construction with a large diameter thru-shaft. Shown at the right is an E-703 servomotor with and E-796 integrally mounted.


## HOW TO ORDER

To order, you must supply sales model number. For complete tachometer order
$\qquad$ motor winding 1796- $\qquad$ for special

The E-796 modular tachometer can be supplied with all E-700 series, E-5260, and $\mathrm{M}-1700$ servomotors.

Typical oscilloscope picture of E-796 tachometer output showing A.C. ripple component

## STANDARD RIPPLE FILTER SCHEMATIC


D.C. voltage $=3.74$


## PERFORMANCE SPECIFICATIONS

| $\underline{\text { Parameter }}$ | Value | Units |
| :---: | :---: | :---: |
| Linearity | . 2 | \% Max. Deviation from Perfect Linearity |
| Ripple | 5 | Max. Percentage Peak to Peak AC Component in DC output at 500 RPM |
| Ripple Frequency | 21 | Cycles/Revolution |
| Speed Range | $1-4000$ | RPM |
| Temperature Coefficient | . 05 | \%/ ${ }^{\text {c }}$ C |
| Armature Inertia | . 04 | oz. in. $\mathrm{sec}^{2}$ |
| Insulation Resistance | 10 | Megohm |
| Friction Torque | 7 | oz. in., max. |
| Rated Life | 10,000 | Hours at 4 KRPM |
| Operating Temperature Range | 0.75 | ${ }^{\circ} \mathrm{C}$ |

WINDING VARIATIONS
Winding

## Winding Constants

## Output Voltage Gradient (Volts/KRPM)

Armature Resistance (ohms)
(includes temperature compensation network)
Armature Inductance (mhy)
Optimum Load Impedance (ohms)

| A | B | C |  |
| :---: | :---: | :---: | :---: |
| 3.0 | 7.0 | 14.2 | 21 |
| 535 | 550 | 570 | 590 |
| . 5 | 2.0 | 8 | 25 |
| 5,000 | 5,000 | 5,000 | 10,000 |

## Servomotor Controllers

Based upon broad applications experience, Electro-Craft has assembled a selection of predesigned servomotor control systems. Consisting of a sophisticated four quadrant servo control amplifier and a high performance DC motor, these systems are available for immediate application where their specifications meet your design criteria.

In the following section, you will find complete specifications and performance data on the Electro-Craft motor control line.

| RATINGS | P6100 | P6200 | P6300 |
| :--- | :--- | :--- | :--- |
| Voltage at Rated |  |  |  |
| $\quad$ Current | $\pm 60 \mathrm{~V}$ | $\pm 55 \mathrm{~V}$ | $\pm 55 \mathrm{~V}$ |
| Rated Current | 7 A | 15 A | 15 A |
| Peak Current | 12 A | 20 A | 25 A |
| Rated Power | 420 W | 825 W | 825 W |
| Peak Power | 670 W | 1000 W | 1150 W |
| Bandwidth | 1000 Hz | 1000 Hz | 1000 Hz |



## FEATURES

Unique Open Modular Construction
Completely self contained
Plug in Standard or Custom Options
Regenerative Energy Protected

Quiet Operation

115/230 V AC or DC Power Options<br>Independent Average and Peak Current Limits<br>Crow-Bar Protected AC Power Supply

## DESCRIPTION

The P6000 series are the fastest response pluse width modulated servo controls competitively available, providing bandwidths at full load exceeding 1000 Hz . The unique fixed frequency design gives quiet operations free from annoying audible "chirps". The standard unit, which is available for operation from AC line, has the following features: three input fully adjustable summing amplifier with independent gain and lead-lag adjustment to enable operation with a wide range of motor and load types; a dual stage current limiter providing independent adjustment of peak and average current limit as well as pulse duration; unique crowbar protection which protects the amplifier from damage due to short circuits, motor failure or misuse; space for two 4-5/16 $\times 2-3 / 4^{\prime \prime}$ option cards which can be selected from a wide range of standard options or custom designed options; logic compatible end of travel limit and clamping inputs; all major subassemblies are mounted on plug-in connectors to enable replacement without the need to change the entire unit; and all major test points are brought out to a 14 pin array for fast, easy troubleshooting and set up. This, combined with the unique open construction, makes the P6000 series the most accessible and serviceable units of their type available. The P6000 series amplifiers can be readily adapted, by means of plug-in options, for interfacing with an unlimited range of remote inputs, programmers, and computer controls. Large volume users can purchase the basic modules and harnesses for use in applications where space and cost are critical.
The P6000 amplifiers are also available in Electro-Craft's D.P.S. range of Digital Positioning Systems which typically can provide fully programmable step motion drives capable of completing single steps of $1.8^{\circ}$ in 8 ms including settling, and slew speeds of 15,000 steps per second with inertial loads of 0.5 oz . in. $\sec ^{2}{ }^{\circ}$ and friction loads of 100 oz . in. The D.P.S. series are available in a wide range of step sizes; consult our factory for further details.

## HOW TO ORDER

Specify model number followed by enclosure option code in parenthesis e.g. P6200 (O/S). List other options from pages 70 and 71 separately.


## INDUSTRIAL NEMA 12 ENCLOSURE



| PERFORMANCE CONTINUED |  | P6100 | P6200 | P6300 |
| :---: | :---: | :---: | :---: | :---: |
| Parameter | Conditions | Value | Value | Value |
| Frequency |  |  |  |  |
| Response | Open loop - 3db point | 1 Khz nom | 1 kHz nom | 1 kHz nom |
| Gain | Open loop (adjustable) reference input | 0-3750 Amps/volt | 0-7500 Amps/volt | 0-7500 Amps/volt |
|  | Open loop (adjustable) velocity tach input | 0-1380 Amps/volt | 0-2760 Amps/volt | 0-2760 Amps/volt |
|  | Open loop (selectable) auxiliary | 0-3750 Amps/volt | 0.7500 Amps/volt | 0-7500 Amps/valt |
| Offset Voltage |  | adjustable to zero | adjustable to zero | adjustable to zero |
| Deadband |  | negligible | negligible | negligible |
| Drift | Maximum gain referred to input, auxiliary input not in use <br> For details consult 741 op-amp data. | $20{ }^{\circ} \mathrm{CO}$ nom | $20{ }^{\circ}{ }_{\mathrm{C}}{ }^{\text {c }}$ nom | $20{ }^{\circ} \mathrm{CV}$ uv ${ }^{\text {c }}$ |
| STANDARD FEATURES |  |  |  |  |
| Parameter | Conditions | Value | Value | Value |
| Current Limit <br> Peak Current | Dual stage, independent peak and average limits |  |  |  |
|  | Range (adjustable) | Average setting to 12 A nom | Average setting to 25 A nom | Average setting to 25 A nom |
|  | Duration (adjustable) | 10 msec to 100 msec nom | 10 msec to 100 msec nom | 50 msec to 400 msec nom |
| Average Current | Range (adjustable) | 3 to 7 A nom | 6 to 16 A nom | 6 to 15 A nom |
| Cro-Bar |  |  |  |  |
| Time to Clear Fuse Time to Reduce | Amplifier output shorted + to - or to ground | 4 msec nom | 4 msec nom | 4 msec nom |
| Power Input to 10 v | Amplifier output shorted + to - or to ground | $500 \mu \mathrm{sec}$ nom | $500 \mu \mathrm{sec}$ nom | $500 \mu \mathrm{sec}$ nom |
| Current Monitor |  |  |  |  |
| Output Signal |  | . $4 \mathrm{v} / \mathrm{A}$ | .2v/A | .2v/A |
| Remote Current | Peak | 10k $\Omega$ | 10k $\Omega$ | 10k $\Omega$ |
| Limits Pots | Average | 20k $\Omega$ | 20k $\Omega$ | 20k $\Omega$ |
| End of Travel <br> Limts and <br> Amplifier Clamp | Enable | switch closure switch open | switch closure switch open | switch closure switch open |
|  | Inhibit |  |  |  |
|  | or |  |  |  |
|  | Enable | TTL low | TTL low | TTL low |
|  | Inhibit | TTi high | TTL high | TTL high |

[^5]
## P6100 MAXIMUM OUTPUT VOLTAGE VS CURRENT



## Rack Enclosure

The motor control can be supplied in a standard 19" rack enclosure complete with power switch and lamp mounted on front panel. The enclosure provides mechanical protection, protection from shock hazards should be provided by the rack cabinet in which the unit is mounted. Electro-Craft can provide a variety of rack cabinets and enclosures; for details contact the factory sales office.
(enclosure option code R19)


## VOLTAGE VS CURRENT



## Industrial Enclosure

This Nema 12 enclosure is recommended for industrial environments. Standard finish is light blue (DuPont 93-63189). Special finishes can be provided. Cable entry glands are provided loose for customer mounting.
(enclosure option code N12)

## Bench Enclosure

This enclosure is designed for free standing laboratory type applications. The enclosure provides mechanical and shock hazard protection. A power switch and lamp are provided on the front of the unit.
(enclosure option code BEN)



## OPTIONS FOR USE WITH P6000 SERVO MOTOR CONTROLS



Speed Meter (Option Number SM505)
Center zero meter with scale 5000-0-5000 rpm to indicate motor speed $\pm 5 \%$. Available as a loose kit or mounted in controller panel on enclosed units.

## Torque Meter (Option Number TM505)

Center zero meter with scale 5-0-5 oz. in. x 10 or 100 to indicate total torque generated by motor $\pm 5 \%$. Available as a loose kit or mounted in controller panel on enclosed units.

## Speed/Torque Meter (Option STM505)

Combined meter with, scales for both speed and torque as previously described. Option includes toggle switch for selecting speed or torque read out. Available as a loose kit or mounted in controller panel on enclosed units.

## Ramp Generator (Option RGI)

Plug in printed circuit card providing adjustable acceleration and deceleration rate. Solid state inputs permit the selection of two acceleration rates and two deceleration rates. Two run/stop inputs permit external selection of two running speeds or control by an external analog speed command derived from a potentiometer or an automatic control source. To order specify option number, if ordered with a P6000 amplifier the ramp generator will be supplied installed in the amplifier. Switches and speed controls for use with the ramp generator can be ordered from the control options following.


