

RECOMP II USERS' PROGRAM NO. 1069

PROGRAM TITLE: AGC-109 EIGENVALUES AND EIGENVECTORS BY JACOBI'S
METHOD (Floating Point)

PROGRAM CLASSIFICATION: General

AUTHOR: Mrs. Marcella J. Wulff
Aerojet-General Corporation
Sacramento, California

PURPOSE: This program uses an iterative procedure to find
the eigenvalues and eigenvectors of a symmetric
matrix.

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(Floating Point)

LOCATIONS USED: 0000-0007, 0150-0527, 0770-1557 are used for commands and constants. Data and results are stored in 3000-7757; the exact number of locations used depends upon the size of the matrix. The following subroutines are punched on the tape: AN-002 (0010-0147), AN-014 (0530-0767), and AN-007 (2310-2667).

Thus, the program and input-output subroutines occupy locations 0000-1557 and 2310-2667. Notice that the Input-Output Control is not destroyed (locations 1650-1763).

INTRODUCTION: Given a symmetric matrix A (one for which $A^t = A$), its eigenvalues (characteristic roots) and eigenvectors may be found by Jacobi's Method. Since the matrix is symmetric and all transformations preserve this symmetry, only the elements on and below (or above) the diagonal are necessary. This conserves memory space and increases speed. It has no effect upon the accuracy of the results.

The output consists of n (the order of the matrix) eigenvalues with their associated eigenvectors, followed by the number of iterations. The nixie tubes also display the iteration being performed in octal @ 38.

METHOD: The elements of A, a_{ij} ($1 \leq i \leq j \leq n$), are searched to find the element of greatest magnitude. Call this element a_{xy} . Element a_{xy} is annihilated by the following matrix multiplication:

$R^t \cdot A \cdot R$, where the elements of R are:

$$r_{xx} = \cos \theta$$

$$r_{xy} = \sin \theta$$

$$r_{yx} = -\sin \theta$$

$$r_{yy} = \cos \theta$$

$$r_{ii} = 1$$

$$r_{iy} = r_{xj} = r_{ij} = 0 \quad i \neq j; i, j \neq x, y$$

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{-a_{xy}}{\frac{1}{2} (a_{xx} - a_{yy})} \right]$$

This is equivalent to the following transformations:

$$\begin{aligned}
 b_{xi} &= b_{ix} = a_{xi} \cos \theta - a_{yi} \sin \theta \\
 b_{yi} &= b_{iy} = a_{xi} \sin \theta + a_{yi} \cos \theta \quad i, j \neq x, y \\
 b_{ij} &= a_{ij} \\
 b_{xx} &= a_{xx} \cos^2 \theta + a_{yy} \sin^2 \theta - 2 a_{xy} \sin \theta \cos \theta \\
 b_{yy} &= a_{xx} \sin^2 \theta + a_{yy} \cos^2 \theta + 2 a_{xy} \sin \theta \cos \theta \\
 b_{xy} &= b_{yx} = (a_{xx} - a_{yy}) \sin \theta \cos \theta + a_{xy} (\cos^2 \theta - \sin^2 \theta)
 \end{aligned}$$

By setting $b_{xy} = 0$, it may be seen how θ was determined. Let

$$\lambda = -a_{xy} \text{ and } \mu = \frac{1}{2} (a_{xx} - a_{yy}); \quad \omega = \operatorname{sgn}(\mu) \frac{\lambda}{\sqrt{\lambda^2 + \mu^2}}$$

Then $\sin \theta = \frac{\omega}{\sqrt{2(1 + \sqrt{1 - \omega^2})}}$ and $\cos \theta = \sqrt{1 - \sin^2 \theta}$.

Next, replace the elements a_{ij} by the elements b_{ij} . This gives a new matrix A, for which the process may be repeated. In annihilating a "new" element of A, it is likely that the elements that are currently zero will be made non-zero. Thus, this is an iterative procedure, which is carried out until the non-diagonal element having greatest magnitude is within a specified range. The diagonal elements of A are the eigenvalues.

Next, consider the matrix $S_i = S_{i-1} \cdot R_i$, where $S_1 = R_1$. This matrix multiplication is equivalent to the following transformations:

$$\begin{aligned}
 t_{ij} &= s_{ij} \quad i, j \neq x, y \\
 t_{ix} &= s_{ix} \cos \theta - s_{iy} \sin \theta \\
 t_{iy} &= s_{ix} \sin \theta + s_{iy} \cos \theta
 \end{aligned}$$

We then replace the elements s_{ij} by the elements t_{ij} . When the desired accuracy has been reached, S will contain the eigenvectors desired. The i^{th} column of S corresponds to element a_{ii} of A.

RESTRICTIONS: The matrix must be real, symmetric, and of order n such that $3 \leq n \leq 27$. Logically, the program could handle any order greater than 2 that the memory capacity would permit. Practically, the program would be very slow for large n (see section titled "TIME"). The original matrix is destroyed.

USAGE: Enter the program tape and hit start button 1. The typewriter will type N. Type as an integer the order of the matrix; enter with a C/R. It will then type E (epsilon). Type, in AN-007 format, the greatest permissible absolute value that any non-diagonal element of A may have, and enter it with a C/R. When the machine next stops, enter the elements of A in AN-007 format as follows:

```

a11 C/R
a21 tab    a22 C/R
a31 tab    a32 tab    a33 C/R
      .           .           .
      :           :           :
      .           .           .
an1 tab    an2 tab    an3 tab    an4 C/R
an5 tab           .           .
                               ann C/R

```

When $\frac{n}{2}(n+1)$ elements have been entered, computation will begin. The first iteration requires more time than do subsequent iterations, as it includes initialization.

The console will display, as an octal number at binary 38, the number of the iteration being performed. If one sees that he has requested too much accuracy (that is, the process will use too much time), he may set the pre-set stop for location 00041. Then use the input-output control to enter a new value of E in locations 0266-0267 (floating point). Set location counter to 00041 and press regular start. This procedure may be used as often as desired.

If the answers have already been printed but greater accuracy is desired, enter a new E in 0266-0267, as described above, and start at location 00041. However, the diagonal elements change less rapidly than do the non-diagonal elements; E may have to be 1/1000 the size of the "old" E in order to gain one more digit of accuracy in the eigenvalues.

CODING
INFORMATION:

a_{ij}	3000-4363	$c(3000) = a_{11}$	
b_{ij}	4364-4713	$c(4364) = b_{x1}$	$c(4540) = b_{1y}$
r_{ij}	4714-7577	$c(4714) = r_{11}$	
s_{ij}	7600-7757	$c(7600) = s_{1x}$	$c(7666) = s_{1y}$
iteration counter @ 39	1270		
$x @ 38$	0206		
$y @ 38$	0207		
$N @ 38$	0227		
$N^+ (N @ 38 + 1 @ 39)$	0210		
$2N + 2 @ 38$	0314		
$2N @ 38$	1265		
E	0266-0267		
a_{xy}	1266-1267		
λ	1240-1241		
λ^2	1242-1243		
μ	1246-1247		
ω	1250-1251		
$\sin \theta$	1252-1253		
$\sin^2 \theta$	1254-1255		
$\cos^2 \theta$	1256-1257		
$\cos \theta$	1260-1261		
$2 \sin \theta \cos \theta$	1262-1263		

The output automatically follows computation. The eigenvalues are not necessarily in descending order. Each eigenvector follows its associated eigenvalue.

TIME:

The following times, for computation only, have been observed. Output may require several minutes if N is large.

N	E	Element Having Largest Absolute Value	Time		Iterations
			Min.	Sec.	
3	10^{-6}	4	1	0	7
3	10^{-15}	2	1	5	9
4	10^{-7}	3	2	15	15
4	10^{-7}	4	2	27	17
5	10^{-7}	3	4	50	28
6	10^{-3}	20×10^6	8	55	44
7	10^{-3}	10.7×10^6	15	35	66
8	10^{-3}	22.5×10^6	22	15	82

A well-conditioned matrix will require less iterations than will an ill-conditioned matrix.

REFERENCE:

"The Determination of the Characteristic Roots of a Matrix by the Jacobi Method," by John Greenstadt; Mathematical Methods for Digital Computers, edited by Anthony Ralston and Herbert S. Wilf; John Wiley & Sons, Inc., 1959.

