HONEYWELL EDP

APPLICATION BULLETIN

MANUFACTURING

WEIBULL DISTRIBUTION APPLICATIONS AT MICRO SWITCH

PURPOSE

To acquaint Honeywell customers and sales and service personnel with some of the practical applications of the Weibull distribution which are possible in a manufacturing environment.

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A SURVEY OF WEIBULL DISTRIBUTION APPLICATIONS

DEFINITION OF WEIBULL DISTRIBUTION

The Weibull distribution was first documented about 1950 as a tool for assessing the strength of cotton fibers and analyzing the fatigue life of steel. The flexible nature of this distribution soon resulted in its application to a wide variety of problems.

Statisticians are often confronted with the problem of finding the appropriate distribution to describe the pattern of variation of empirical data. Generally, the interpreter of experimental data attempts to relate the information to some known frequency function in order to obtain knowledge about certain characteristics of the population. The validity of the techniques employed depends upon the assumption of the distribution, and is often sensitive to departures from the assumed frequency curve.

By assuming the Weibull distribution when analyzing experimental data, an attempt is made to reduce the risk of making an incorrect assumption about the underlying distribution. This technique is a family of frequency functions which can range in shape - dependent upon the sample values - from an exponential distribution through a normal curve.

There are three parameters which determine a Weibull distribution.

ν = location parameter

β = shape parameter

η = scale parameter

The equational forms which describe the Weibull frequency curve indicate to some extent its flexibility and its empirical nature.

f (x) = probability density function

F(x) = cumulative density function

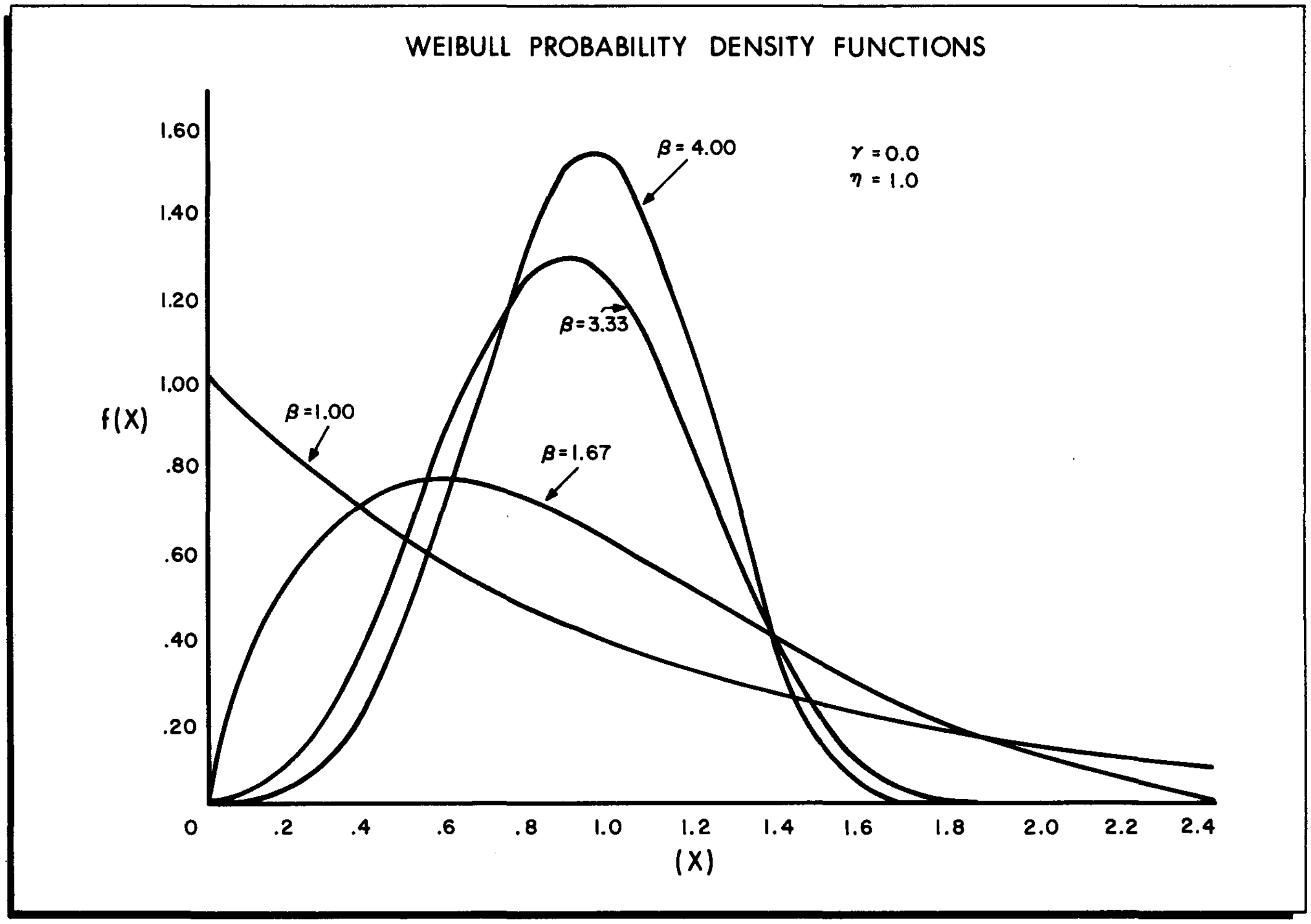
$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}$$