Speed and Torque Controls

|  |  | 10 Turn | 1 Turn |
| :--- | :--- | :--- | :--- |
| Option Numbers | Speed | SP10 | SP1 |
|  | Torque | TP10 | TP1 |

Optional controls for external adjustment of motor speed or torque. These options are available as single or ten turn controls and are complete with knob or turn counting dial where appropriate. On enclosed controllers these options can be supplied mounted in the controller front panel. The option includes a reversing switch with off position.

## Run Stop Switch (Option Number RSSI)

This three position switch enables the motor to run at the set speed, stop under full control using the full braking capacity of the drive or stop in a mode where the motor is free to rotate by hand. This option can be supplied loose or mounted in the front panel on enclosed models.

## 9000 AMPLIFIER

OUTPUT RATINGS

$\frac{\text { Parameter }}{\text { Output }}$| Voltage |
| :--- |

Current

Power

Temperature
Range

Conditions
Maximum
At continuous rated current (nominal) At continuous rated current (low line)
At peak rated current (nominal)
At peak rated current (low line)

| Current | Continuous rated (independent of line) <br> Peak rated (500 msec maximum) <br> (independent of line) |
| :--- | :--- |
| Power | Maximum continuous |
|  | (nominal line) <br> (low line) <br> (nominal line) <br> (low line) |
| Temperature <br> Range | Operating <br> Storage |


| 9215 | 9230 |
| :---: | :---: |
| Value | Value |
| $\pm 90 \mathrm{~V}$ | $\pm 90 \mathrm{~V}$ |
| $\pm 72 \mathrm{~V}$ | $\pm 66 \mathrm{~V}$ |
| $\pm 64 \mathrm{~V}$ | $\pm 60 \mathrm{~V}$ |
| $\pm 67 \mathrm{~V}$ | $\pm 56 \mathrm{~V}$ |
| $\pm 60 \mathrm{~V}$ | $\pm 50 \mathrm{~V}$ |
| 7 A | 15 A |
| 15 A | 30 A |
| 500 W | 990 W |
| 450 W | 900 W |
| 1000 W | 1680 W |
| 900 W | 1500 W |
| $\begin{aligned} & 0-50^{\circ} \mathrm{C} \\ & -30^{\circ} \text { to } 80^{\circ} \mathrm{C} \end{aligned}$ | ${ }_{-30}-50{ }^{\circ} \mathrm{C}$ |

INPUT REQUIREMENTS

| Parameter | Conditions |  | Value | Value |
| :---: | :---: | :---: | :---: | :---: |
| Voltage | Nominal | (single phase) | 115/230v RMS | 115/230v RMS |
|  | High line | (single phase) | 125/250v RMS | 125/230v RMS |
|  | Low line | (single phase) | 105/210v RMS | 105/210v RMS |
| Frequency |  | (single phase) | $50 / 60 \mathrm{~Hz}$ | $50 / 60 \mathrm{~Hz}$ |

## PERFORMANCE

| $\frac{\text { Parameter }}{\text { Frequency }}$ | Conditions |
| :--- | :--- |
| Response | Open loop -3 db point |
| Gain | Open loop (adjustable) reference input <br> Open loop (adjustable) velocity tach input |
|  | Open loop (selectable) auxiliary |

Value
1 Khz nom
0-3750 Amps/V
0-1380 Amps/V
0-3750 Amps/V

## Value

1 Khz nom
$0-7500 \mathrm{Amps} / \mathrm{V}$
0-2760 Amps/V
0-7500 Amps/V


## PERFORMANCE CONTINUED

| Parameter | Conditions | Value | Value |
| :---: | :---: | :---: | :---: |
| Offset Voltage |  | adjustable to 0 | adjustable to 0 |
| Deadband |  | negligible | negligible |
| Drift | Maximum gain referred to input, auxiliary input not in use For details consult 741 op-amp data | $20{ }_{{ }^{\mathrm{o}}}^{\mathrm{C}} \text { nom }$ | ${ }^{20}{ }_{\mathrm{o}}^{\mathrm{uv}} \text { nom }$ |

## STANDARD FEATURES

## Parameter

Current Limit
Cro-Bar
Time to Clear Fuse
Time to Reduce
Power Input to 10 V
Current Monitor
Output Signal
Remote Current
Limit Pot
End of Travel
Limits and Amplifier Clamp

| $\frac{\text { Conditions }}{\text { Range (adjustable) }}$ | $\frac{\text { Value }}{15 \mathrm{~A} \mathrm{Max}}$ | $\frac{\mathrm{Value}}{30 \mathrm{~A} \mathrm{Max}}$ |
| :--- | :--- | :--- |
| Amplifier output shorted + to - or to ground | 4 msec nom | 4 msec nom |
| Amplifier output short + to - or to ground | $500 \mu \mathrm{sec}$ nom | $500 \mu \mathrm{sec}$ nom |
|  |  |  |
|  | $.125 \mathrm{~V} / \mathrm{A}$ | $.25 \mathrm{~V} / \mathrm{A}$ |
|  | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ |
| Enable | 0 V | 0 V |
| Inhibit | 0 V | 0 V |
| or | $5-15 \mathrm{~V}$ | $5-15 \mathrm{~V}$ |
| Enable | $0-0 \mathrm{~V}$ | $0-0 \mathrm{~V}$ |
| Inhibit | $5-15 \mathrm{~V}$ | $5-15 \mathrm{~V}$ |

## EXTERNAL CONNECTIONS

Pin \& Socket connectors (mating connectors are supplied with each unit).

## PROTECTION

| $1^{2} \mathrm{~T}$ | Over Voltage |
| :--- | :--- |
| Logic Supplies | Buss Fault |
|  | Regenerative Energy |

## Servomotor Control Systems

The following pages describe off-the-shelf combinations of high performance servomotors and regenerative servo controllers.

The systems are suitable for all types of high performance speed and position controls where bi-directional (four quadrant) operation is required.

The servo motor controllers are described fully in the previous section (starting on page 63) and are available with all of the options described in that section.

These control packages are fully preadjusted and run-in for 48 hours prior to shipment.

## 0400-00-049



## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Motor Control Model | P6100AP | P6200AP | P6300AP |  |
| Maximum Speed (No Load) | 6000 | 6000 | 6000 | rpm |
| Maximum Continuous Torque Output | 25 | 25 | 25 | oz. in. |
| Maximum Continuous Power Output | 45 | 45 | 45 | Watts |
| Maximum Peak Torque Output | 54 | 100 | 130 | oz. in. |
| ${ }^{*}$ Speed Regulation (Typical) | -2 | -2 | -2 | \% of set speed |
| ${ }^{*}$ Speed Range (Typical) | 1000:1 | 1000:1 | 1000:1 |  |
| Motor Specifications (Nominal) |  |  |  |  |
| Torque Constant |  |  | 5.6 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  |  | 4.2 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  |  | 1.25 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  |  | . 004 | oz. in. s. ${ }^{2}$ |
| Rotational Loss Torque |  |  | 1.0 | oz. in./krpm |
| Static Friction Torque |  |  | 3.0 | oz. in. |
| Armature Inductance |  |  | 2.25 |  |
| Armature to Ambient Thermal Resistance |  |  | 4.0 | oc/Watt |
| Maximum Armature Temperature |  |  | 155 |  |
| Tachometer Specifications (Nominal) |  |  |  |  |
| Voltage Gradient |  |  | 3 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  |  | 150 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  |  | 4 |  |
| Ripple Amplitude |  |  | 2 | \% of DC output |
| Ripple Frequency |  |  | 19 | Cycles per rev. |
| Temperature Stability |  |  | -. 01 | \%/ ${ }^{\circ} \mathrm{C}$ |
| Linearity |  |  | 0.2 | \% max error |

[^6]

| CONNECTION COOE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FUNCTION |  | TAB | $\begin{aligned} & \text { TAB } \\ & \text { S12E } \\ & \hline \end{aligned}$ | COLOR CODE |
| MTR | * | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | $\begin{array}{r} .250 \\ .250 \\ \hline \end{array}$ | $\begin{aligned} & \text { RED } \\ & B L K \end{aligned}$ |
| TACH | $\begin{aligned} & \text { NOT } \\ & \text { NSED } \\ & \hline \end{aligned}$ | 3 4 5 | . 167 | BLU WHI |
| OPT. ENC. | +15 -15 COM ENC 2 ENC 1 INDEX SHIELD | $\begin{aligned} & \hline 6 \\ & 7 \\ & 8 \\ & 9 \\ & 10 \\ & 11 \\ & 12 \\ & \hline \end{aligned}$ | , | $\begin{aligned} & \text { RED } \\ & \text { BLU } \\ & \text { BLK } \\ & \text { GRN } \\ & \text { WHT } \\ & \text { ORN } \end{aligned}$ |
| $\begin{array}{\|l\|} \hline \text { MTR } \\ \text { OND } \\ \hline \end{array}$ | FRAME | 13 | 250 | - |

System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 1000:1

## MT SERIES GENERAL INFORMATION



## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face $\quad 0.010 \mathrm{in} / \mathrm{in}$
Shaft concentricity to mounting pilot
Shaft end play
Shaft radial play
Maximum shaft radial load 0.005 in $\pm 0.004$ in maximum $\pm 0.0015$ in maximum $\pm 30$ lbs $\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of the MT series servo motors is
$3 \times 10^{8}$ revolutions at $60 \%$ rated load, and;
$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load.