EXAMPLE 1

N 3

E 1.0-6

2
3
1

1
2

4

EIGENVALUE

+ .63871996134 + 1

EIGENVECTOR

+ .51841480090 + 0 + .53604246596 + 0 + .66626163698 + 0

EIGENVALUE

- .16955886857 + 1

EIGENVECTOR

- .59228155534 + 0 + .78707313220 + 0 - .17239038197 + 0

EXAMPLE 1 (Cont'd)

EIGENVALUE

+ .23083890724 + 1

EIGENVECTOR

- .61680519895 + 0 - .30524475307 + 0 + .72551842658 + 0

7 ITERATIONS

EXAMPLE 2

N 4

E 0.0000001

1
1
1
1

2
2
2

3
3
3

4

EIGENVALUE

+ .28311858285 + 0

EIGENVECTOR

+ .42852507314 + 0 - .65653850203 + 0 + .57735026918 + 0 - .22801342887 + 0

EIGENVALUE

+ .10000000000 + 1

EIGENVECTOR

+ .57735036016 + 0 + .57735023762 + 0 - .79956924581 - 7 - .57735020982 + 0

EXAMPLE 2 (Cont'd)

EIGENVALUE

+0.42602204776 + 0

EIGENVECTOR

-0.65653842203 + 0 +0.22801350885 + 0 +0.57735026921 + 0 -0.42852515312 + 0

EIGENVALUE

+0.82908593694 + 1

EIGENVECTOR

+0.22801342887 + 0 +0.42852507312 + 0 +0.57735026921 + 0 +0.65653850205 + 0

17 ITERATIONS

EXAMPLE 3

N 6

E 0.1-3

476120.496			
-460009.900	+9367626.854		
0	-4640920.727	+11571428.517	
0	0	-8897391.737	+12602451.779
0	0	0	-7024112872-3
+19966905.655	0	0	0
0	0	0	0
-14399832.963	+27642276.381		

EIGENVALUE

+ .49592431927 + 6

EIGENVECTOR

+ .95572298717 + 0	- .41144699542 - 1	- .17338489889 + 0	- .19436885973 + 0
- .11538233878 + 0	- .61204776299 - 1		

EXAMPLE 3 (Cont'd)

EIGENVALUE

$$+.72453449864 + 7$$

EIGENVECTOR

$$\begin{array}{cccc} -.44816370967 - 1 & +.65949031943 + 0 & +.30602555066 + 0 & -.19519767697 + 0 \\ -.53651245014 + 0 & -.37876725255 + 0 & & \end{array}$$

EIGENVALUE

$$+.48808409794 - 4$$

EIGENVECTOR

$$\begin{array}{cccc} +.28963436800 + 0 & +.29977802421 + 0 & +.57638864149 + 0 & +.59325182847 + 0 \\ +.33428734251 + 0 & +.17414202171 + 0 & & \end{array}$$

EIGENVALUE

$$+.22402610884 + 8$$

EIGENVECTOR

$$\begin{array}{cccc} -.47801220158 - 2 & +.22784574730 + 0 & -.63947801492 + 0 & +.65961903468 + 0 \\ -.11028938402 + 0 & -.30310116331 + 0 & & \end{array}$$

EXAMPLE 3 (Cont'd)

EIGENVALUE

$$+.11978815047 + 8$$

EIGENVECTOR

$$\begin{array}{cccc} -.25964134577 - 1 & +.64924148219 + 0 & -.36271852774 + 0 & -.32203950257 + 0 \\ +.43086055469 + 0 & +.39610146732 + 0 & & \end{array}$$

EIGENVALUE

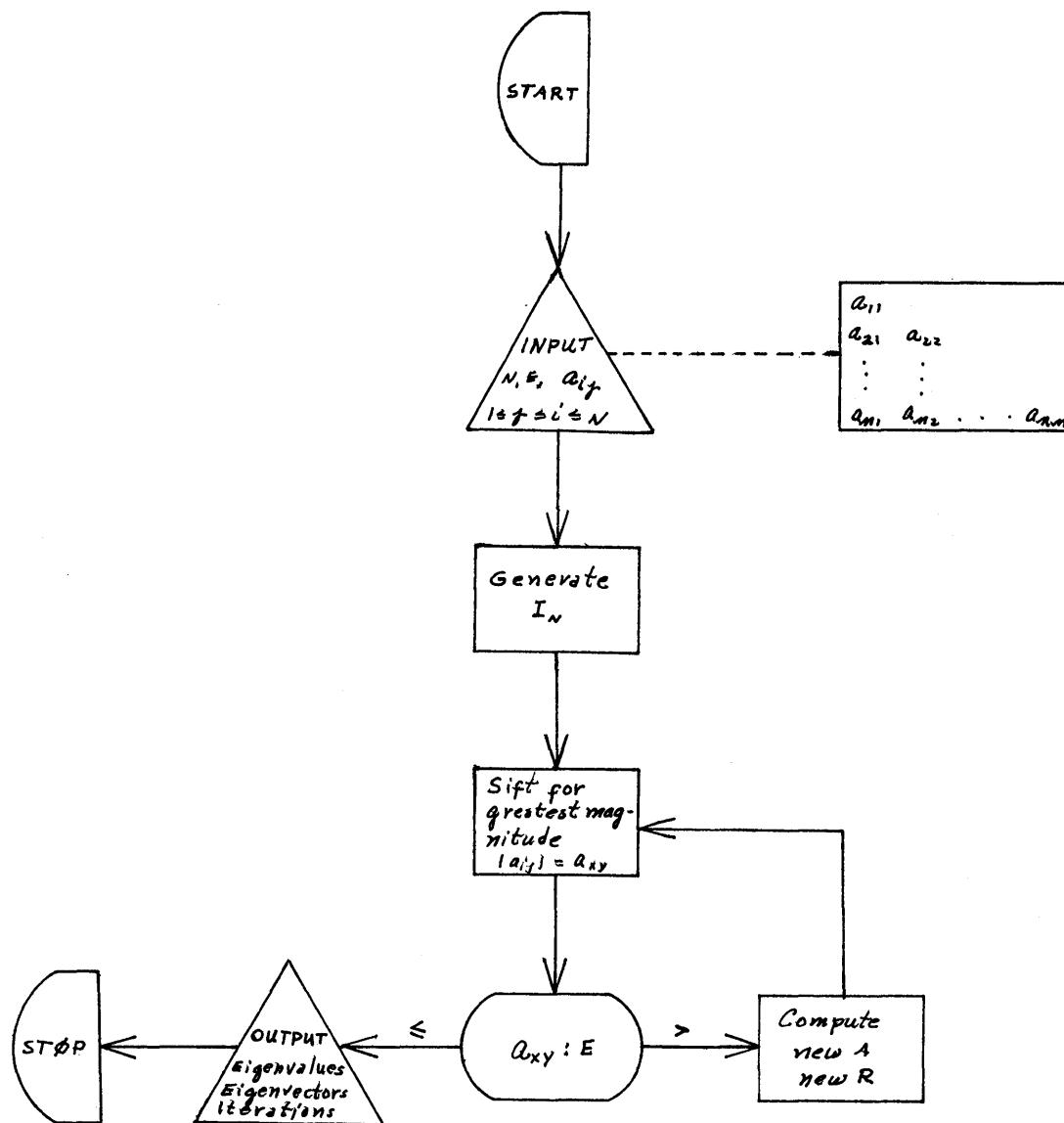
$$+.39504114445 + 8$$

EIGENVECTOR

$$\begin{array}{cccc} -.10840001244 - 3 & +.91968347409 - 2 & -.59710227180 - 1 & +.18265861385 + 0 \\ -.62393005507 + 0 & +.75742802461 + 0 & & \end{array}$$