where $\eta = \alpha^{\frac{1}{\beta}}$ for $x \geq \gamma;\ \eta, \beta > 0$

$$F(x) = 1 - e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}$$

The Weibull function yields a discrete curve form for each value of the shape parameter (see Exhibit 1). For example, a shape parameter of 1 gives an exponential distribution, a value of 2 for the shape parameter yields a Rayleigh distribution, and a shape parameter of about 3.25 gives a good approximation for a normal curve.

Exhibit 1



Several methods for determining the values of the Weibull distribution parameters are available to the statistician. Three analytical techniques — the variance method, the maximum likelihood method, and the method of moments — are being used to solve specific problem types. All indicative information — such as the proportion of the population exceeding a specified value, the mean, the median, and the standard deviation — can be calculated once the three parameters are known.

A number of application areas in which the Weibull distribution may be used are being explored at Honeywell's MICRO SWITCH Division.

INVENTORY CONTROL APPLICATION

Objectives Of Inventory Control

Specific objectives within the above definition can be enumerated as follows:

1. An inventory savings should be realized due to the elimination of unusual orders from the forecasting model.
2. The total number of assembly work orders should be reduced since only truly unusual orders would be processed on a "make" basis.
3. Customer service should be improved because of coordination between the incoming order classification and the forecasting model.

Problem Definition

Creating a filter to eliminate large, extraneous customer orders from a demand distribution is a problem which has continually plagued computer-oriented forecasting and inventory control systems. At MICRO SWITCH, the problem involves analysis of sales data on 14,000 catalogue listings of precision, snap-action, mercury and manually operated switches. Approximately 100,000 customer sales orders are received which result in 250,000 line item orders per year. Deliveries are made from inventory on about 70 percent of the customer sales orders, 91 percent of the line item orders, and 56 percent of the dollar volume. The remainder of the orders and the resulting dollar volume are shipped on a make-to-order basis. Customers request delivery within one week on 88 percent of the line item orders while the remaining 12 percent are scheduled beyond one week from the date of receipt.

It is known that no single, predefined frequency function can describe adequately the nuances and variations in the demand for several thousand products. A product demand curve can generally be described by a Weibull distribution, which provides the greatest flexibility and delivers the best results yet possible.

All customer orders that are considered extraneous or extreme and do not fit the product demand distribution must be identified and separated. The Weibull distribution accurately identifies and separates these orders. Such orders should be processed on a "make" basis provided that an excess inventory condition does not exist. The chosen truncation point on the product demand distribution should minimize a cost equation composed of assembly work order processing costs and inventory carrying costs. Detail customer line item order sizes for the past year form the product data to be analyzed.

The detail line items and the summary items are described on a data tape file. Exhibit 2 illustrates the necessary data items that are included in the file.