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

RECOMMENDED WIRE SIZES

| Motor armature | 14 gage |
| :--- | :--- |
| Tachometer | 18 gage |
| Position encoder | 24 gage (optional) |

## 0660-06-011

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Motor Control Model | P6100AP | P6200AP | P6300AP |  |
| :---: | :---: | :---: | :---: | :---: |
| Maximum Speed (No Load) | 4000 | 4000 | 4000 | rpm |
| Maximum Continuous Torque Output at Stall | 120 | 120 | 120 | oz. in. |
| Maximum Continuous Power Output | 270 | 270 | 270 | Watts |
| Maximum Peak Torque Output | 240 | 410 | 515 | oz. in. |
| * Speed Regulation (Typical) | -2 | -2 | -2 | \% of set speed |
| ${ }^{*}$ Speed Range (Typical) | 1000:1 | 1000:1 | 1000:1 | - |
| Motor Specifications (Nominal) |  |  |  |  |
| Torque Constant |  |  | 21 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  |  | 16 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  |  | 1.15 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  |  | . 03 | oz. in. s. ${ }^{2}$ |
| Rotational Loss Torque |  |  | 1.0 | oz. in./krpm |
| Static Friction Torque |  |  | 7.0 | oz. in. |
| Armature Inductance |  |  | 4 |  |
| Armature to Ambient Thermal Resistance |  |  | 2.8 | ${ }^{\circ} \mathrm{C} /$ Watt |
| Maximum Armature Temperature |  |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |  |
| Voltage Gradient |  |  | 14.2 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  |  | 750 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  |  | 140 | mH |
| Ripple Amplitude |  |  | 5 | \% of DC output |
| Ripple Frequency |  |  | 11 | Cycles per rev. |
| Temperature Stability |  |  | -0.05 | \%/ ${ }^{\circ} \mathrm{C}$ |
| Linearity |  |  | 0.20 | \% max error |



System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 1000:1

## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face Shaft concentricity to mounting pilot Shaft end play Shaft radial play Maximum shaft radial load Maximum shaft thrust load
$0.010 \mathrm{in} / \mathrm{in}$ 0.005 in $\pm .01$ in maximum $\pm 0.0015$ in maximum $\pm 30 \mathrm{lbs}$ $\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of these servo motors is:

$$
3 \times 10^{8} \text { revolutions at } 60 \% \text { rated load, and; }
$$

$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load. Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

## 0660-08-014

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |
| :---: | :---: | :---: | :---: |
| Motor Control Model | P6200AP | P6300AP |  |
| Maximum Speed (No Load Peak) | 6000 | 6000 | rpm** |
| Maximum Continuous Torque Output at Stall | 120 | 120 | oz. in. |
| Maximum Continuous Power Output | 270 | 270 | Watts |
| Maximum Peak Torque Output | 280 | 350 | oz. in. |
| ${ }^{*}$ Speed Regulation (Typical) | -2 | -2 | \% of set speed |
| *Speed Range (Typical) | 1000:1 | 1000:1 | - |
| Motor Specification (Nominal) |  |  |  |
| Torque Constant |  | 14.5 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  | 11 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  | 0.75 | $\Omega$ at $250 \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  | 0.03 | oz. in. s ${ }^{2}$ |
| Rotational Loss Torque |  | 1.0 | oz. in./krpm |
| Static Friction Torque |  | 7.0 | oz. in. |
| Armature Inductance |  | 1.6 | mH |
| Armature to Ambient Thermal Resistance |  | 2.8 | oc/Watt |
| Maximum Armature Temperature |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |
| Voltage Gradient |  | 14.2 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 750 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  | 140 |  |
| Ripple Amplitude |  | 5 | \% of DC output |
| Ripple Frequency |  | 11 | Cycles per rev. |
| Temperature Stability |  | $-0.05$ | \%/ ${ }^{\circ} \mathrm{C}$ |
| Linearity |  | 0.20 | \% max error |

*Speed range represents the speed range for which the specified regulation applies.

[^7]

Typical speed regulation $2 \%$ of set speed over 1000:1

## TYPICAL SHAFT TOLERANCES

bhaft perpendicularity to mounting face shaft concentricity to mounting pilot Shaft end play

## shaft radial play

Vaximum shaft radial load Vaximum shaft thrust load
$0.010 \mathrm{in} / \mathrm{in}$
0.005 in
$\pm .01$ in maximum $\pm 0.0015$ in maximum $\pm 30 \mathrm{lbs}$ $\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of these servo motors is:
$3 \times 10^{8}$ revolutions at $60 \%$ rated load, and;
$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load.
Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

## 0660-08-015

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |
| :---: | :---: | :---: | :---: |
| Motor Control Model | P6200AP | P6300AP |  |
| Maximum Speed (No Load Peak) | 6000 | 6000 | rpm** |
| Maximum Continuous Torque Output at Stall | 120 | 120 | oz. in. |
| Maximum Continuous Power Output | 270 | 270 | Watts |
| Maximum Peak Torque Output | 280 | 350 | oz. in. |
| ${ }^{*}$ Speed Regulation (Typical) | 2 | 2 | \% of set speed |
| *Speed Range (Typical) | 1000:1 | 1000:1 | - |
| Motor Specification (Nominal) |  |  |  |
| Torque Constant |  | 14.5 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  | 11 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  | 0.75 | $\Omega$ at 250C $\pm 15 \%$ |
| Armature Moment of Inertia |  | 0.03 | oz. in. s ${ }^{2}$ |
| Rotational Loss Torque |  | 1.0 | oz. in./krpm |
| Static Friction Torque |  | 7.0 | oz. in. |
| Armature Inductance |  | 1.6 | mH |
| Armature to Ambient Thermal Resistance |  | 2.8 | ${ }^{\circ} \mathrm{C} /$ Watt |
| Maximum Armature Temperature |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |
| Voltage Gradient |  | 14.2 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 750 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  | 140 | mH |
| Ripple Amplitude |  | 5 | \% of DC output |
| Ripple Frequency |  | 11 | Cycles per rev. |
| Temperature Stability |  | $-0.05$ | \%/0C |
| Linearity |  | 0.20 | \% max error |
| *Speed range represents the speed range for which the specified regulation applies. |  |  |  |



System if fully regenerative and can operate in all four speed/torque quadrants

MAXIMUM SPEED VS TORQUE OUTPUT


Typical speed regulation $2 \%$ of set speed over 1000:1

## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face Shaft concentricity to mounting pilot
Shaft end play
Shaft radial play Maximum shaft radial load Maximum shaft thrust load
$0.010 \mathrm{in} / \mathrm{in}$ 0.005 in $\pm 0.004$ in maximum $\pm 0.0015 \mathrm{in} / \mathrm{in}$ $\pm 30 \mathrm{lbs} / \mathrm{in}$ $\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of these servo motors is:
$3 \times 10^{8}$ revolutions at $60 \%$ rated load, and;
$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load.
Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

## 0660-10-029

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' $S$ ' to the end of the part number and describe your special requirement.

Performance Ratings

| Motor Control Model | P6100AP | P6200AP | P6300AP |  |
| :---: | :---: | :---: | :---: | :---: |
| Maximum Speed (No Load) | 4200 | 4200 | 4200 | rpm |
| Maximum Continuous Torque Output at Stall | 120 | 120 | 120 | oz. in. |
| Maximum Continuous Power Output | 270 | 270 | 270 | Watts |
| Maximum Peak Torque Output | 240 | 410 | 515 | oz. in. |
| Speed Regulation (Typical) | -2 | -2 | -2 | \% of set speed |
| Speed Range (Typical) | 1000:1 | 1000:1 | 1000:1 | - |
| Motor Specifications (Nominal) |  |  |  |  |
| Torque Constant |  |  | 21 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  |  | 16 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  |  | 1.2 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  |  | 0.03 | oz. in. s. ${ }^{2}$ |
| Rotational Loss Torque |  |  | . 7 | oz. in./krpm |
| Static Friction Torque |  |  | 7 | oz. in. |
| Armature Inductance |  |  | 4 | mH |
| Armature to Ambient Thermal Resistance |  |  | 2.8 | oc/Watt |
| Maximum Armature Temperature |  |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |  |
| Voltage Gradient |  |  | 14.2 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  |  |  | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Ripple Amplitude |  |  | <6 | \% of DC output |
| Ripple Frequency |  |  |  | Cycles per rev. |
| Temperature Stability |  |  | 0.05 | \%/0'C |

*Speed range represents the speed range for which the specified regulation applies.


| CONNECTION CODE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FUNCTION |  | TAB | $\begin{aligned} & \text { TAB } \\ & \text { SIZE } \end{aligned}$ | $\begin{gathered} \hline \text { COLOR } \\ \text { CODE } \\ \hline \end{gathered}$ |
| MTA | + | 1 | $\begin{array}{r} 250 \\ .250 \end{array}$ | $\begin{aligned} & \text { RED } \\ & \text { BLLK } \\ & \hline \end{aligned}$ |
| TACH | $\begin{gathered} \text { NOT } \\ \text { USED } \\ \hline \end{gathered}$ | 3 4 5 | 187 | $\begin{gathered} \hline \text { BLU } \\ \overline{W H T} \end{gathered}$ |
| OPT. ENC. |  | $\begin{array}{r} \hline 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ \hline \end{array}$ | + |  |
| $\begin{array}{\|l} \hline \text { MTR } \\ \text { GND } \end{array}$ | frame | 13 | 250 | - |

System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 1000:1

## MT SERIES GENERAL INFORMATION



## TYPICAL SHAFT TOLERANCES

| Shaft perpendicularity to mounting face | 0.010 in $/ \mathrm{in}$ |
| :--- | :--- |
| Shaft concentricity to mounting pilot | 0.005 in |
| Shaft end play | $\pm 0.004$ in maximum |
| Shaft radial play | $\pm 0.0015$ in maximum |
| Maximum shaft radial load | $\pm 30 \mathrm{lbs}$ |
| Maximum shaft thrust load | $\pm 30 \mathrm{lbs}$ |

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of the MT series servo motors is:

$$
\begin{aligned}
& 3 \times 10^{8} \text { revolutions at } 60 \% \text { rated load, and; } \\
& 2 \times 10^{7} \text { start stop cycles at } 60 \% \text { rated load. }
\end{aligned}
$$

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

RECOMMENDED WIRE SIZES
$\begin{array}{ll}\text { Motor armature } & 14 \text { gage } \\ \text { Tachometer } & 18 \text { gage } \\ \text { Position encoder } & 24 \text { gage (optional) }\end{array}$
-

## 0670-01-009

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add 'S' to the end of the part number and describe your special requirement.
Performance RatingsMotor Control Model
Maximum Speed (No Load)
Maximum Continuous Toruqe Output at StallMaximum Continuous Power OutputMaximum Peak Torque Output*Speed Regulation (Typical)
*Speed Range (Typical)
Motor Specification (Nominal)
Torque Constant ..... 19
Voltage Constant ..... 14
Terminal Resistance ..... 0.65
Armature Moment of Inertia ..... 0.05
Rotational Loss Torque ..... 1.0
Static Friction Torque
Armature Inductance
Armature to Ambient Thermal Resistance
Maximum Armature Temperature
Tachometer Specifications (Nominal)
Voltage Gradient ..... 14
Terminal Resistance ..... 750
Armature Inductance
Ripple Amplitude ..... 5.0
Ripple Frequency ..... 11
Temperature StabilityLinearity82.62.1155

| P6200AP | P6300AP |  |
| :--- | :--- | :--- |
| 4600 | 4600 | rpm |
| 175 | 175 | oz. in. |
| 440 | 440 | Watts |
| 370 | 460 | oz. in. |
| -2 | -2 | \% of set speed |
| $1000: 1$ | $1000: 1$ | - |

oz. in. $/ A \pm 10 \%$
V/krpm $\pm 10 \%$
$\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$
oz. in. s²
oz. in./krpm
oz. in.
mH
${ }^{\circ} \mathrm{C} /$ Watt
${ }^{\circ} \mathrm{C}$

V/krpm $\pm 10 \%$
$\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$
mH
\% of DC output
Cycles per rev.
\%/ ${ }^{\circ} \mathrm{C}$
\% max error
*Speed range represents the speed range for which the specified regulation applies.