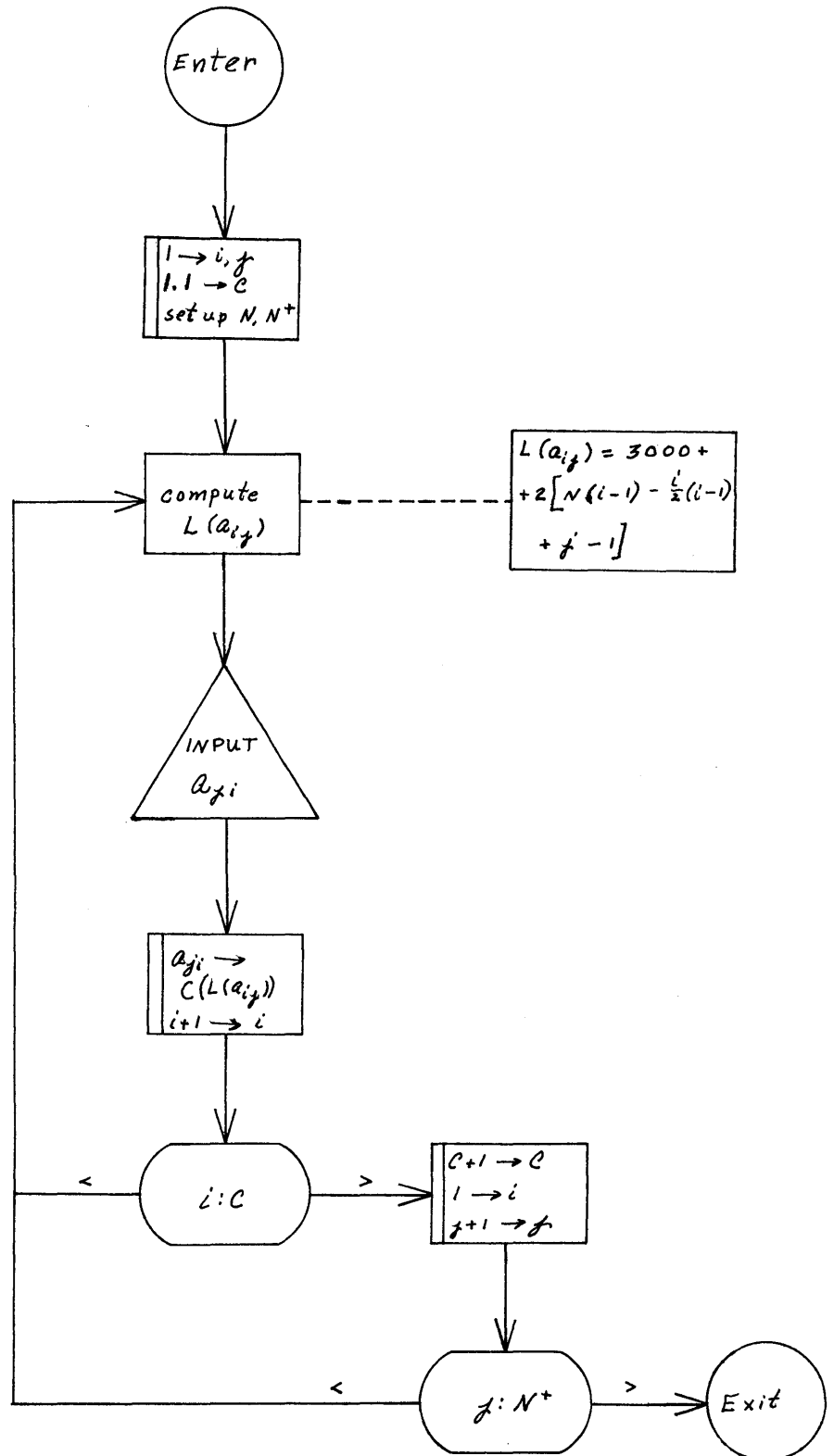
44 ITERATIONS

AGC-109 CONTROL



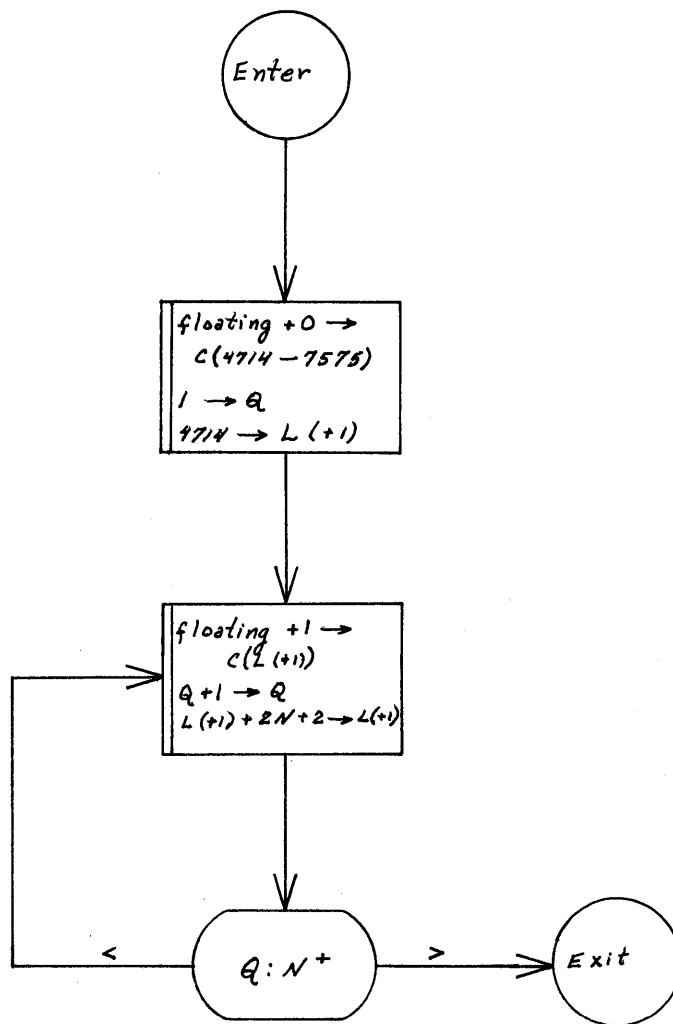
AGC-109

Sifting for element with greatest magnitude

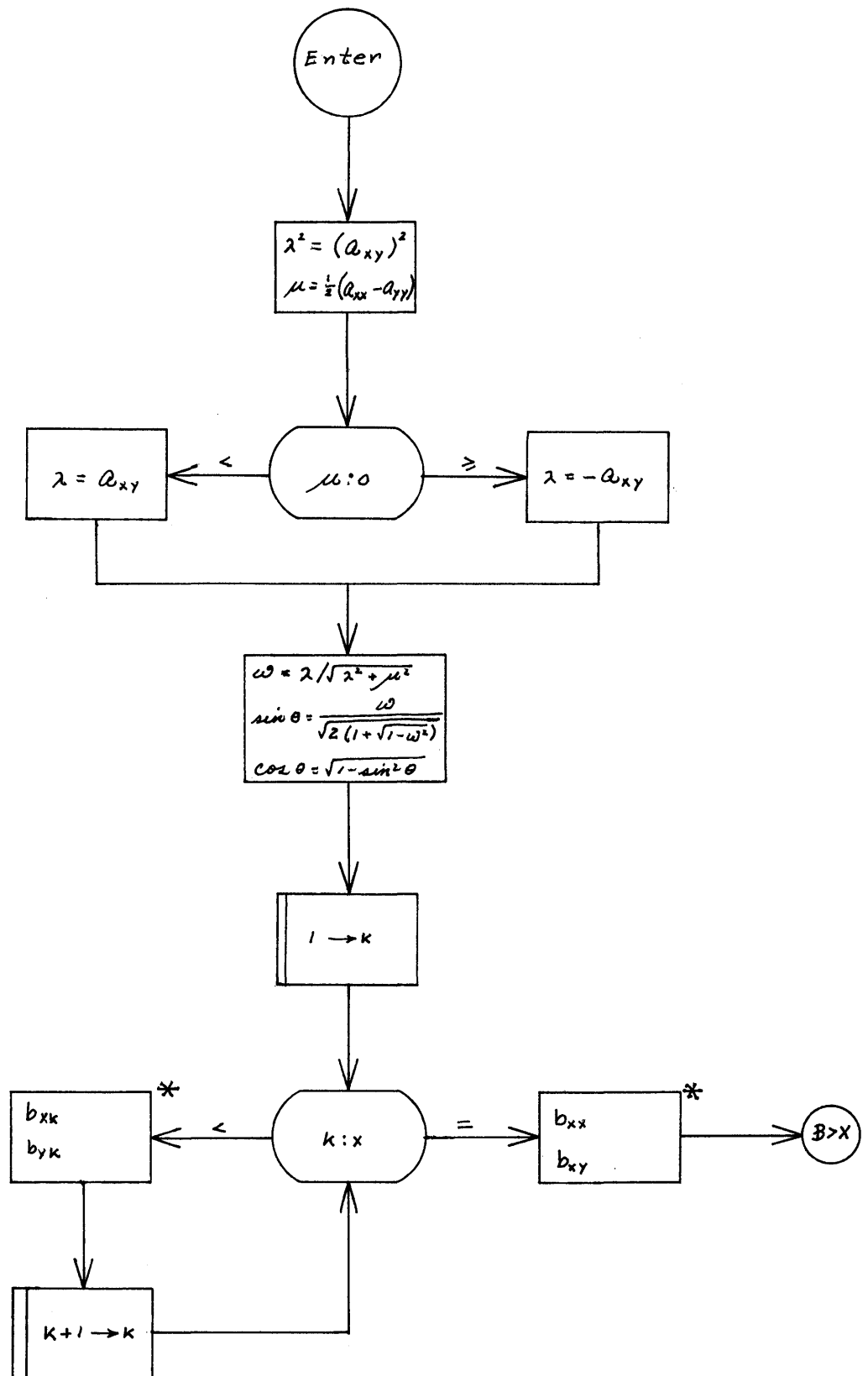


AGC-109

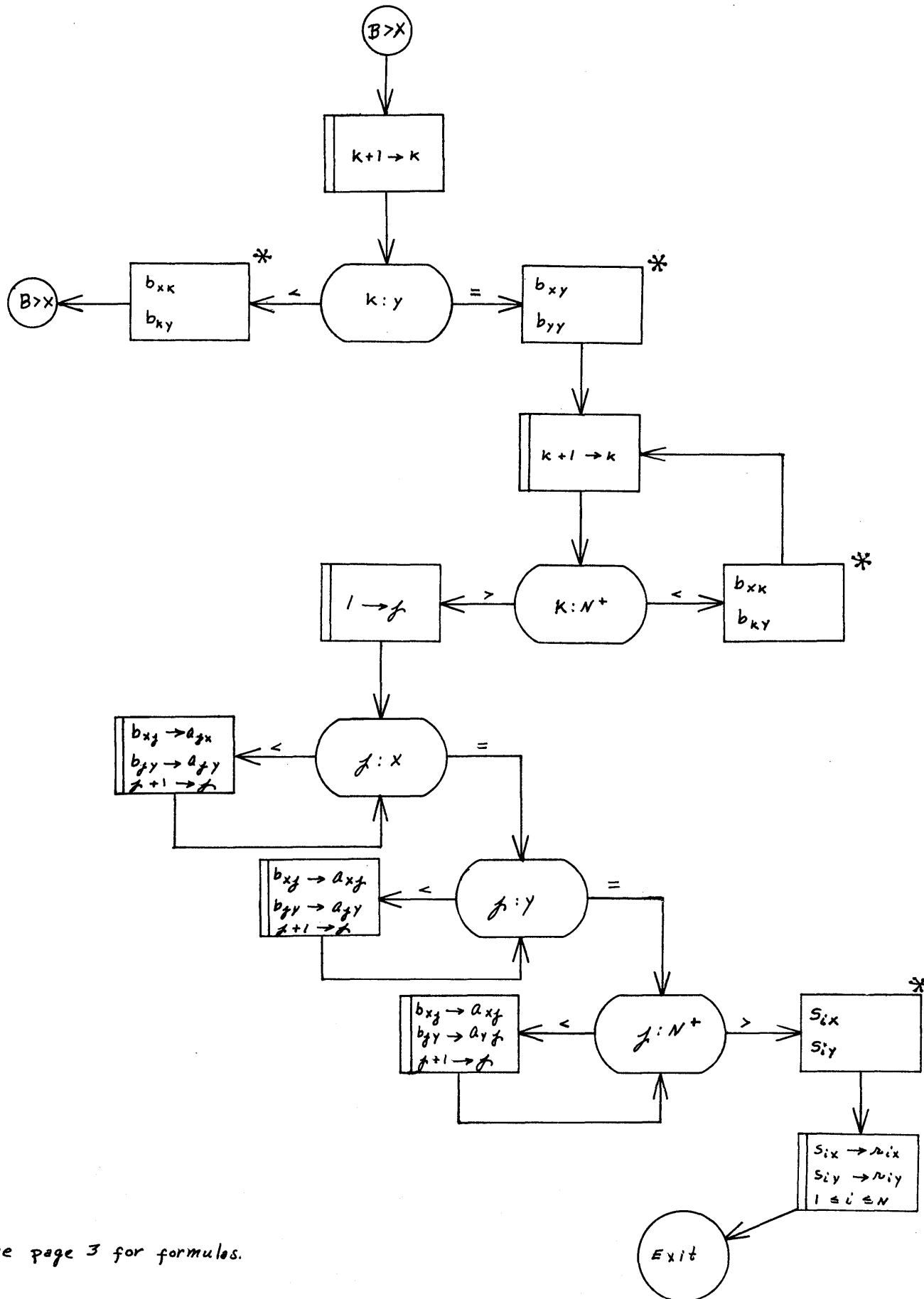
Generation of I_N ($N \times N$ identity matrix)



AGC-109 Computation of new A and new R.



* See page 3 for formulas.



* See page 3 for formulas.

AGC-109 New A and new R (p. 3).

$$b_{xk} = \begin{cases} a_{kx} \cos \theta - a_{ky} \sin \theta & 1 \leq k < x \\ a_{xx} \cos^2 \theta + a_{yy} \sin^2 \theta - 2a_{xy} \sin \theta \cos \theta & k = x \\ a_{xk} \cos \theta - a_{ky} \sin \theta & x < k < y \\ 0 & k = y \\ a_{xk} \cos \theta - a_{yk} \sin \theta & y < k \leq N \end{cases}$$

