A U T O N E T I C S

A DIVISION OF NORTH AMERICAN AVIATION, INC.

INDUSTRIAL PRODUCTS
3584 Wilshire Blvd., Los Angeles 5, Calif.

December 14, 1959

RECOMP TECHNICAL BULLETIN NO. 4

TITLE: DIVISION IN RECOMP

PURPOSE: To develop a more explicit definition of the operations occurring during execution of fixed point division command.

EFFECTIVE DATE: December 14, 1959

CONTENTS: See the attached pages.

REFERENCES:

INFORMATION TO: All Concerned

WRITTEN BY: Professor R. A. Harling
U.S.A.F. INSTITUTE OF TECHNOLOGY
Wright-Patterson Air Force Base
Dayton, Ohio

1c

Prof. Harling
24 Nov 59

DIVISION IN RECOMP

The rules usually given for unrounded, fixed point division are (1) a 78 bit dividend in the A and the R registers, divided by a 39 bit divisor in memory, yields a 39 bit quotient in A, with a remainder in R. (2) The binary scale of the quotient is the scale of the dividend minus the scale of the divisor. (3) The remainder, in R, is at the same binary scale as the divisor.

These rules are of course, correct, but confusion can still arise concerning the exact nature of the remainder after a division. The method of determining the remainder given in the following examples may, therefore, be of interest.

EXAMPLE (1). To divide 40_{10} at b 39 by 3_{10} at b2.

$$\frac{40_{10} \text{ at b } 39}{3_{10} \text{ at b } 2} = 13\ 1/3_{10} \text{ at b } 37 \text{ if this number will hold.}$$

$$= 53\ 1/3_{10} \text{ at b } 39 \text{ if this number will hold.}$$

(13 1/3 has been multiplied by 4 to compensate for the change of 2 in the binary scale.)

Now 53_{10} at b 39 will hold in A, and so this will be the content of the A register; it is equivalent at b 37 to 13.25_{10}.

The remainder, in R, will be 1 at b 2 (the "1" is the numerator of the 1/3 in 53 1/3 above).

c

Thus, the actual contents of the A and the R registers after this division will be:

A register	R register
. . . 0 1 1 0 1 $_\wedge$0 1	[+] 0 1 $_\wedge$0 . . . .
(A) = 13.75 at b 37	R = 1 at b 2

This may be checked by multiplying back as follows:

$$\frac{40_{10} \text{ at b } 39}{3_{10} \text{ at b } 2} = 13.25_{10} \text{ at b } 37 + \frac{1 \text{ at b } 2}{3 \text{ at b } 2}.$$

Multiply by 3_{10} at b 2:-

40_{10} at b 39 = 39.75_{10} at b 39 + 1 at b 2

1 at b 2 in the R register is the equivalent of .25 at b 0 in R, or of .25 at b 39 in A and in R together, so

40_{10} at b 39 = 39.75_{10} at b 39 + .25 at b 39

= 40_{10} at b 39, as is expected.

__EXAMPLE__ (2). To divide 41_{10} at b 38 by 6_{10} at b 3

$$\frac{41_{10} \text{ at b } 38}{6_{10} \text{ at b } 3} = (6\ 5/6)_{10} \text{ at b } 35 \text{ if this number will hold}$$

$\equiv (109\ 2/6)_{10}$ at b 39 if this number will hold

(6 5/6) has been multiplied by 16 to compensate for the shift in scale).

Now 109 at b 39 will hold in A, and so this will be the content of the A register; it is equivalent to $(6\ 13/16)_{10}$ at b 35.

The remainder, in R, will be 2 at b 3 (the numerator of the fraction in 109 2/6). Thus, the contents of A and of R will be:

1 1 0 ∧ 1 1 0 1	+ 0 1 0 ∧ 0 0 0
$(A) = (6\ 13/16)_{10}$ at b 35	$(R) = 2$ b 3.

Checking this:

$$\frac{41_{10} \text{ at b } 38}{6_{10} \text{ at b } 3} = (6\ 13/16)_{10} \text{ at b } 35 + \frac{2 \text{ at b}3}{6 \text{ at b}3} .$$

Multiply by 6_{10} at b 3:

41_{10} at b 38 = $(40\ 14/16)_{10}$ at b 38 + 2 at b 3

> 2 at b 3 in the R register is the equivalent of 2 at b 42 in the A and the R registers together, or of 2/16 at b 38, so

41_{10} at b 38 = $(40\ 14/16)_{10}$ at b 38 + $(2/16)_{10}$ at b 38

= 41_{10} at b 38, as is expected.