Exhibit 2

Data File Structure

Detail Items	1. Product code 2. Customer number 3. Sales order number 4. Customer order classification 5. Date of customer request 6. Quantity 7. Dollar amount
Summary Item	1. Product code 2. Inventory type 3. Total quantity 4. Total dollar amount 5. Alpha constant 6. Three-track exponential smoothing averages 7. Three-track mean absolute deviations 8. Three-track sums of the errors

PRODUCT CLASSIFICATION

Internal systems classify finished products into four inventory types.

Type 1 (3300 products) — General sale items which are computer controlled for forecast, inventory control parameters, and lot-size order placement.

Type 2 and 3 (300 products) — New listings undergoing market development which are manually controlled for forecast and inventory control parameters.

Type 4 (10,400 products) — These items do not have forecast and inventory control parameters and are manufactured strictly for customer order sizes.

Incoming line item orders are divided into three categories.

MAKE orders presuppose delayed delivery and indicate that a specific work order must be processed for the customer demand.

RESERVATION orders provide for customer requested delayed delivery or delayed delivery due to low product availability. Such customer line item orders are allocated against in-process stock replenishment orders.

STOCK orders denote immediate shipment from inventory.

Theoretically, all customer orders for Type 4 products should be shipped on a "make" basis. Customer orders for Type 1, 2, and 3 products should be shipped on a stock or reservation basis unless they are of unusually large size or have special requirements.

Problem Solution

Weibull parameters are calculated for the product and a distribution truncation point is determined. As an example, a distribution truncation point of 99 percent indicates that only 1 percent of the customer orders are expected to be larger than the given value.

Exhibit 3

Weibull Distribution Product Example

Product code 01-00900
Original Sample Size = 56

Sample Values*

1	1	1	1	2	2	2
2	2	2	2	2	2	2
2	2	3	3	3	3	4
4	4	4	6	6	6	6
6	10	10	10	10	10	10
10	10	10	10	10	10	10
10	10	10	12	20	20	20
30	36	40	40	50	50	705

*Number of units requested per order.

Sample size is reduced by eliminating redundant values.

Reduced Sample Size = 13

Sample Values*

1	2	3	4	6	10	12
20	30	36	40	50	705	

Shape Parameter, β ,	=	.749
Scale Parameter, η ,	=	29.325
Mean, μ ,	=	34.967
Median, m ,	=	17.970
Standard deviation, σ,	=	46.399

Notice that in the following example, any of the probable truncation points achieve the desired result of truncating the distribution just prior to the final sample value. Therefore, the example serves to indicate the problems which can arise in inventory provision due to extraneous demands rather than as an explicit solution to finding the proper distribution truncation point.

	Probable Truncation Points	
10%	of values exceed	89.342
7%	of values exceed	108.292
4%	of values exceed	139.761
2%	of values exceed	181.350
1%	of values exceed	225.500
.5%	of values exceed	271.945
.1%	of values exceed	387.575

The set of demands screened by the Weibull distribution are input to an exponential smoothing forecasting model and an inventory control simulator. Simulations are conducted using various Weibull distribution truncation points and the results are compared through a cost equation composed of order processing costs and inventory carrying costs.

*Order Size

Exhibit 4

Cost Equation Product Example

n = number of demands eliminated

M = make-order processing cost

N = number of stock replenishment orders

S = stock replenishment order processing costs

i = inventory carrying cost

C = standard cost per unit

I = average inventory

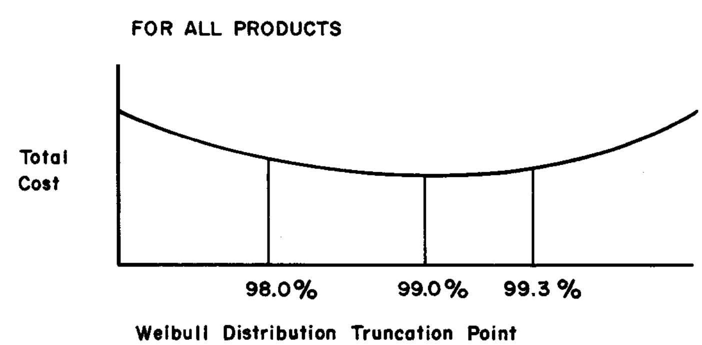
TC = nM + NS + iCI

Product code 01-00900
M = S = $4.75
i = .25
C = $ .52

	nM	+	NS	+	iCI	=	Total Cost
Total Sample	(0) ($4.75)	+	(3) ($4.75)	+	(.25) ($.52) (328)	=	$56.89
Truncated Sample	(1) ($4.75)	+	(4) ($4.75)	+	(.25) ($.52) (117)	=	$40.96