$$b_{ky} = \begin{cases} a_{kx} \sin \theta + a_{ky} \cos \theta & 1 \leq k < x \\ 0 & k = x \\ a_{xk} \sin \theta + a_{ky} \cos \theta & x < k < y \\ a_{xx} \sin^2 \theta + a_{yy} \cos^2 \theta + 2a_{xy} \sin \theta \cos \theta & k = y \\ a_{xk} \sin \theta + a_{yk} \cos \theta & y < k \leq N \end{cases}$$

$$\left. \begin{array}{l} b_{xk} \rightarrow a_{kx} \quad 1 \leq k \leq x \\ b_{xk} \rightarrow a_{xk} \quad x < k \leq N \\ b_{ky} \rightarrow a_{ky} \quad 1 \leq k \leq y \\ b_{ky} \rightarrow a_{yk} \quad y < k \leq N \end{array} \right\} \text{replace equivalent of 1 row and 1 column.}$$

$$b_{if} \equiv b_{fi}$$

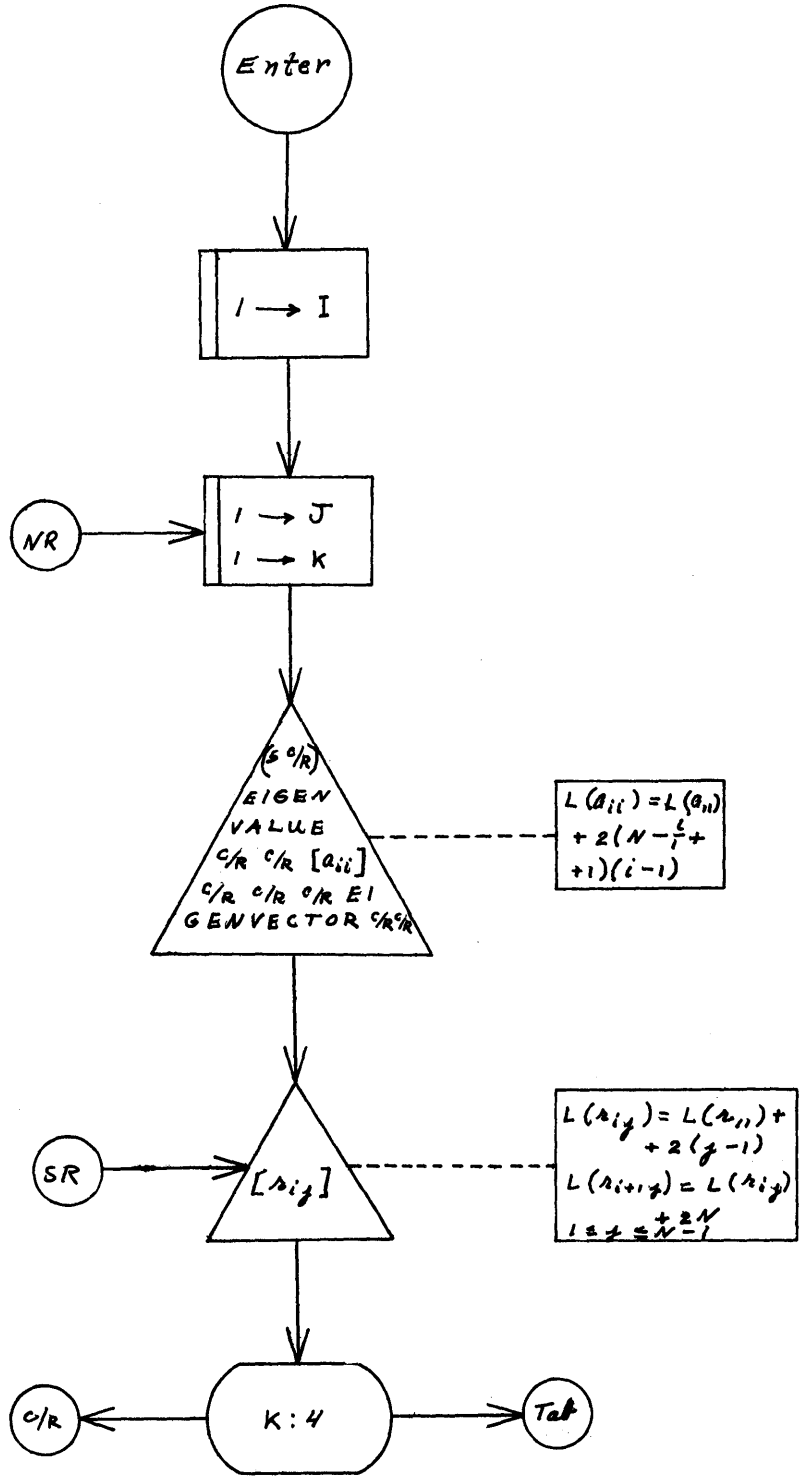
$$\begin{aligned} s_{ix} &= r_{ix} \cos \theta - r_{iy} \sin \theta & 1 \leq i \leq N \\ s_{iy} &= r_{ix} \sin \theta + r_{iy} \cos \theta & 1 \leq i \leq N \end{aligned}$$