System if fully regenerative and can operate in all four speed/torque quadrants

MAXIMUM SPEED VS OUTPUT TORQUE


Typical speed regulation $2 \%$ of set speed over 1000:1

## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face Shaft concentricity to mounting pilot Shaft end play Shaft radial play Maximum shaft radial load Maximum shaft thrust load
$0.010 \mathrm{in} / \mathrm{in}$
0.005 in
$\pm .01$ in maximum $\pm 0.0015$ in maximum $\pm 30 \mathrm{lbs}$
$\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of these servo motors is:
$3 \times 10^{8}$ revolutions at $60 \%$ rated load, and;
$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load.
Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

## 0670-08-022

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

## Performance Ratings

## Motor Control Model

Maximum Speed (No Load)
Maximum Continuous Torque Output at Stall
Maximum Continuous Power Output
Maximum Peak Torque Output
*Speed Regulation (Typical)
*Speed Range (Typical)

|  |  |  |
| :--- | :--- | :--- |
| P6100AP | P6200AP | P6300AP |
| 3600 | 3600 | 3600 |
| 175 | 175 | 175 |
| 360 | 360 | 360 |
| 280 | 470 | 590 |
| -2 | -2 | -2 |
| $1000: 1$ | $1000: 1$ | $1000: 1$ |

## Motor Specifications (Nominal)

Torque Constant24
Voltage Constant ..... 18
Terminal Resistance ..... 1
Armature Moment of Inertia 05
Rotational Loss Torque1
Static Friction Torque ..... 8
Armature Inductance4.0
Armature to Ambient Thermal Resistance2.1
Maximum Armature Temperature ..... 155
Tachometer Specifications (Nominal)
Voltage Gradient14.2
Terminal Resistance
Ripple Amplitude$<6$
Ripple Frequency
Temperature Stability ..... 0.05
Linearity ..... 0.20
*Speed range represents the speed range for which the specified regulation applies.


| CONNECTIONCODE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FUN | TION | TAB | $\begin{aligned} & \text { TAB } \\ & \text { SIZE } \end{aligned}$ | COLOR CODE |
| MTR | + | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & .250 \\ & .250 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { RED } \\ & \text { BLK } \end{aligned}$ |
| TACH | $\begin{array}{r} + \\ \text { NOT } \\ \text { USED } \end{array}$ | $\begin{aligned} & 3 \\ & 4 \\ & 5 \\ & \hline \end{aligned}$ | . 187 | BLU WHT |
| OPT, ENC. | +15 -15 COM ENC 2 ENC 1 INDEX SHIELD | $\begin{array}{r} 6 \\ 7 \\ 8 \\ 9 \\ 90 \\ 11 \\ 12 \\ \hline \end{array}$ |  | BED <br> BLU <br> BLK <br> GRN <br> WHT <br> ORN |
| $\begin{aligned} & \text { MTR } \\ & \text { GND } \\ & \hline \end{aligned}$ | FRAME | 13 | 250 |  |

System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 1000:1

## MT SERIES GENERAL INFORMATION



## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face Shaft concentricity to mounting pilot Shaft end play
Shaft radial play
Maximum shaft radial load
Maximum shaft thrust load
$0.010 \mathrm{in} / \mathrm{in}$
0.005 in
$\pm 0.004$ in maximum $\pm 0.0015$ in maximum $\pm 30 \mathrm{lbs}$
$\pm 30 \mathrm{lbs}$

MOTOR LIFE
Motor life is limited mainly by brush life; the design brush life of the MT series servo motors is:

$$
\begin{aligned}
& 3 \times 10^{8} \text { revolutions at } 60 \% \text { rated load, and; } \\
& 2 \times 10^{7} \text { start stop cycles at } 60 \% \text { rated load. }
\end{aligned}
$$

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

RECOMMENDED WIRE SIZES

| Motor armature | 14 gage |
| :--- | :--- |
| Tachometer | 18 gage |
| Position encoder | 24 gage (optional) |

## 0702-06-006

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |
| :---: | :---: | :---: | :---: |
| Motor Control Model | P6200AP | P6300AP |  |
| Maximum Speed (No Load Peak) | 3800 | 3800 | rpm** |
| Maximum Continuous Torque Output at Stall | 330 | 330 | oz. in. |
| Maximum Continuous Power Output | 350 | 350 | Watts |
| Maximum Peak Torque Output | 430 | 550 | oz. in. |
| ${ }^{*}$ Speed Regulation (Typical) | -2 | -2 | \% of set speed |
| *Speed Range (Typical) | 1000:1 | 1000:1 | - |
| Motor Specification (Nominal) |  |  |  |
| Torque Constant |  | 23 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  | 17 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 0.4 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  | 0.15 | oz. in. s² |
| Rotational Loss Torque |  | 7.0 | oz. in./krpm |
| Static Friction Torque |  | 10.0 | oz. in. |
| Armature Inductance |  | 2.0 | mH |
| Armature to Ambient Thermal Resistance |  | 1.6 | oc/Watt |
| Maximum Armature Temperature |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |
| Voltage Gradient |  | 14.2 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 750 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  | 140 | mH |
| Ripple Amplitude |  | 5.0 | \% of DC output |
| Ripple Frequency |  | 11 | Cycles per rev. |
| Temperature Stability |  | 0.05 | \%/ ${ }^{\circ} \mathrm{C}$ |
| Linearity |  | 0.20 | \% max error |

*Speed range represents the speed range for which the specified regulation applies.
** For maximum continuous no load speed, see speed vs torque curve


System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 1000:1

## TYPICAL SHAFT TOLERANCES

shaft perpendicularity to mounting face shaft concentricity to mounting pilot Shaft end play
shaft radial play
Vaximum shaft radial load laximum shaft thrust load

$$
\begin{aligned}
& 0.010 \mathrm{in} / \mathrm{in} \\
& 0.005 \text { in } \\
& \pm .01 \text { in maximum } \\
& \pm 0.0015 \text { in maximum } \\
& \pm 30 \mathrm{lbs} \\
& \pm 30 \mathrm{lbs}
\end{aligned}
$$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of these servo motors is:

$$
\begin{aligned}
& 3 \times 10^{8} \text { revolutions at } 60 \% \text { rated load, and; } \\
& 2 \times 10^{7} \text { start stop cycles at } 60 \% \text { rated load. }
\end{aligned}
$$

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

## ZECOMMENDED WIRE SIZES

Votor armature - 14 gage; Tachometer - 18 gage

## 0703-02-060

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

## Performance Ratings

Motor Control Model
Maximum Speed (No Load)
Maximum Continuous Torque Output at Stall
Maximum Continuous Power Output
Maximum Peak Torque Output
*Speed Regulation (Typical)
*Speed Range (Typical)


Motor Specification (Nominal)
Torque Constant 49
Voltage Constant
37
$\begin{array}{ll}\text { Terminal Resistance } & 0.85\end{array}$
Armature Moment of Inertia 0.20

Rotational Loss Torque
10.0

Static Friction Torque 14.0

Armature Inductance
4.4

Armature to Ambient Thermal Resistance
1.3

Maximum Armature Temperature
155
Tachometer Specifications (Nominal)
Voltage Gradient 14.2
Terminal Resistance 550
Ripple Amplitude
5.

Ripple Frequency
Temperature Stability
21 Linearity0.05

Linearity 0.20
*Speed range represents the speed range for which the specified regulation applies.


System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation 2\% of set speed over 500:1

## MT SERIES GENERAL INFORMATION



## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face Shaft concentricity to mounting pilot Shaft end play
Shaft radial play
Maximum shaft radial load

## $0.010 \mathrm{in} / \mathrm{in}$

0.005 in
$\pm 0.004$ in maximum $\pm 0.0015$ in maximum $\pm 30 \mathrm{lbs}$
$\pm 30 \mathrm{lbs}$

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of the MT series servo motors is:
$3 \times 10^{8}$ revolutions at $60 \%$ rated load, and;
$2 \times 10^{7}$ start stop cycles at $60 \%$ rated load.

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

RECOMMENDED WIRE SIZES
Motor armature
Tachometer
Position encoder 24 gage (optional)

## 1450-00-031

## HOW TO ORDER

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |
| :---: | :---: | :---: | :---: |
| Motor Control Model | P6200AP | P6300AP |  |
| Maximum Speed (No Load) | 3500 | 3500 | rpm |
| Maximum Continuous Torque Output at Stall | 170 | 170 | oz. in. |
| Maximum Continuous Power Output | 300 | 300 | Watts |
| Maximum Peak Torque Output | 480 | 610 | oz. in. |
| * Speed Regulation (Typical) | -2 | -2 | \% of set speed |
| *Speed Range (Typical) | 200:1 | 200:1 |  |
| Motor Specification (Nominal) |  |  |  |
| Torque Constant |  | 25 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  | 19 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 1.25 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  | 0.01 | oz. in. s ${ }^{2}$ |
| Rotational Loss Torque |  | 8.0 | oz. in./krpm |
| Static Friction Torque |  | 9 | oz. in. |
| Armature Inductance |  | . 3 | mH |
| Armature to Ambient Thermal Resistance |  | 1.5 | OC/Watt |
| Maximum Armature Temperature |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |
| Voltage Gradient |  | 3 | V/krpm $\pm 10 \%$ |
| Terminal Resistance |  | 150 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  | 4 | mH |
| Ripple Amplitude |  | 2 | \% of DC output |
| Ripple Frequency |  | 19 | Cycles per rev. |
| Temperature Stability |  | -. 01 | \%/oc |
| Linearity |  | 0.2 | \% max error |
| *Speed range represents the speed range for which the specified regulation applies. |  |  |  |



System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation 2\% of set speed over 200:1

MT SERIES GENERAL INFORMATION


TYPICAL TERMINAL
BOARD ARRANGEMENT

TYPICAL SHAFT TOLERANCES
Shaft perpendicularity to mounting face
$0.010 \mathrm{in} / \mathrm{in}$ Shaft concentricity to mounting pilot
Shaft end play
.005 in
$\pm .01$ in maximum
$\pm 0.0015$ in maximum
$\pm 30 \mathrm{lbs}$
$\pm 30 \mathrm{lbs}$
Maximum shaft radial load

## MOTOR LIFE

Motor life is limited mainly by brush life; the design brush life of the MT series servo motors is:

$$
\begin{aligned}
& 3 \times 10^{8} \text { revolutions at } 60 \% \text { rated load, and; } \\
& 2 \times 10^{7} \text { start stop cycles at } 60 \% \text { rated load. }
\end{aligned}
$$

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual.