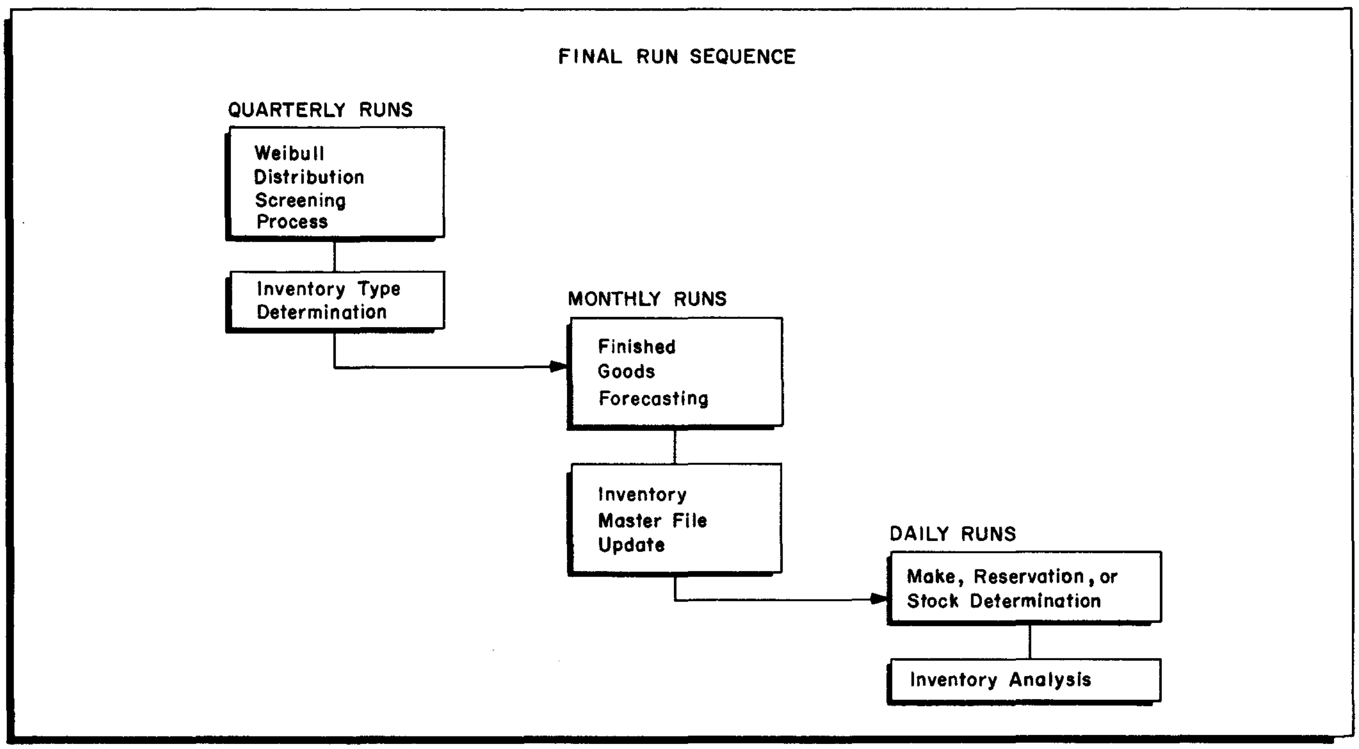
A relatively insensitive cost region is found when total cost is compared with percentage points on the Weibull distribution. The stable cost region seems to be between 98.0 and 99.3 percent on the Weibull distribution with a minimum cost very near 99.0 percent.