$$\left. \begin{array}{l} s_{ix} \rightarrow r_{ix} \\ s_{iy} \rightarrow r_{iy} \end{array} \right\} 1 \leq i \leq N \quad \text{replace 2 columns.}$$

$$L(a_{if}) = L(a_{ii}) + 2 \left[N(i-1) - \frac{i(i-1)}{2} + f - 1 \right]$$

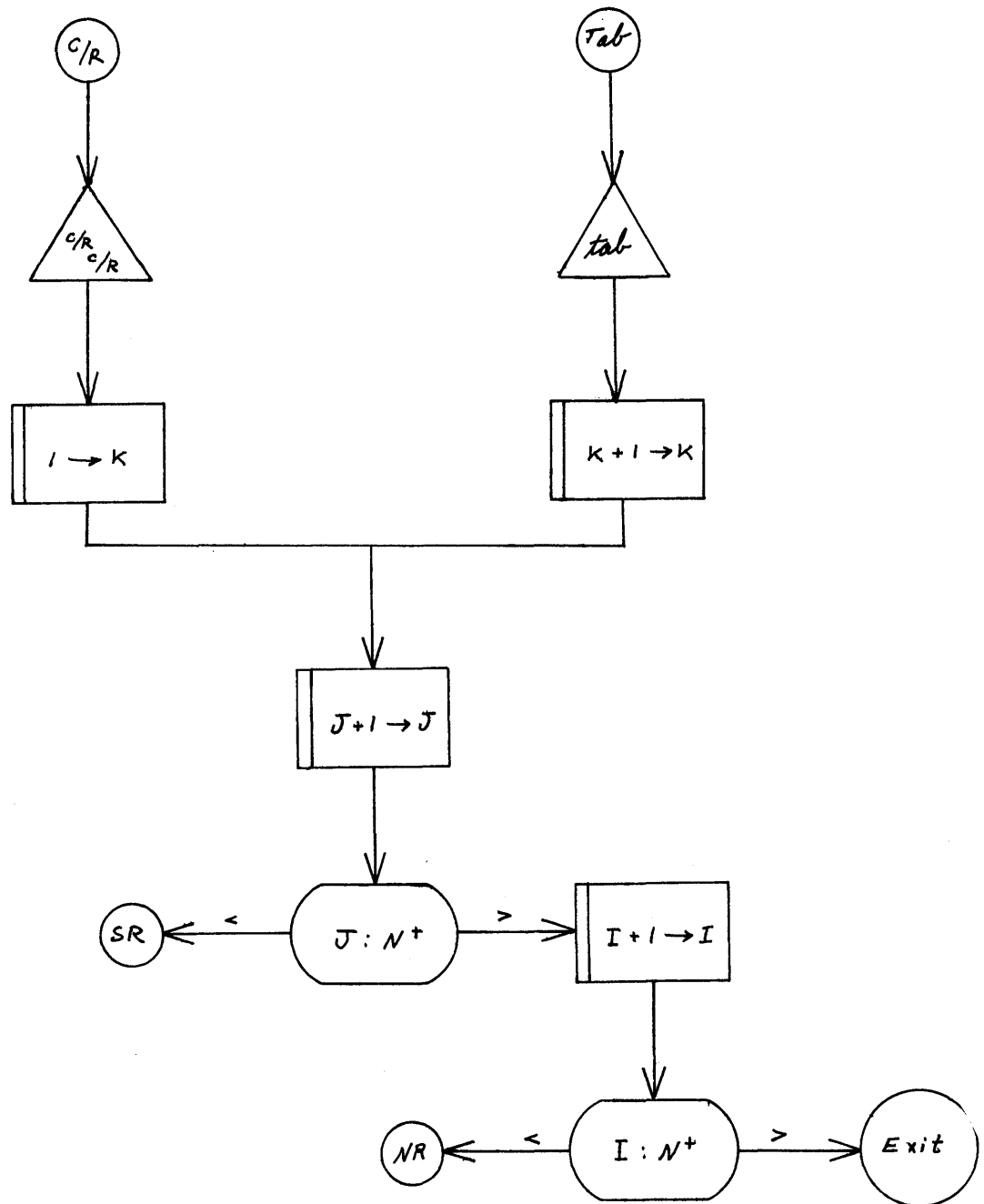
$$L(r_{if}) = L(r_{ii}) + 2 \left[N(i-1) + f - 1 \right]$$

AGC-109 OUTPUT



AGC-109

OUTPUT (p.2)



L00000	C+1513440+7700000	+0003100+6012700	+5702120+4100010	+6012650+5700041
L00040	C+1502040+6401500	+5777610+4201640	+6401600+6601700	+5777600+4000000
L00100	C+1500270+6400300	+6600400+5777600	+6601300+5777740	+4100010+4277761
L00140	C+0077630+5277760	+0277640+5777761	+0077640+4000000	+5601431+5702160
L00200	C+0077660+5077730	+0077640+2177660	+5300120+6077640	+0077650+0377670
L00240	C+0377770+5177760	+5077760+5701061	+6600100+5777730	+0000000-0000230
L00300	C+0177670+4201061	+0177670+4200171	+5777741+0000500	+0200000-0000000
L00340	C+0000000-0000000	+0000000-0000000	+0000000-0000000	+0000000-0000001
L00400	C+3577600+3077640	+0701400+0477600	+3577640+3077660	+5077741+0701400
L00440	C+3577660+0077630	+7177610+4100010	+0177620+4277771	+0077640+5700000
L00500	C+5777741+4000000	+3001140+5777700	+5701061+4000000	+5001420+5701061
L00540	C+5700600+4000000	+5701061+4000000	+3001260+5777700	+3001240+5777700
L00600	C+0077660+5701450	+5701061+4000000	+3001160+5777700	+5701061+4000000
L00640	C+5701061+4000000	+5701061+4000000	+5701061+4000000	+5701061+4000000
L00700	C+3001200+5777700	+5077741+5701061	+5701061+4000000	+3001120+5777700
L00740	C+5701061+4000000	+3001220+5777700	+3000340+5777700	+3001320+5777700

L 01000	C+3001100+5777700	+5701061+4000000	+5701061+4000000	+5777741+4000000
L01040	C+0077660+5701300	+5701061+4000000	+4000000+5702151	+5777741+4000000
L01100	C+4400000-0000000	+0000000-0000020	+4000000-0000000	+0000000-0000010
L01140	C+6000000-0000000	+0000000-0000010	+4000000-0000000	+0000000-0000011
L01200	C+5000000-0000000	+0000000-0000011	+6000000-0000000	+0000000-0000011
L01240	C+7000000-0000000	+0000000-0000011	+4000000-0000000	+0000000-0000020
L01300	C+5001310+5701061	+3001320+5777740	+4000000-0000000	+0000000-0000001
L01340	C+0077650+0177730	+6077650+0077640	+1177720+2377660	+6600200+5777721
L01400	C+5000000-0000000	+0000000-0000020	+0277630+6077630	+5777741+6077600
L01440	C+3677600+5700171	+5001461+6600200	+5777700+3001320	+3577660+5701451
L01500	C+0102010+4201721	+0002000+6002060	+6002040+0102000	+6002070+0102000
L01540	C+6002050+3030020	+0030021+3512660	+0002110+4201601	+4201631+5700051
L01600	C+3012660+0630041	+5277650+0002040	+6002060+0002050	+6002070+3030041
L01640	C+0030041+3512660	+0002050+0102000	+6002050+0302100	+5177740+0002040
L01700	C+0102000+6002040	+0102000+6002050	+0302100+5212710	+0002030+5777741
L01740	C+0002020+0177600	+4277601+4277631	+4100240+4277640	+5777600+4000000