RECOMMENDED WIRE SIZES

| Motor armature | 14 gage |
| :--- | :--- |
| Tachometer | 18 gage |
| Pasition encoder | 24 gage (optional) |

Position encoder 24 gage (optional)

## 1450-02-033

## HOW TO ORDER **

To order a motor control system specify the motor type using the nine digit part number and the motor control using the ordering code described on page 65.
If you require an optical encoder or other special configuration on the motor add ' S ' to the end of the part number and describe your special requirement.

| Performance Ratings |  |  |  |
| :---: | :---: | :---: | :---: |
| Motor Control Model | P6200AP | P6300AP |  |
| Maximum Speed (No Load Peak) | 4700 | 4700 | rpm*** |
| Maximum Continuous Torque Output at Stall | 170 | 170 | oz. in. |
| Maximum Continuous Power Output | 300 | 300 | Watts |
| Maximum Peak Torque Output | 345 | 440 | oz. in. |
| *Speed Regulation (Typical) | -2 | -2 | \% of set speed |
| *Speed Range (Typical) | 300:1 | 300:1 | of |
| Motor Specification (Nominal) |  |  |  |
| Torque Constant |  | 18.8 | oz. in./A $\pm 10 \%$ |
| Voltage Constant |  | 14.3 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  | . 65 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Moment of Inertia |  | 0.01 | oz. in. $\mathrm{s}^{2}$ |
| Rotational Loss Torque |  | 8.0 | oz. in./krpm |
| Static Friction Torque |  | 9 | oz. in. |
| Armature Inductance |  | . 3 |  |
| Armature to Ambient Thermal Resistance |  | 1.5 | ${ }^{\circ} \mathrm{C} /$ Watt |
| Maximum Armature Temperature |  | 155 | ${ }^{\circ} \mathrm{C}$ |
| Tachometer Specifications (Nominal) |  |  |  |
| Voltage Gradient |  | 3 | $\mathrm{V} / \mathrm{krpm} \pm 10 \%$ |
| Terminal Resistance |  | 150 | $\Omega$ at $25^{\circ} \mathrm{C} \pm 15 \%$ |
| Armature Inductance |  | 4 | mH |
| Ripple Amplitude |  | 2 | \% of DC output |
| Ripple Frequency |  | 19 | Cycles per rev. |
| Temperature Stability |  | -. 01 | \%/0' |
| Linearity |  | 0.2 | \% max error |
| * Speed range represents the speed range for which the specified regulation applies. |  |  |  |



System if fully regenerative and can operate in all four speed/torque quadrants


Typical speed regulation $2 \%$ of set speed over 300:1

MT SERIES GENERAL INFORMATION


TYPICAL TERMINAL BOARD ARRANGEMENT


10001 MAB Rev. PA

## TYPICAL SHAFT TOLERANCES

Shaft perpendicularity to mounting face $\quad 0.010 \mathrm{in} / \mathrm{in}$ Shaft concentricity to mounting pilot
Shaft end play
Shaft radial play
Maximum shaft radial load
Maximum shaft thrust load
$\pm 01$ in maximum $\pm 0.0015$ in maximum
$\pm 30 \mathrm{lbs}$
$\pm 30 \mathrm{lbs}$

MOTOR LIFE
Motor life is limited mainly by brush life: the design brush life of the MT series servo motors is

$$
\begin{aligned}
& 3 \times 10^{8} \text { revolutions at } 60^{\circ} \text { rated load, and. } \\
& 2 \times 10^{7} \text { start stop cycles at } 60^{\circ} \text { rated load. }
\end{aligned}
$$

Many factors affect brush life and in most applications actual life results in the order of five to ten years are not unusual

RECOMMENDED WIRE SIZES

| Motor armature | 14 gage |
| :--- | :--- |
| Tachometer | 18 gage |
| Position encoder | 24 gage (optional) |

BRUDIC brushless direct current drives represent a revolutionary improvement in industrial servo drive technology. By offering all solid state control and eliminating all brushes, commutators, and slip rings from the motor, the user can expect minimum downtime and maintenance; and the servo performance is absolutely remarkable. User evaluations have demonstrated thesuperiority of BRUDIC over other drive systems.


## SYSTEM 1000

BRUDIC System 1000 is the highest power model in the present range. It is designed for computerized control in the most demanding industrial applications, 24 hours a day continuous operation where downtime must be minimized without sacrificing repeatability, precision, and performance.

BRUDIC System 1000 consists of a specially designed three-phase pulse width modulated (PWM) four-quadrant servo amplifier and matching brushless DC servomotor-tachometer-encoder package. There are no periodic maintenance requirements, no forced air cooling requirements, and self-diagnostic indicators are provided as standard.

## BRUDIC SYSTEM 1000 PERFORMANCE

Velocity Servo Bandwidth - DC to 200 Hz max. System can be operated at zero speed.

Speed Regulation $- \pm 2 \%$ or $\pm 5 \mathrm{rpm}$ of the set speed, whichever is greater.

## Dynamics (no load)

Acceleration from stall $a_{\mathrm{MAX}}=3800 \mathrm{rad} / \mathrm{s}^{2}$

| Acceleration times | $0-1800 \mathrm{rpm}$ 50 ms <br> $0-3600 \mathrm{rpm}$ 125 ms |  |
| :--- | :--- | ---: |
|  |  |  |
| Deceleration times | $1800 \mathrm{rpm}-0$ | 45 ms |
|  | $3600 \mathrm{rpm}-0$ | 90 ms |



SYSTEM 1000 TORQUE-SPEED CHARACTERISTICS


ENVIRONMENTAL DATA - SYSTEM 1000 will perform continuously under following conditions:

Ambient temperature 0 to $50^{\circ} \mathrm{C}$
Max. relative humidity 95\%
High oil content atmosphere
No auxiliary cooling of the amplifer or the motor.

RELIABILITY - Design life of 10 years in normal operation. Life test system accumulated to date more than 3 million operating cycles with maximum load.

## SPECIAL FEATURES

System Efficiency - Pulse-width modulated switching amplifier combined with permanent magnet brushless motor yields the most energy efficient industrial drive system available today.

Modular Design - Each module (amplifier subassemblies or the motor) can be replaced in less than 3 minutes. There is no loose hardware during replacement procedure and the only tool required is a straight-blade screwdriver.

Diagnostic and Protection System - BRUDIC SYSTEM 1000 has built-in protection circuits and an automatic diagnostic system.

In a case of any abnormal, external or internal condition or malfunctioning, the protection circuits will immediately disable the whole system, preventing a catastrophic failure, and signal this condition to the mircrocomputer. Diagnostic system display will indicate LOCATION of the problem.

A faulty module can then be easily replaced by unskilled maintenance personnel.

Installation Flexibiltiy - Convection cooling eliminates any need for special installation layouts and maintenance of cooling fans.

Wiring distance between the amplifier and the motor can be up to $200 \mathrm{ft}(60 \mathrm{~m})$. Motor power and signal cables do not require a special raceway. High DC bus voltage reduces size of motor power cable conductors.

Compliance - BRUDIC SYSTEM 1000 complies with applicable JIC manufacturing standards.

BSA-2580 Brushless Servo Amplifier - High-power, 3-phase, pulse-width modulated (PWM) transistor switching amplifier with complete servo control and current limiting circuitry, and with built-in short circuit protection and fault diagnostic system.



BSA-2580 Outline
Enclosure-Single-door, oil-tight, NEMA 12 enclosure $36^{\prime \prime} \times 36^{\prime \prime} \times 12^{\prime \prime}(915 \times 915 \times 305 \mathrm{~mm})$. Power amplifier heatsinks extend $4.5^{\prime \prime}(115 \mathrm{~mm})$ from the enclosure rear wall. Control microcomputer installed in the enclosure. Main 3-phase power transformer is in a separate enclousre, $\mathrm{W} \times \mathrm{D} \times \mathrm{H}: 12^{\prime \prime} \times 15^{\prime \prime} \times 12.5^{\prime \prime}$ ( $305 \times 380 \times 318 \mathrm{~mm}$ ).

Input Line Power - Input power is 3-phase, 460/230V (+10\%, $-15 \%$ ), 60 Hz . Other voltages and/or 50 Hz optional.

Recommended Control Micromcomputer is a modified version of Allen-Bradley PLC-2:

Controller: A-B Model 1771-UC
Servo Interface: A-B Model 1771-SR


The Microcomputer-Servo System Interface is defined in terms of inputs to and outputs from the position control microcomputer:

## To position control microcomputer

From position encoder: 2 quadrature pulse trains 1 index marker

From servo system: 1 velocity control status signal 1 current threshold signal

## From position control microcomputer to servo system

1 analog velocity command voltage, $2.5 \mathrm{~V} / \mathrm{krpm}$, $\pm 9 \mathrm{~V}$ max.
1 velocity control inhibit command
1 current limit enable command


## B-2101

Motor-Tachometer-Encoder Assembly - Highperformance, permanent magnet brushless DC servo motor with an integral brushless tachomter for velocity feedback, a solid state commutation encoder, and an incremental encoder for digital position feeback.

B-2100 Motor - Fully enclosed heavy-duty aluminum housing, $11^{\prime}$ DIA $\times 18.25^{\prime \prime}$ overall length ( $\phi 280 \times .464 \mathrm{~mm}$ ). Front end compatible with EX-CELL-O CORPORATION, CONE DRIVE DIVISION worm gearbox P/N M51557. Motor front shaft end has SAE 1" 6B female spline per CONE DRIVE P/N 51557-140. Mounting with "marmon" type V-ring clamp. (See outline drawing.)