Exhibit 5



The Weibull distribution fosters the integration of many diverse elements — inventory type determination, finished goods forecasting, incoming order classification, and inventory analysis — within the inventory control system (see Exhibit 6). Once a quarter, this technique is used in conjunction with the inventory type determination, and the results become input to finished goods forecasting. When forecasts are developed, a record containing the inventory type, the forecast usage, the economic order quantity, the safety stock, and the Weibull distribution truncation point is issued to the Inventory Master File. This file is utilized in daily operations to classify incoming orders as make, reservation, or stock according to the Weibull truncation point and existing inventory availability conditions. Therefore, the same criteria are being used to eliminate customer demands from the monthly forecast and to separate unusual customer orders for processing on a make basis.

Exhibit 6



Summary

Initial estimates indicate that all of the objectives stated earlier can be achieved by utilizing the Weibull distribution, the cost equation, and the inventory simulator:

1. Finished goods inventory will be reduced by 6 percent.
2. The number of assembly work orders will be reduced by 8 percent.
3. Type 1 dollar volume shipments from inventory will increase by 4 percent.

RELIABILITY APPLICATION

The Weibull distribution has considerable empirical justification as a failure pattern in product life analysis for many mechanical and electrical devices. The merit of this frequency function is found in the fact that it is the simplest mathematical expression having the required three-parameter form and that it often fits the observations better than other known distributions.

Indicative product life criteria such as the reliability, reliable life, failure rate, and hazard rate are easily calculated from the estimates of the Weibull parameters. These characteristics involve generally accepted definitions of criteria which help to describe the distribution of the test data.

The failure pattern of the life data can be determined and plotted by the computer to further facilitate understanding of the techniques, if such information is desired. The curves which are scaled and plotted are the probability density function and the cumulative density function. (See Exhibits 7 and 8: the data in this example is a generated exponential distribution.)

Test results illustrate that the advantages of the Weibull Distribution include:

1. More certain estimation of the parent distribution than is possible with a predetermined or less empirical model;
2. More reasonable methods for calculating performance criteria such as the failure rate; and
3. An implicit solution to the problem of quoting failure patterns to customers interested in system reliability for multiple component circuits.

The last two advantages concern the fact that for the Weibull distribution the assumption of a constant hazard rate or instantaneous failure rate is valid only for the special case of an exponential distribution (shape parameter = 1.0). A definition explaining that the failure rate is correct only for the specific, quoted interval is included in all customer quotations. The hazard rate or instantaneous failure rate is defined as being valid only around the given point. These explanations indicate that the Weibull distribution has the ability to describe hazard rates which increase, remain constant, or decrease with time according to the parameter values.

Suggestions are often made to customers indicating that circuit performance can be evaluated, without confusion, from reliability computations - rather than by working backward from failure rate figures. Variable failure rates and hazard rates as derived by the Weibull distribution can sometimes be confusing to those heavily schooled in exponential reliability analysis. Therefore, the direct evaluation of circuit reliability at a specific point offers the advantage of correct results - irrespective of the underlying distribution assumptions made by the component supplier.

Exhibit 7

PERSON 141 PROBLEM 1 WEIBULL

C = 1.01368, B = 14115.27697 9/3/65 JAGDISH PATEL

N = NUMBER OF KNOWN SAMPLE VALUES = 100 NTOTAL = TOTAL NUMBER OF SAMPLE VALUES = 100

X (SAMPLE VALUES)

405.69000	463.59000	479.99000	559.25000	642.12000	756.09000	799.53000
888.41000	966.41000	1221.34000	1307.05000	1423.93000	1864.78000	1984.10000
2060.20000	2353.28000	2416.55000	2675.02000	2718.06000	2812.43000	2985.81000
3118.94000	3158.18000	3766.66000	3906.39000	3915.08000	4021.13000	4185.72000
4350.68000	4456.13000	4499.58000	4547.68000	4573.23000	4825.45000	5022.12000
5214.46000	5385.09000	5449.38000	6211.92000	6371.37000	6447.87000	6463.27000
6644.49000	6644.49000	7228.63000	7232.06000	7647.96000	7675.47000	7774.05000
7779.65000	8017.26000	8626.20000	9037.32000	9255.92000	9620.18000	9897.20000
11489.57000	11531.12000	11681.38000	11911.57000	12676.16000	12848.44000	12854.81000
14570.15000	15194.13000	15215.63000	15420.91000	17091.85000	17512.82000	17908.18000
17936.81000	18420.25000	19729.41000	21386.81000	22603.68000	23649.07000	23685.73000
23748.47000	23909.74000	24050.61000	24685.68000	27526.14000	29548.87000	29576.35000
29876.54000	30242.28000	30278.79000	31821.61000	33529.50000	33721.11000	41712.85000
41865.66000	46359.81000	50358.67000	52742.10000	59150.94000	59275.84000	66282.90000
67521.53000	70196.24000					