L02000	C+0000000-0000010	+0000000-0000001	+0000000-0000020	+0000000-0000040
L02040	C+0000000-0000050	+0000000-0000050	+0000000-0000020	+0000000-0000040
L02100	C+0000000-0000041	+0030041-0030041	+7200370+7200100	+7200140+7200040
L02140	C+7200330+0002260	+5700100+5702141	+6402200+5777600	+6402300+5777600
L02200	C+6002270+0102010	+6002100+7200370	+7200100+7200100	+0002000+6002040
L02240	C+6002050+0102010	+6002440+5702600	+0000000-0000230	+0000000-0000040
L02300	C+0002040+0302000	+6002450+1102270	+4300000+4000010	+6002460+0202040
L02340	C+1102450+4300000	+4000020+0102050	+0302000+0102460	+4100010+6602400
L02400	C+0177770+4202421	+5702411+5723100	+5702411+3530220	+6402500+5777600
L02440	C+0000000-0000010	+0000000-0000030	+0000000-0000100	+0000000-0030000
L02500	C+0002040+0102000	+6002040+0302440	+5102170+0002000	+6002040+0102440
L02540	C+6002440+0002050	+0102000+6002050	+0302100+5102170	+6402700+5777600
L02600	C+7200010+7200040	+7200330+5723100	+5702611+3502660	+4000000+7200370
L02640	C+7200100+7200100	+6402300+5777600	+5075340-5075340	-0000000-0000030
L02700	C+3003100+3577700	+3577720+3577740	+3577760+3547160	+0002000+6002040
L02740	C+4000000+6747200	+0077640+0103120	+4277641+0303130	+5177641+6603000

L03000	C+0002270+4100010	+0102020+6003140	+3403150+3547140	+0077720+0103140
L03040	C+4277721+0002040	+0102000+6002040	+0302100+5177720	+0002270+5700021
L03100	C+0000000-0000000	-0000000-0000230	+0000000-0000100	+4000000+6775771
L03140	C+0000000-0000120	-4000000-0000000	+0000000-0000001	+6403300+6603400
L03200	C+5777600+0077710	+4204761+0102020	+4205221+0077750	+4203261+0102020
L03240	C+4205261+6404600	+6604700+5777600	+6405000+3545400	+6605200+5777600
L03300	C+0002060+0302000	+6012150+1102270	+4300000+4000010	+6002460+0202060
L03340	C+1112150+4300000	+4000020+0102460	+0302000+6012350	+0102070+4100010
L03400	C+0177770+4203601	+4203611+0012350	+0102060+4100010	+0177770+4203621
L03440	C+0002070+0302000	+6012160+1102270	+6403500+5777600	+0000000-0030000
L03500	C+4300000+4000010	+6002460+0202070	+1112160+4300000	+4000020+0102460
L03540	C+0302000+6010270	+0102070+4100010	+0177770+4203631	+6403600+5777600
L03600	C+6603700+3030040	+3512400+0730040	+3512420+3030000	+4000000+0630160
L03640	C+0512440+3512460	+5077660+5177670	+3412400+3512400	+3012460+0712460
L03700	C+0412420+3512360	+4412360+3512360	+3012400+0512360	+3512500+0703150
L03740	C+0712500+0603150	+3512360+5714020	+0603150+6404000	+6604100+5777600

L04000	C+0712440+3512360	+4412360+3512360	+3012500+0512360	+3512520+0712520
L04040	C+3512540+0703150	+0603150+3512560	+4412560+3512600	+0712520+0712440
L04100	C+3512620+0002000	+6002040+6404200	+6604400+5777600	+0002050+0302070
L04140	C+5011350+6411100	+0077700+4211221	+0077720+4211241	+6611200+5777600
L04200	C+0302060+5003201	+0002040+0302000	+6002440+1102270	+4300000+4000010
L04240	C+6002450+0202040	+1102440+4300000	+4000020+0102450	+6404300+5777600
L04300	C+0302000+6012170	+0102060+4100010	+0103470+4277661	+4277721+0012170
L04340	C+0102070+4100010	+0103470+4277701	+4277731+3000000	+0712600+3512360
L04400	C+3412520+0700000	+0412360+3543640	+3012520+0700000	+3512360+3000000
L04440	C+0712600+0412360	+4000000+3545400	+0077710+0102020	+6404500+5777600
L04500	C+4277711+0077750	+0102020+4277751	+0002040+0102000	+6002040+0302060
L04540	C+5003201+4000000	+6404200+5777610	+0302100+5210260	+5777620+4000000
L04600	C+0012350+0102060	+4100010+0103470	+4277711+4210051	+0012350+0102070
L04640	C+4100010+0103470	+4277741+4210111	+0010270+0102070	+4100010+0103470
L04700	C+4277721+4210061	+3012560+0700000	+3512360+3000000	+0712540+0412360
L04740	C+3512360+3400000	+0712620+0412360	+4000000+3543640	+3003100+5703260

L05000	C+0002040+0102000	+6002040+0302070	+5007770+0012350	+0102040+4100010
L05040	C+0103470+4277731	+4205171+0002040	+0302000+6002440	+6405100+5777600
L05100	C+1102270+4300000	+4000010+6002450	+0202040+1102440	+4300000+4000020
L05140	C+0102450+0102070	+0302000+4100010	+0103470+4277741	+4277711+3030020
L05200	C+0712600+3512360	+3412520+0700000	+0412360+3543660	+3012520+0700000
L05240	C+3512360+3000000	+0712600+0412360	+4000000+3545420	+6407700+5777600
L05300	C+1505400+6605500	+6405400+5777700	+5077760+5277740	+6077700+0277700
L05340	C+6077650+4000000	+6605700+5777700	+0077630+4100430	+6606300+5777701
L05400	C+6645720+5261350	+0000000-0000000	+0000000-0000000	+0000000-0000001
L05440	C+0000000-0000050	+0000000-0000000	+0000000-0000000	+1277771+5700010
L05500	C+0177670+6077670	+4277721+4300000	+6077610+0000000	+4000250+6077620
L05540	C+7200330+0077600	+5277761+7200030	+5777770+7200210	+6605600+5777700
L05600	C+7200340+5277720	+0277600+6077600	+5077770+0077610	+1177760+0177630
L05640	C+4300000+3577650	+6607600+5777741	+2321010-6502370	+6606700+5777710
L05700	C+1377760+4000010	+0177770+1377650	+4000010+0177750	+1377650+4000010
L05740	C+6606000+5777700	+0024721-0475071	+0002071-5234230	+0003721-2076741

L06000	C+0177760+1377650	+4000010+0177770	+1377650+4000010	+0177750+1377650
L06040	C+6606100+5777700	+1050240-1055150	+0105271-5031700	+0324031-3304770
L06100	C+4000010+0177750	+1377650+4000010	+0177760+1377650	+4000020+0177770
L06140	C+6606200+5777700	+2257270+7051121	+4043401-6076151	+2515220+1435700
L06200	C+1377650+4000010	+0177770+1377650	+0177760+6077700	+1377700+6077650
L06240	C+0077610+4000000	+6606300+5277700	+2000000-0000000	+2232730-3356650
L06300	C+0077650+1177600	+6077650+4100040	+4000040+2277750	+6606400+5777700
L06340	C+4300000+6606400	+0400000-0000000	+0077750+2177650	+1377600+5777740
L06400	C+4300000+0077650	+4000430+5077750	+2377640+4300000	+0077620+4100010
L06440	C+6606500+5777700	+0077660+0377630	+6077660+4300000	+1177640+5777711
L06500	C+0177710+6077710	+4300000+0106550	+5377730+5777750	+0077660+0177630
L06540	C+6077660+0077760	+6606700+5777710	+0631460+4631461	+0024361-7024361
L06600	C+0002030-2233511	+0000150-3334271	+0000010+1742651	+0000000-4143370
L06640	C+0000000-0326600	+0000000-0025361	+0000000-0002111	+0000000-0000160
L06700	C+0000000-0000011	+4300000+0077620	+0377770+5277750	+0077700+4000020
L06740	C+6077650+5777761	+6077650+0077770	+6077620+6607000	+0000000-0000040