B-2101 Motor - Same motor as above with shaft, front mounting and front bearing. (See outline drawing.)
Other front end configurations optional.
Motor weight 141 lb . ( 64 kg ).

## B-2100 and B-2101 Motors

Rated top speed $\mathrm{n}_{\text {MAX }}=3600 \mathrm{rpm}$
Peak output torque $\mathrm{T}_{\text {MAX }}=30 \mathrm{lb}$. ft. ( 41 Nm )

Rated (continuous or RMS ) torque for $50^{\circ} \mathrm{C}$ motor housing temperature rise (convection cooling)
$\mathrm{T}_{\mathrm{RMS}}=22 \mathrm{lb} . \mathrm{ft} .(30 \mathrm{Nm})$
Note:
Motor constants, and consequently the peak torque and top speed can be altered to match individual application requirements.

Peak mechanical output power at 2500 rpm
$P_{\text {MAX }}=9 \mathrm{~kW}(12 \mathrm{hp})$
Moment of inertia

$$
\begin{aligned}
\mathrm{J}_{\mathrm{M}}= & 1.53 \mathrm{oz} . \mathrm{in} . \mathrm{s}^{2} \\
& 0.256 \mathrm{lb} . \mathrm{ft}^{2} \\
& \left(1.08 \times 10^{-2} \mathrm{~kg}-\mathrm{m}^{2}\right)
\end{aligned}
$$

Thermal resistance
(winding-ambient, convection cooling only)

$$
\mathrm{R}_{\mathrm{TH}}=0.27^{\circ} \mathrm{C} / \mathrm{W}
$$

Thermal time constant $\tau_{\text {TH }}=90 \mathrm{~min}$.

## Position Encoder

The position encoder is a non-contacting rotational transducer that generates two pulse trains, in quadrature, with 250 cyçles per revolution. After decoding, it provides 1000 transitions per revolution of the motor shaft. In addition, a "once-per revolution" index marker pulse is provided.

## Speed Control Systems

## The Motomatic Concept

MOTOMATIC is a unique* transistorized motor speed control system designed, engineered and manufactured by ElectroCraft Corporation. MOTOMATIC is a complete closed-loop system consisting of a fast response, permanent magnet, inte-grally-wound motor-generator and highgain transistorized servo feedback amplifier. MOTOMATIC systems offer precise speed control, even in the presence of changing load and line voltage conditions. Using MOTOMATIC speed can be accurately controlled from a potentiometer or from any transducer sensing heat, light, tension, etc.

# Motomatic Speed Controls 



## SPEED REGULATION

Speed regulation is defined as the ability of a motor to maintain a set speed when a load torque is applied．A DC motor with＂open loop＂control has a relatively con－ stant power input so if the load increases，the speed decreases．In order to maintain a constant speed with increasing load，the power input to the motor must， therefore，be increased．
The MOTOMATIC closed－loop control system is a self－ balancing system which automatically adjusts the input to the motor so that an essentially constant speed is main－ tained even while the load on the motor is changing． The integral tachometer－generator senses that the load wants to change the motor speed and calls for more or less power to maintain constant speed．From no－load to full－ load conditions，the variation in speed is typically better than $1 \%$ of set speed．It may be noted that similar claims made for certain SCR variable speed drives are frequently based upon＂percent regulation of maximum speed．＂ MOTOMATIC＇s speed regulation can be considerably better than $1 \%$ if less than the full load or the 1000：1 speed range is utilized．

## DESIGN AND CONSTRUCTION

Electro－Craft motor－generators are quality engineered and manufactured to exacting specifications．Typical life cycle，on the basis of over 500，000 MOTOMATIC systems in operation，is in the range of 5000 hours at rated speed before brush changes are necessary．All rotors are epoxy resin insulated by a spray－coating process before winding and all finished armatures are impregnated to NEMA
standards．Both the motor insulation and construction are U．L．recognized．Commutators are turned with a diamond cutter for a 20 micro－inch surface finish，assuring perfect brush seating and minimum brush bounce．Finally，all rotors are dynamically balanced for smooth operation and long bearing life．Motors are rated at a maximum armature temperature rise of 130 degrees C ，which is better than the limits of Class＂$F$＂insulation．All units are＂run in＂before 100\％final testing and shipment．

## SOME MOTOMATIC APPLICATIONS

Computer Peripheral Equipment $⿴ 囗 十$ Welding Feeds $⿴ 囗 十$ Paper Chart Drives Textile Machines Office Copying Ma－ chines | Micro－Film Readers－Printers $■$ Medical Equipment |
| :---: | －Accurate Metering Pumps ■ Photographic and Film Processing Equipment $\quad$ Cameras $\quad$ Viscosimeters $\quad$ Lab－ oratory Instruments ■ Material Handling ■ Automatic Assembly Machines $\quad$ Machine Tool Programmers $⿴ 囗 十$ Pro－ cess Control ■ Automation Devices $\square$ Semiconductor Manufacturing Tension Sensing Winders over 300 different applications in total．

## E-350 Control

## E-350 MGH

## STANDARD FEATURES

1500:1 speed range 5 to 7500 rpm
Cloosed-loop completely transistorized velocity feedback amplifier
Single polarity power supply is easily adaptable to DC power source
Adjustable torque limit
IR compensation (allows setting optimum regulation at speeds below 10 rpm )
Maintains full rated torque across entire speed range 115 VAC $50 / 60 \mathrm{~Hz}$ - fuse protection
230 VAC $50 / 60 \mathrm{~Hz}$ - fuse protection

## OPTIONAL FEATURES

High accuracy speed torque readout (motor friction torque and motor damping factor are nulled out) 10 turn potentiometer
Reversing switch

## E350 MG - E350 MGH

1/70 hp 2 oz inch @ 7500 rpm
Shielded ball bearings
Silver brushes for stable operation
Maximum output torque 40 oz inches with gearhead

| how to read the speed regulation ZONE GRAPHS: | at speed |
| :---: | :---: |
|  | \% of set speed |
|  |  |
| Select the torque and rom | 6\% of set speed |
| range the data $\%$ of | [ [1] Illill Intermittent duty |
| regulation) is stated as a | only-consult ECC Engi- |
| percentage of the set | neering Application Dept. |
| speed | for additional data. |

SPEED REGULATION ZONES FOR E-350 CONTROL


## E-552 M Master

## E-552-0 Open



## E-552 MASTER \& E-552 OPEN STANDARD FEATURES

1000:1 speed range 5 to 5000 rpm
Closed-loop completely transistorized velocity feedback amplifier
Adjustable torque limit 0-10 oz. in.
IR compensation (allows setting optimum regulation at speeds below 10 rpm )
Single polarity power is easily adaptable to DC power` source
Maintains full rated torque across entire speed range $115 / 230$ VAC $50 / 60 \mathrm{~Hz}$ line fuse and thermal circuit breaker protection

## E-552-M STANDARD FEATURES

High accuracy speed-torque readout (motor friction torque and motor damping factor are nulled out)
Forward-reverse switch (push button)
Dual concentric speed control potentiometer
Remote input - (external control of speed by a potentiometer or voltage source)

## E-552-O STANDARD FEATURES

Single turn speed control potentiometer
Adapatable to a external voltage to control speed

## E-552-O OPTIONS

10 turn potentiometer
Forward-stop-reverse switch (rotary)
High accuracy speed-torque readout

## E-552 MG

## E-552 MGH



## E-552 MG STANDARD FEATURES

1/20 HP
Stainless steel shaft
Separate motor and tach-generator windings
Temperature compensated generator
Shielded ball bearings
Flange or base mounting
Unique negator brush design applies constant brush pressure
Fully enclosed all metal housing Bi-directional

## E-552 MGH STANDARD FEATURES

Precision low backlash gearhead (typical $1^{0}$ )
Flange mounting
Precision die cast gear housing
Ball bearings
Output torque 9 to 90 inch-lbs.


## SPEED REGULATION ZONES FOR E-552-M, E552-O



## E-586

## E586-O CONTROL STANDARD FEATURES

1000:1 speed range 5 to 5000 rpm
Closed-loop completely transistorized velocity feedback amplifier
IR compensation (allows setting optimum regulation at speed below 10 rpm )
Single turn speed control potentiometer $115 / 230$ VAC, $50 / 60 \mathrm{~Hz}$ line fuse and thermal circuit breaker protection.

## OPTIONS

High accuracy or (standard) speed torque readout 10 turn potentiometer
Forward-stop-reverse switch (rotary)
Remote toruqe limit potentiometer

## E-586 MG



## E-586 MGH



## 1/10 HP

Die cast end caps
Nickel chrome plated motor and tachometer housing Stainless steel shaft
Flange or servo clamp mounting Separate motor and tach generator windings Temperature compensated generator
Shielded ball bearings
Bi-directional

## E586 MGH STANDARD FEATURES

Precision low backlash gearhead (typical $1^{\mathrm{O}}$ ) Flange mounting Precision die cast gear housing Ball bearings
Output torque 9 to 90 inch-lbs

| HOW TO READ THE | 99\% of set speed |
| :---: | :---: |
| SPEED REGULATION | - |
| ZONE GRAPHS | $\mathbf{9 7 \%}$ of set speed |
| Select the torque and rpm | <96\% of set speed |
| range, the data $1 \%$ of |  |
| regulation) is stated as a | only-consult ECC Engi- |
| percentage of the set | neering Application Dept. |
| speed. | for additional data. |

SPEED REGULATION ZONES FOR E-586 CONTROL


## E－586 BVC Control



## E586－BI DIRECTIONAL VELOCITY OR POSITION CONTROL

## STANDARD FEATURES：

100：1 speed range 50 to 5000 rpm
Closed－loop completely transistorized velocitv feed－ back amplifier
Adjustable torque limit
Single turn－center tap－speed control potentiometer
Position sensor potentiomer（bi－directional position system only）
115／230 VAC， $50 / 60 \mathrm{~Hz}$ fuse and circuit breaker protection

| HOW TO READ THE | 99\％of set speed |
| :---: | :---: |
| SPEED REGULATION | 98\％of set speed |
| ZONE GRAPHS | $\square$ $97 \%$ of set speed |
| Select the torque and rpm | ＜96\％of set speed |
| range：the data $1 \%$ of | $\square\left\\|\left\\|\int\right\\|\right.$ Intermittent duty |
| regulation）is stated as a | only－consult ECC Engi－ |
| percentage of the set | neering Application Dept． |
| speed． | for additional data． |

## SPEED REGULATION ZONES E－586 BVC CONTROL




E586-BPC BI-DIRECTIONAL POSITION CONTROL SYSTEM

## Brief Description:

The E586-BPC system is a position control system employing a high-gain, closed-loop electronic control. The input command can be any variable resistance or ungrounded reference DC voltage. The motor angular position of a potentiometer or any other error voltage source. Velocity feedback is employed to control the travel rate while the motor is "homing in" on the desired position.