SHAPE PARAMETER C = 1.01368
SCALE PARAMETER B = 14115.27761

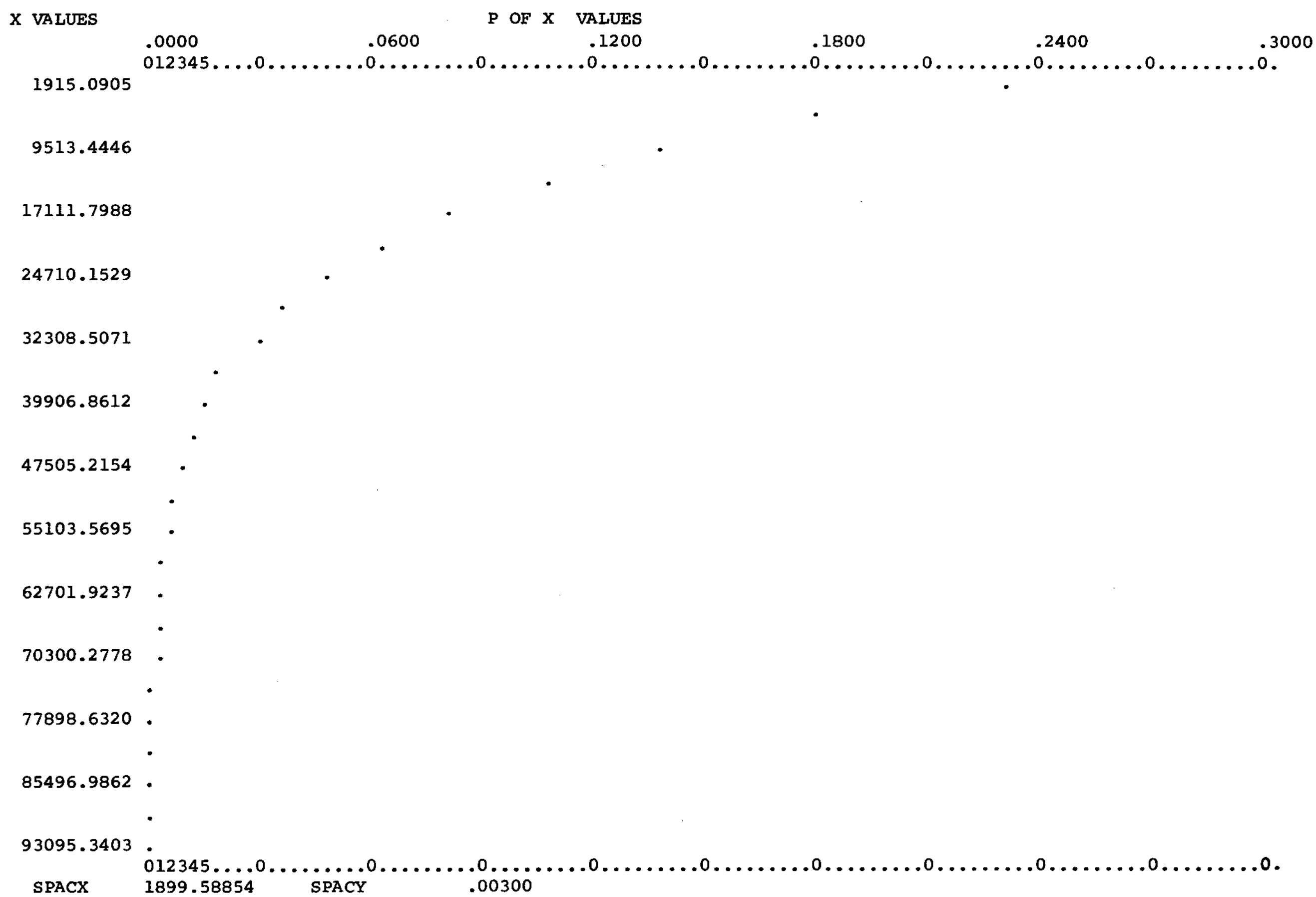
MEAN = 14035.77321
MEDIAN = 9832.51271
STANDARD DEVIATION = 13847.04137