Division by 1 is particularly interesting, and is illustrated by the next three examples. Notice that the numbers used are now octal rather than decimal.

EXAMPLE (3). To divide 7.250_8 at b 39 by 1 at b 1.

$$\frac{7.250_8 \text{ at b } 39}{1 \text{ at b } 1} = 7.250_8 \text{ at b } 38 \text{ if that number will hold.}$$

$$\equiv 16.520_8 \text{ at b } 39 \text{ if that number will hold.}$$

Now 16_8 at b 39 will hold in A, and is the equivalent of 7_8 at b 38.

The remainder is $.520_8$ at b 1 in R.

	A					R						
Before		0	1	1	1	0	1	0	1	0	1	0
After	0	1	1	1	0	0	1	0	1	0	1	0

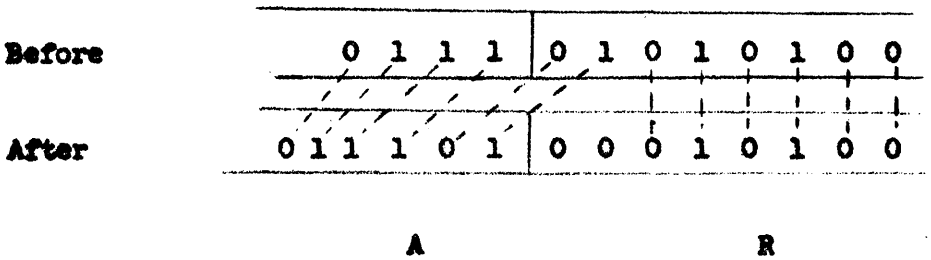
EXAMPLE (4). To divide 7.250_8 at b 39 by 1 at b 2

$$\frac{7.250_8 \text{ at b } 39}{1 \text{ at b } 2} = 7.250_8 \text{ at b } 37 \text{ if that number will hold.}$$

$$\equiv 35.240_8 \text{ at b } 39 \text{ if that number will hold.}$$

Now 35_8 at 39 will hold in A, and is the equivalent of 7.2_8 at b 37.

The remainder is $.240_8$ at b 2. Thus



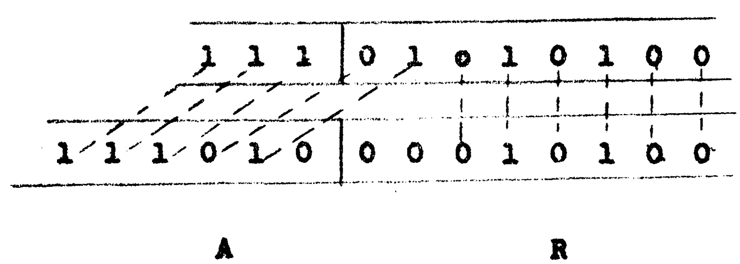
EXAMPLE (5). To divide 7.250_8 at b 39 by 1 at b 3

$$\frac{7.250 \text{ at b } 39}{1 \text{ at b } 3} = 7.250_8 \text{ at b } 36 \text{ if that number will hold.}$$

$$= 72.50_8 \text{ at b } 39 \text{ if that number will hold.}$$

Now 72_8 at b 39 will hold in A, and is the equivalent of 7.2_8 at b 36.

The remainder is $.50_8$ at b 3 in R. Thus:



Notice that a division by 1 at b n produces the same effect as a "Long Left Shift" as far as the content of A after the process is concerned; but it is not a true Long Left Shift, for the bits in R, other than the first n bits, are not shifted at all. The contents of A, and the first n bits of R, are Long Left Shifted n places, the remaining bits in R are left unchanged, and the vacant places at the left end of R are filled with zeros. As a result, a Long Left

Shift of 3 places, produced in this manner, does not give the same result, even in the A register, as three successive Long Left Shifts of one place each. This is illustrated below:

	A	R
Before	1 1 1	1 1 1 1
LLS 1	1 1 1 1	0 1 1 1
LLS 1	1 1 1 0	0 1 1 1
LLS 1	1 1 1 0 0	0 1 1 1

Three separate shifts of one place each, produced by a division by 1 at b 1.

	A	R
Before	1 1 1	1 1 1 1
LLS 3	1 1 1 1 1 1	0 0 0 1

Division by 1 at b 3 to produce a left shift of 3.