System Specifications:

The position accuracy of the system is limited by two variables: (1) the drift characteristics of the reference power supplies, and (2) the accuracy of the command and feedback signals. Using the potentiometer supplied with the standard E586-BPC, system accuracy is approximately $1 \%$ per revolution or $3.6^{\circ}$. Using improved power supply regulation and sensors, the accuracy can easily be improved to $.1 \%$ per revolution or $.36^{\circ}$. For
high accuracy systems, please consult an Electro-Craft representative.

The system travel speed is controlled by velocity feedback from an integral tachometer and closed-loop servo. The standard E-586-BPC controls the motor to run up to $5,000 \mathrm{rpm}$ during travel to the "target" position.

Command input can be accomplished in either of two ways: (1) a position setting on a potentiometer, using the E586-BPC reference power supply voltage, or (2) an ungrounded command voltage of 0 to +15 volts $D C$. Care must be exercised to insure that the input command range is not greater than the feedback sensor output range.

The feedback input requirements are similar to the command requirements and can be either a position on a potentiometer, using the feedback reference voltage source, or an ungrounded 0 to -15 volts $D C$ signals.

## E-652 Master

## E-652 Open Master



## E-652 MASTER AND E-652 OPEN MASTER

## 1000:1 speed range 3 to 3000 rpm

Closed-loop completely transistorized velocity feedback amplifier
Adjustable torque limit 0-5 inch-lbs
IR compensation (allows setting optimum regulation at speed below 10 rpm )
Single polarity power is easily adaptable to DC power source
115/230 VAC, 50/60 Hz line fuse and thermal circuit breaker protection

## E-652 MASTER STANDARD FEATURES

High accuracy speed-torque readout (motor friction torque and the motor damping factor are nulled out)
Forward-reverse switch (push botton)
Dual concentric speed control potentiometer
Remote input (external control of speed by a potentiometer or voltage source 0 to +10 v )

## E-652-O/M STANDARD FEATURES

Single turn speed control potentiometer
Adaptable to an external voltage to control speed Flat plate chassis is designed for a NEMA I enclosure ( $12 \times 10 \times 6$ inches in size)

## E-652-O/M OPTIONS

10 turn potentiometer
10 turn dial
Forward-stop-reverse switch (rotary)
High accuracy speed-torque readout
Remote torque limit terminals

## E-652 MG



## E-652 MGH



## E-652 MG STANDARD FEATURES

1/5 HP
Stainless steel shaft
Die cast end caps
Nickel chrome plated motor housing Mounting

Flange
Base
NEMA 42 CZ
Shielded ball bearings
Bi-directional
Separate motor and tach generator windings Temperature compensated generator

## E-652 MGH STANDARD FEATURES

Precision low backlash gearhead
Flange mounting
Die cast gear housing
Ball bearings
Output torque 26 to 400 inch-lbs


SPEED REGULATION ZONES E-652-M - E652-O/M (E-652-MG)


## E-710

## E-670 MG



## OPTIONS

Speed torque readout *
Reverse-stop-forward switch (rotary)
220 VAC $50 / 60 \mathrm{~Hz}$ transformer
Adaptable to DC power supply
Ten turn potentiometer
Ten turn dial ( $1-3 / 8$ diameter)
Ramp time extended to 1.5 seconds

## SPEED REGULATION ZONES <br> E-710 (670/596-MG)



## E-720

## E-703 MG



## E-703 MG STANDARD FEATURES

$1 / 2 \mathrm{HP}$ (see speed regulation graphs) Stainless steel shaft
Separate motor and tach generator windings
Temperature compensated generator
Shielded ball bearings

## Bi-directional

## Mounting

## Flange

## Nema 56

## Die cast end caps

Nickel chrome plate motor housing

## E-720 STANDARD FEATURES

$400: 1$ speed range 5 to 2000 rpm rated maximum torque 17 inch-lbs
Fixed frequency pulse width modulated amplifier
Adjustable ramp geneator
Adjustable torque limit
Single turn speed command potentiometer
Flat plate chassis is designed for a Nema 1 or 12
enclosure ( $14 \times 12 \times 8$ inches in size)
115 VAC 50/60 Hz.

## OPTIONS

## Speed torque readout

Reverse-stop-forward switch (rotary)
220 VAC $50 / 60 \mathrm{~Hz}$ transformer
Adaptable to DC power supply
Ten turn potentiometer
Ten turn dial (1-3/8 diameter)
Ramp time extended to 1.5 seconds

| HOW TO READ THE |
| :--- | :--- |
| SPEED REGULATION |
| ZONE GRAPHS: |

## SPEED REGULATION ZONES <br> E-720 (703/796-MG)



## Digital Positioning Systems

At Electro-Craft we're always looking for ways to provide the control equipment that you need to improve your products or to upgrade your manufacturing operations.

Now, there's a new dimension in position control available from Electro-Craft - The DPS Series Digital Positioning System. It's a product of our own special brand of creativity, innovation and design excellence.

The system is composed of three basic components.

## THE INDEXER

The core of the system is the indexer itself. We offer a choice of two basic models with a variety of control panel options and position displays. Generally, however, the indexer accepts commands in the form of parallel or serial digital words. These commands can be entered manually be setting the move length, speed and direction with thumbwheel switches and push buttons located on the front panel. Or, they may be generated by a computer, micro processor or a remote control panel.

The indexer loads the commands into memory locations within the unit and generates instructions to the motor control. The position and speed of the motor are fed back to the indexer allowing it to monitor the execution of the commands and make corrections where necessary.

Circuits and inputs in the indexer provide overtravel protection as well as jog and automatic return to home features. Indexers are available with or without digital absolute position displays.

## THE MOTOR CONTROL

Three standard motor control types are available depending on the servo motor you choose and the application that you have in mind.

The motor control takes the signal level instructions from the indexer and provides the power to drive the servo motor. Regenerative energy protection and short circuit protection are standard features of Electro-Craft's motor controls.

## THE SERVO MOTOR

With our Digital Positioning Systems, you can select from five standard high performance DC servo motors. With each you get smooth, jitter-free rotation even at speeds as low as five rpm. These motors, designed and manufactured by ElectroCraft, represent the state of the art in permanent magnet motor design. They are designed for the highest possible torque to inertia ratio, for maximum acceleration capability, and to be free of resonances and cogging at all speeds.

## THE TOTAL SYSTEM

The system design is based on Electro-Craft's vast experience in positioning systems. The design includes the lessons learned from over 1000 positioning systems installed over a six year period. The built-in flexibility enables us to provide, from stock, fifteen different performance ratings and over twentyeight control options. Together with custom modifications, 1500 versions are available almost guaranteeing that we can provide a system to meet your exact needs.

## THE ADVANTAGES

Electro-Craft's Digital Positioning Systems provide many advantages over other types of positioning systems. A few of the more important ones are:

No external ramping of motor speed required.
Acceleration and Deceleration are linear and independently adjustable.

Motor rotation is smooth and jitter-free even at speeds as low as 5 rpm .

Position repeatability is better than four times the system resolution.

The system is stable and free of resonances at all speeds.
Shaft stiffness is very high, providing the full rated torque of the system for position errors of a fraction of a degree.

The motor shaft does not vibrate or hunt between adjacent positions in the stopped mode.

Torque output is independent of speed and load inertia.
In combination, these components provide extremely accurate motion and position control for applications such as cut to length systems, scanning, machine tool feed drives and coil winders. All this provides you with standard, off the shelf positioning systems that can speed your production, increase your yield and improve the performance of your product.

## CONSTRUCTION AND DESIGN

INDEXERS. Electro-Craft positioning indexers utilize a unique printed circuit card construction that eliminates all wiring from the equipment with the exception of the AC power cord. All external connections are made to connectors directly mounted on the printed circuit mother board. For convenience, pre-assembled cables with connectors are available.

The design uses high noise immunity logic and a unique optoelectronic position feedback encoder that provides much higher immunity to noise pickup and false positioning than
other conventional devices. All systems are thoroughly tested and "burned in" for 48 hours prior to shipment.

MOTOR CONTROLS. These units employ a unique open modular construction designed to give maximum accessibility for set up and system debugging. All adjustment and compensation changes are accomplished using variable controls, eliminating the need to unsolder and change electronic components. The control is self-contained. It requires standard AC power rather than a DC power supply and external transformer as with other systems.

The circuit design provides complete short circuit protection and a unique trip circuit to protect power components and minimize damage due to misuse or circuit failure. These controls are also "burned in" for 48 hours at full current prior to shipment.

MOTORS. Electro-Craft servo motors include the latest in magnet, winding and insulation technology. Featuring U L Listed insulation systems, most motors are wound on automatic machinery to maintain consistent performance and quality levels. All motors are thoroughly inspected and run-in for 24 hours in both directions prior to shipment.

## DESIGNED FOR PROFIT

The Electro-Craft DPS Series is designed with just one thing in mind: increasing your profits. Whether you use it in your plant or build it into your product, DPS can provide the following benefits.

Improved Accuracy - This means less waste, higher yields, higher consistency in output. If you build DPS into your product, it probably means that you can upgrade the performance specifications and get an edge on your competition or increase the value of your product.

Higher Speeds - Increased throughput and higher production rates. Often replacing an existing system with DPS will eliminate a production bottleneck and help delay the need for a more expensive capital outlay. As with increased accuracy, increased speed adds to the marketability of your product when DPS is used as a component subsystem.

Less Maintenance - A DPS will typically run for many years with no preventive maintenance and, in many cases, considerably longer. This means reduced downtime, less maintenance and fewer service visits to your customers.

The following case histories outline three examples of how DPS was able to increase the profits by waste reduction, increased throughput and product enhancement.