L07000	C+4100270+0177750	+6077600+0277650	+4100270+0177760	+6077610+0077620
L07040	C+6607100+5777740	+4100430+5777710	+4100430+5777761	+0277620+5777700
L07100	C+4300000+1177640	+0177770+4100040	+6077770+0077620	+0377630+6077620
L07140	C+5077750+5777700	+0077770+4000000	+6607200+5777700	+0000000-0000000
L07200	C+0177760+5777600	+6077600+0077650	+6607300+5777740	+1277601+7200040
L07240	C+0077600+4000430	+6607400+5777700	+0000000-0000071	+6077610+5777730
L07300	C+4300000+1177640	+0177620+4100040	+6077620+0077650	+0377630+6077650
L07340	C+5077750+5777700	+0077620+0177770	+5777610+6607200	+0000000-0000071
L07400	C+5077760+0077660	+5077760+5277740	+7200030+6077660	+0277660+5777741
L07440	C+7200210+4300000	+6607500+5777700	+7200210+5777670	+5017000-0000000
L07500	C+0077650+2277770	+5077730+7200340	+7200340+5777671	+2277640+5077750
L07540	C+4100430+5777760	+0077640+5777740	+6607600+5777710	+0000000-0000620
L07600	C+0017000-0000000	+6077770+4300000	+4100370+0177770	+0177700+6077770
L07640	C+5777670+0077610	+1377770+0177650	+6605300+5777720	+7477700+0000000
L07700	C+0077720+0102020	+4277721+0077760	+0102020+4277761	+0002040+0102000
L07740	C+6002040+0302070	+5077670+4000000	+6405000+5777621	+6410000+5777600

L10000	C+0077720+4277641	+0102020+4210331	+0077760+4210121	+0102020+4210371
L10040	C+3003100+3500000	+3012540+0730000	+3512360+3030160	+0712560+6610100
L10100	C+0412360+3512360	+3012620+0730040	+0412360+3545440	+0002040+0102000
L10140	C+6002040+0302100	+5210541+0012350	+0102040+4100010	+6410200+5777600
L10200	C+0103470+4210301	+4210341+0010270	+0102040+4100010	+0103470+4210311
L10240	C+4210351+6610400	+6410300+5777600	+6411600+5777600	+0000000-0000040
L10300	C+3012600+0730060	+3512360+3430200	+0712520+0412360	+4000000+3543720
L10340	C+3012520+0730060	+3512360+3030200	+0712600+0412360	+4000000+3545460
L10400	C+0077600+0102020	+4277601+4277641	+0077610+0102020	+4277611+4277651
L10440	C+0077630+0102020	+4277631+0077670	+0102020+4277671	+5710500+4000000
L10500	C+0002040+0102000	+6002040+0302100	+5210541+4000000	+5777600+4000000
L10540	C+4000000+0002000	+6002050+0302060	+5015520+6611000	+6410600+5777600
L10600	C+0002050+0302000	+6002440+1102270	+4300000+4000010	+6002450+0202050
L10640	C+1102440+4300000	+4000020+0102450	+0302000+6012170	+6410700+5777600
L10700	C+0102060+4100010	+0103470+4277711	+0012170+0102070	+4100010+0103470
L10740	C+4277731+5777701	+0102000+6002050	+0302060+5004130	+6410600+5777600

L11000	C+4000000+3043640	+4000000+3500000	+4000000+3045400	+4000000+3500000
L11040	C+0077700+0102020	+4277701+0077720	+0102020+4277721	+0002050+5777650
L11100	C+0012350+0102050	+4100010+0103470	+4277731+0002050	+0302000+6002440
L11140	C+0202050+4000010	+0102270+1102440	+4300000+4000010	+0102070+0302000
L11200	C+4000000+4000000	+4100010+0103470	+4277751+3043640	+4000000+3500000
L11240	C+4000000+3045400	+6411300+3500000	+0077720+0102020	+4277721+5777600
L11300	C+0077740+0102020	+4277741+0002050	+0102000+6002050	+0302070+5077650
L11340	C+6411100+5777600	+0002050+0302100	+5210260+0077720	+6411400+5777600
L11400	C+4277671+0077740	+4211511+6611500	+0012350+0102050	+4100010+0103470
L11440	C+4277701+0010270	+0102050+4100010	+0103470+4277721	+4000000+3000000
L11500	C+4000000+3500000	+4000000+3045440	+4000000+3500000	+0077670+0102020
L11540	C+4277671+0077710	+0102020+4277711	+0002050+0102000	+6002050+5704560
L11600	C+0002000+6002050	+0012150+4100010	+0112640+4211701	+4211741+4212211
L11640	C+0012160+4100010	+0112640+4211711	+4211751+4212231	+6411700+5777600
L11700	C+3012600+0747140	+3512360+3447200	+0712520+0412360	+6612000+3576000
L11740	C+3012520+0747140	+3512360+3047200	+0712600+0412360	+4000000+3576660

L12000	C+0077600+0112650	+4277601+4277641	+0077610+0112650	+4277611+4277651
L12040	C+0077630+0102020	+4277631+0077670	+0102020+4277671	+6612100+5777700
L12100	C+0002050+0102000	+6002050+0302100	+5177600+6412200	+0002000+6002050
L12140	C+6612300+5777601	+0000000-0000000	+0000000-0000020	-0000000-0000010
L12200	C+4000000+3076000	+4000000+3547140	+4000000+3076660	+4000000+3547200
L12240	C+0077600+0102020	+4277601+0077620	+0102020+4277621	+0077610+0112650
L12300	C+4277611+0077630	+0112650+4277631	+0002050+0102000	+6002050+0302100
L12340	C+5177601+5700041	-0000000-0000010	+5624100-2361660	-0000000-0000041
L12400	C-6450160+5044100	-0000000-0000030	+5321601-6212310	-0000000-0000060
L12440	C+4000000-0000000	+0000000-0000010	+5316170+7435210	+0000000-0000001
L12500	C-4675440+3516220	-0000000-0000030	-4675460-5324260	-0000000-0000031
L12540	C+5734110-6763440	-0000000-0000071	+7777720-2173320	-0000000-0000000
L12600	C+7777750-1075331	+0000000-0000000	-4675440+3516220	-0000000-0000030
L12640	C+0000000-0047140	+0000000-0000100	+5027011-5552430	-0000000-0000030
L12700	C+0000000-0000050	+3012660+0602660	+5012750+5112750	+0012700+0102010
L12740	C+6012700+5713450	+0002000+6002040	+6413000+6613100	+5777600+4000000

L13000	C+0002000+6002050	+6002440+7200370	+7200100+7200100	+7200100+7200100
L13040	C+7200100+7200010	+7200060+7200320	+7200010+7200140	+7200360+7200030
L13100	C+7200220+7200070	+7200010+4000000	+7200100+7200100	+0002040+0302000
L13140	C+6002450+0202040	+4000010+0102000	+0102270+1102450	+6413200+5777600
L13200	C+4300000+0103470	+4277641+0002040	+0302000+1102000	+4300000+0112640
L13240	C+4213401+3000000	+5713251+5705300	+0000130+7200370	+6413300+5777600
L13300	C+7200100+7200100	+7200100+7200010	+7200060+7200320	+7200010+7200140
L13340	C+7200360+7200010	+7200160+7200200	+7200300+7200120	+7200100+5713400
L13400	C+7200100+3047520	+4000000+5705300	+0000130+6413500	+6613600+5777600
L13440	C+5077721+5177701	+4100010+6013470	+3613470+5703170	+0000000-0000120
L13500	C+0002440+0302030	+5077640+7200330	+7200100+0002440	+0102000+5777660
L13540	C+7200370+7200100	+7200100+0002000	+6002440+0002050	+0102000+6002050
L13600	C+0302100+5177740	+0002040+0102000	+6002040+0302100	+5112760+5713660
L13640	C+0013400+0112650	+4213401+5713401	+7200370+7200100	+7200100+7200100
L13700	C+7200100+7200100	+7200330+0012700	+5714100+7200370	+7200040+7200060
L13740	C+7200200+7200010	+7200120+7200030	+7200200+7200060	+7200300+7200140