## CASE HISTORY \#1 WASTE REDUCTION

Customer A was using a mechanical system to feed wire plated with a precious metal into a machine which weaves it into a wire fabric. The mechanical system permitted a variation in the length of the wire of 1-2 inches; consequently the feed length had to
be set 2 inches longer than required and the wire trimmed after the weaving process. Result: thousands of dollars worth of the wire was scrapped every year. The customer installed an Electro-Craft Digital Positioning System which enabled a feed accuracy of $\pm 0.005$ inches which eliminated the trimming operation and eliminated the scrap. The Bottom Line- the system paid for itself in less than six months and generated increased profits of thousands of dollars per year.


# CASE HISTORY \#2 INCREASED PRODUCTION 

Customer B used automatic embroidery equipment to place monograms and custom emblems on garments. The system used a mechanical drive to position the garment under the embroidery needles and was limited to a rate of 300 stitches per minute. Result: a typical garment took more than five minutes to embroider. An Electro-Craft Digital Positioning System with a positioning rate of 1000 stitches per minute was installed, speeding up production by more than a factor of three. The Bottom Line -the Electro-Craft System increased the number of garments per hour by a factor of three and the solid state nature of the system enabled the machines to be connected directly to a computer, thus decreasing set-up time and yielding even more increase in profits.

# THE WALL STREET JOORNAL. 

C 2970 Dim Dowes Company In all Rybher Revered


# CASE HISTORY \#3 PRODUCT ENHANCEMENT 

Customer C manufactures automation equipment for newspaper and magazine composing rooms. The biggest bottleneck in type composition was the conversion of the electronic signals representing type size, spacing, line length and actual alphanumeric characters generated by a computer into hard copy.

Working with Electro-Craft's engineering staff, the customer developed a system which used a custom DPS to position a mirror which
reflects the optically projected characters onto a photographic paper. The DPS was able to position the characters within thousandths of an inch at a rate of about 100 characters per second. The Bottom Line - the customer was able to bring to the market the fastest machine of its type and enjoyed a level of success that would not have been possible with a slower system. As a side effect, thousands of newspapers and magazines around the world were able to reduce the time involved and the cost of type setting.

## PERFORMANCE SUMMARY

Max Speeds up to 4000 rpm Max load inertia up to 0.500 oz . in. $\mathrm{s}^{2}$ Max peak torque up to 1200 oz. in. Max RMS torque up to 375 oz . in. Max resolution up to 1250 units per revolution Position accuracy and repeatability up to 4 times the resolution.

This data represents the best of what can be provided, there are trade-offs between the various performance parameters. Consult the buying guide for detailed specifications of individual systems.

MINIMUM MOVE TIME vs MOVE LENGTH


MOVE LENGTH (revolutions)

REMOTE ONLY INDEXERS


These units take their position commands from a remote source provided by you. They can be remote mounted switches and push buttons or the output from a computer.

REMOTE ONLY MODELS __ RO _

| Function | Front Panel Control | Front Panel Indicator | Remote Input | Remote Output |
| :---: | :---: | :---: | :---: | :---: |
| Power | $\times \times$ | $\times \times$ | $N / A$ | N/A |
| Standby/Run | $N / A$ | $N / A$ | XX | N/A |
| Move Length | N/A | $N / A$ | $\times \times$ | N/A |
| Speed | N/A | N/A | $x \times$ | N/A |
| Direction | N/A | $N / A$ | $x \times$ | N/A |
| Start | N/A | N/A | $\times \times$ | N/A |
| Motion Complete | N/A | N/A | N/A | $\times \times$ |
| Jog | N/A | N/A | $\times \times$ | N/A |
| Return Home | N/A | $N / A$ | $\times \mathrm{X}$ | $\times \times$ |
| Remote/Local | N/A | N/A | N/A | N/A |
| Digital Speed | N/A | N/A | $\times \mathrm{X}$ | N/A |
| Reset | N/A | N/A | $\times \times$ | N/A |
| Over Travel | N/A | N/A | $\times \times$ | N/A |
| Optional Position Display | N/A | N/A | N/A | $\times \times$ |

N/A Control or indicator not available on this model.


All control functions can be commanded from switches mounted on the front of the INDEXER. Lighted push buttons are used to indicate the status of the system. A choice of models allows various combinations of the input commands to be transferred from the front panel control to a remote source by a local/remote switch.
LOCAL ONLY MODEL $\qquad$ LO

| Function | Front Panel Control | Front Panel Indicator | Remote Input | Remote Output |
| :---: | :---: | :---: | :---: | :---: |
| Power | X $\times$ | N/A | N/A | N/A |
| Standby/Run | $x \times$ | X $\times$ | X $\times$ | N/A |
| Move Length | X $\times$ | $N / A$ | N/A | N/A |
| Speed | $\times$ | N/A | X | N/A |
| Direction | $\times$ | X $\times$ | $\times$ | X $\times$ |
| Start | $\times$ | N/A | $\times$ | N/A |
| Motion Complete | N/A | X $\times$ | N/A | X $\times$ |
| Jog | X | N/A | $\times$ | N/A |
| Return Home | $\times$ | XX | $\times$ | X $\times$ |
| Remote/Local | $\times$ | $x \times$ | N/A | X $\times$ |
| Digital Speed | N/A | $N / A$ | N/A | N/A |
| Reset | X $\times$ | N/A | X $\times$ | N/A |
| Over Travel | N/A | N/A | XX | N/A |
| Optional Position Display | N/A | $\times \times$ | N/A | XX |

$x$ Control or indicator that functions only when remote/local switch is in appropriate position.
$\times \times$ Control or indicator that functions regardless of position of remote/local switch.
N/A Control or indicator not available on this model.
LOCAL/REMOTE MODEL $\qquad$ LR _

| Function | Front Panel Control | Front Panel Indicator | Remote Input | Remote Output |
| :---: | :---: | :---: | :---: | :---: |
| Power | x $\times$ | N/A | N/A | N/A |
| Standby/Run | XX | X $\times$ | x $\times$ | $\times \times$ |
| Move Length | $\times$ | N/A | $\times$ | N/A |
| Speed | $\times$ | N/A | $\times$ | N/A |
| Direction | $\times$ | X $\times$ | $\times$ | $\times \times$ |
| Start | $\times$ | N/A | $\times$ | N/A |
| Motion Complete | N/A | XX | N/A | X $\times$ |
| Jog | $\times$ | N/A | $\times$ | N/A |
| Return Home | $\times$ | X $\times{ }^{\text {' }}$ | $\times$ | $\times \times$ |
| Remote/Local | $\times \times$ | X $\times$ | N/A | $\times \times$ |
| Digital Speed | N/A | N/A | $\times$ | N/A |
| Reset | $\times$ | N/A | $x \times$ | N/A |
| Over Travel | N/A | N/A | $x \times$ | N/A |
| Optional Position Display | N/A | XX | N/A | XX |

$\times$ Control or indicator that functions only when remote/local switch is in the appropriate position.
$X \times$ Control or indicator that functions regardless of position of remote/local switch.
N/A Control or indicator not available on this model.

## MOTOR CONTROLS

The standard version of the motor control is an open unit designed to be built into a machine or enclosure.

Optional bench type, rack type and Nema 12 industrial enclosures are available.

The motor controls are completely self-contained for operation from standard AC power and require no external transformer or power supply.


## ENCLOSURES

Rack mount adapters and Nema 12 industrial enclosures can be provided for both the motor controls and indexers.


## POSITION DISPLAYS

Bright L.E.D. digital displays are available to indicate the absolute position of the system. Two displays can be driven from each indexer and they can be mounted in the indexer enclosure or remote from it.


## MOTORS

Five different frame sizes are available to match your applications. Detailed performance data is available in the DPS buying guide.


## MT1450

This is the highest performance motor in the line and is designed specifically for those applications where rapid acceleration is essential.


## MT400

Like the MT1450, this motor is designed for rapid acceleration; however, the MT400 is suited to applications where the load to be positioned is quite light.


## MT703

This is the work horse of the line, It provides the highest torque rating and is used in applications where the load to be positioned is relatively heavy.


## MT670 AND MT660

These two motors are mid-range performance and are used where a light load must be positioned at a higher speed.

## Digital Positioning Systems precise motion and control and more.

At Electro-Craft, we believe that providing you with dependable, precise motion control systems is only part of the job. You need a source you can rely upon; a manufacturer that can deliver what you need, when you need it. And, a manufacturer that stands behind the products it sells with service and technical support.

## Precision when you need it.

All of the basic Digital Positioning Systems described in this brochure are in stock and can be delivered to you in a few weeks or less. Systems requiring modifications can usually be delivered within six to twelve weeks, depending upon design.

## Service and support, in depth.

The Electro-Craft Applications Engineering Department is at your disposal to help you select the Digital Positioning System you need. They are experts who draw upon thousands of field applications and are thoroughly familiar with all ElectroCraft products. And, you will also find that your local ElectroCraft sales engineer can provide you with knowledgeable, on-the-spot assistance.

Once you have found the system that fits your application, Electro-Craft field engineers can provide you with complete
start-up assistance. And, after your system is in operation, you'll find that replacement parts and a complete field repair service are always available.

## WANT TO KNOW MORE?

Send for the Electro-Craft Digital Position Systems buying guide. It's packed with detailed specifications and a complete price list. Or, if you are ready to get to work on your system, Applications Engineering assistance is available by contacting your local Electro-Craft Sales Office or by calling the plant direct at 612/931-2700.

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[^9]
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[^0]:    *MCM is a registered trade mark for Electro-Craft Moving Coil Motors.

[^1]:    *Moving Coil Motor, or MCM is a registered trademark of Electro-Craft Corporation.

[^2]:    $\mathrm{t}_{1}$ is time of 20 oscillations of master cylinder and collet

[^3]:    *The negative sign here indicates that the friction torque is higher than the torque required to stop the load in time $t_{3}$. However, when the motor inertia is later added, we shall see that a positive torque is, indeed, needed to stop the motor and load in time.

[^4]:    - open loop, on-off applications
    - unidirectional speed-control motor systems

[^5]:    External Connections Pin \& socket connectors (mating connectors are supplied with each unit).

[^6]:    *Speed range represents the speed range for which the specified regulation applies.

[^7]:    * For maximum continuous no load speed, see speed vs torque curves.

[^8]:    Case histories described in this section represent actual examples where DPS is being used or is under evaluation.

[^9]:    Telephone \#