L14000	C+7200050+7200100	+7200100+7700010	+3012360+5114040	+4412360+5777760
L14040	C+3014050+5777760	+0000000-0000000	+0000000-0000000	-0552271-3423560
L14100	C+1514350+0114140	+4214321+0014350	+5014160+6414200	+6614300+5777600
L14140	C+0000000-0000001	+0170000-0000000	+7200260+5714321	+0000000-0000010
L14200	C+0277750+2214420	+3314150+6077740	+5077700+0014400	+6077620+0077740
L14240	C+4100050+0177760	+6077770+1277771	+0077740+1114420	+0177750+6077750
L14300	C+0077600+0114170	+4277601+4277661	+0314410+5213721	+5177600+4000000
L14340	C+0000000-0000000	+0000000-0000050	+0360000-0000000	+0000000-0000000
L14400	C+5777640+0014400	+0277750+2214511	-0046110+5000000	-0003640-4400000
L14440	C-0000300+5200000	-0000020+6100000	-0000000-7640000	-0000000-0620000
L14500	C-0000000-0050000	-0000000-0004000	+4477740+3577760	+5713300+6414470
L14540	C+5777641+3535000	+3077600+0635000	+4000000+5730021	+2514340-7407601
L14600	C+1515270+6414700	+6615000+5777600	+0000000-0000000	+0000000-0014600
L14640	C+0014600-0000000	+0000000-0000000	+0000000-0000000	+0000000-0000000
L14700	C+0115350+4277631	+0177720+4215301	+0115350+4277731	+4000000+0000000
L14740	C+4277670+4277651	+4000000+3000000	+3577740+4000000	+3000000+3577760

L15000	C+0577740+5315220	+6715000+5715100	+0000000-0000010	+4000000+5700000
L15040	C+0000000-0000000	+0000000-0000000	+0000000-0000000	+0000000-0000000
L15100	C+5713300+6415120	+6615000+5777620	+3577760+5077631	+5177651+3077740
L15140	C+5177661+3077760	+5777731+3077740	+5277600+3077760	+0415250+5777731
L15200	C+3077760+0415230	+5777731+4000000	+6415300+5777600	+6220770+5242110
L15240	C+0000000-0000011	+6220770+5242110	+0000000-0000010	+0000000-0000000
L15300	C+3077760+5000000	+5177621+3077640	+5777731+3077660	+5777731+4000000
L15340	C+6220770+5242110	+0000000-0000001	+4554571-1771470	+0000000-0000011
L15400	C+1515670+4000000	+6415500+6615600	+0177700+5777600	+0000000-0000000
L15440	C+0000000-0000000	+0000000-0000000	+0000000-0000000	+0000000-0000000
L15500	C+6077700+0077770	+5077701+0577750	+6415500+5777630	+0077650+4211221
L15540	C+0077670+4211241	+6611200+0043640	+6411100+5777600	+0000000-0045400

L23100	C+1577630+6423100	+6623700+5777620	+0177670+4224301	+0177670+4224271
L23140	C+3077720+0077730	+3524450+7200330	+6423600+5777700	+0000000-0000001
L23200	C+5724300-0000000	+5724300-0000000	+5724300-0000000	+0023250+5724350
L23240	C+5777600-0000000	+5724300-0000000	+5724300-0000000	+0024310+5777630
L23300	C+0024320+5777630	+0024330+5777630	+0024340+5777630	+5724300-0000000
L23340	C+0024360+5777630	+5724300-0000000	+0024400+5724650	+0024410+5777630
L23400	C+5724300-0000000	+5724300-0000000	+5724300-0000000	+5724300-0000000
L23440	C+5724300-0000000	+0024470+5777630	+5724300-0000000	+3023730+5724370
L23500	C+0024520+5777630	+0024530+5777630	+5724300-0000000	+3023730+5724370
L23540	C+0024570+5724650	+5724300-0000000	+0024600+5777630	+3023730+5724370
L23600	C+7177610+4100010	+0177640+4277621	+0077720+5700000	+6077730+0077740
L23640	C+1177760+4323570	+0177730+5324300	+6077740+0077771	+6077770+5077600
L23700	C+0077720+0177750	+6077720+5777600	-0000001-0000000	+7777771+7777771
L23740	C+0000000-0000000	+0000001-0000000	+0000000-0000050	-0000000-0000000

L24000	C+6077730+0077770	+5177650+0077740	+3324450+6024630	+0077670+6077740
L24040	C+6077770+6024460	+0077730+6024450	+6423600+5777661	+0000000-0000000
L24100	C+5077660+0077740	+6024630+0277720	+4100120+6624600	+3577760+3077730
L24140	C+4500000+3577730	+6424500+5777640	+0077740+4100240	+0377720+5777620
L24200	C+6077650+3077760	+5077670+4500000	+3377710+6077760	+4300000+4100020
L24240	C+0177650+6077650	+3077730+0725060	+3577730+5777601	+3077730+5702421
L24300	C+7200370+5702420	+0000000-0000041	+0000000-0000001	+0000000-0000000
L24340	C+0000000-0000030	+6077770+5724510	+0000000-0000010	+6424420+5777620
L24400	C+7777771+7777771	+0000000-0000021	+0077740+3377650	+6077740+0077660
L24440	C+6424100+5777600	-7777771+7777771	+7777771+7777771	+0000000-0000020
L24500	C+6424200+5777600	+0077720+5777600	+0000000-0000031	+0000000-0000040
L24540	C+0077760+5277660	+0077720+5777600	+6424200+5777601	-7777771+7777771
L24600	C+0000000-0000011	+3777771+7777771	+3077730+0525060	-0000000-0000001
L24640	C+0000000-0000231	+6424000+5777600	+5000000-0000000	+0000000-0000020
L24700	C+6200000-0000000	+0000000-0000031	+4704000-0000000	+0000000-0000070
L24740	C+5753601-0000000	+0000000-0000151	+4341570-4677010	+0000000-0000330

L25000	C+4734261-6650130	+0000000-0000651	+6047400+7512000	+0000000-0001521
L25040	C+4473510-3711400	+0000000-0003250	+5247731-5773471	+0000000-0006511
L25100	C+1525340+0125310	+4226331+0025340	+5125141+0625320	+5225121+0425320
L25140	C+5725160+0425320	+5125141+0625320	+6025340+4300000	+0325350+4100010
L25200	C+4225211+4000000	+0025340+4000000	+6025340+5125241	+6425400+6625500
L25240	C+5777600+0225450	+6025450+0225460	+6025460+0225550	+6025550+0025340
L25300	C+5725230+4000000	+0000000-0000001	+6220770+5242101	+0000000-0000011
L25340	C+0000000-0000000	+0000000-0000011	+0000000-0000000	+0000000-0000000
L25400	C+1177670+6077600	+4100110+1177770	+6077610+4100110	+1177770+6077620
L25440	C+0077600+5777700	+0000500-0240000	+0300000-7400000	+7122731-0323071
L25500	C+4000360+4300000	+2277760+4100430	+0177650+6077630	+0077750+6077670
L25540	C+6625600+5777700	+0000000-0000000	+0000000-0000620	+0740000-0000000
L25600	C+2277770+4100370	+0177630+6077630	+4300000+4100330	+0177630+6077630
L25640	C+0077610+4000360	+4300000+4000000	+6625700+5777700	+0000000-0000050
L25700	C+0077670+2277770	+4100230+0177630	+6077630+4300000	+4100170+0177630
L25740	C+6077630+0077620	+4000360+4300000	+0077670+6626000	+0000000-0000050

L26000	C+4100070+0177630	+6077630+4300000	+4100430+0177660	+6077640+0077620
L26040	C+4100110+1177760	+4100330+6626100	+0000000-0000050	+2277760+5777700
L26100	C+1177750+4100270	+0177640+6077640	+4300000+1177750	+4100230+0177640
L26140	C+6077640+6626200	+0000000-0000050	+0177640+6077640	+4300000+5777700
L26200	C+7200330+7200030	+3477630+3525360	+1225361+0077740	+6426300+5777600
L26240	C+0000500-0240000	+0077630+5177700	+7200330+7200040	+3077630+5777711
L26300	C+6025450+0077640	+6025460+0077650	+6025550+0077660	+6025470+5700000
L26340	C+0300000-7400000	+0000000-0000000	+7122731-0323071	+0000000-0000000
L26400	C+1577630+0127050	+4226541+5723100	+5726411+3526560	+5723100+5726430
L26440	C+3526600+5723100	+5726441+3526620	+6426500+6626600	+3077760+5777600
L26500	C+0777661+0477701	+0777760+0477721	+0777740+6077670	+0077660+5177650
L26540	C+0077670+5700000	+0277670+5777641	+0000000-0000000	+0000000-0000000
L26600	C+0000000-0000000	+0000000-0000000	+0000000-0000000	+0000000-0000000
L26640	C+5052641-6627770	-0000000-0000101	+7400000-0000000	+0000000-0000030