FRACTION OF THE TOTAL THAT EXCEEDS 70000.00000 = .00629
FRACTION OF THE TOTAL THAT EXCEEDS 753.63000 = .95000
FRACTION OF THE TOTAL THAT EXCEEDS 100.00000 = .99340
FRACTION OF THE TOTAL THAT EXCEEDS 90000.00000 = .00144

.00100 IS THE FRACTION OF THE TOTAL THAT EXCEEDS 94994.92885
.99900 IS THE FRACTION OF THE TOTAL THAT EXCEEDS 15.50192
.10000 IS THE FRACTION OF THE TOTAL THAT EXCEEDS 32137.89320
.90000 IS THE FRACTION OF THE TOTAL THAT EXCEEDS 1533.04611

X	F(X)	MID X	P(X)
3814.67900	.23315	1915.09046	.23215
7613.85607	.41425	5714.26754	.18111
11413.03315	.55345	9513.44461	.13920
15212.21023	.66000	13312.62169	.10655
19011.38731	.74137	17111.79877	.08137
22810.56438	.80342	20910.97584	.06204
26609.74146	.85067	24710.15292	.04725
30408.91854	.88663	28509.33000	.03596
34208.09561	.91397	32308.50708	.02734
38007.27269	.93474	36107.68415	.02077
41806.44977	.95052	39906.86123	.01578
45605.62685	.96249	43706.03831	.01198
49404.80392	.97158	47505.21538	.00909
53203.98100	.97847	51304.39246	.00689
57003.15808	.98369	55103.56954	.00522
60802.33515	.98765	58902.74662	.00396
64601.51223	.99066	62701.92369	.00300
68400.68931	.99293	66501.10077	.00227
72199.86638	.99465	70300.27785	.00172
75999.04346	.99595	74099.45492	.00130
79798.22054	.99694	77898.63200	.00099
83597.39761	.99768	81697.80908	.00075
87396.57469	.99825	85496.98615	.00056
91195.75177	.99868	89296.16323	.00043
94994.92884	.99900	93095.34030	.00032

Exhibit 8



The following form describes the use of Weibull calculated failure rates and hazard rates and illustrates the suggestions made to customers concerning their use.

Exhibit 9

Customer Reliability Information
Weibull Distribution

I. Definitions

A. Reliability of a device is defined as the probability of its performing a specified function under given conditions, without failure, for a predetermined length of time.

B. Failure rate is defined as the probability of an item failing in the interval (t, t + h), given that it has survived up to time t. The usual practice is to divide this probability by the length of the interval in order to get the failure rate per unit of time.

C. Hazard rate is defined as the limiting value of the failure rate as the length, h, of the interval approaches zero. Hazard rate is also known as instantaneous failure rate or failure rate around a specified point.

II. Test Conditions

Catalog listing :

Sample size :

Current :

Voltage :

Prepared by :

III. Assumptions

Weibull distribution with location parameter equal to zero.

IV. Estimation

Shape parameter, β , =

Scale parameter , η , =

V. Results

A. 1. Reliability for ________ operations is ________.
2. Reliability for ________ operations is ________.
3. Reliability for ________ operations is ________.

B. 1. Failure rate for the interval ________ to ________ operations is ________.
2. Failure rate for the interval ________ to ________ operations is ________.
3. Failure rate for the interval ________ to ________ operations is ________.

Generally, this statement is true for the given interval and NOT FOR ANY OTHER INTERVAL, EVEN THOUGH THE OTHER INTERVAL MAY BE OF THE SAME LENGTH AS THE ONE GIVEN.

C. 1. Hazard rate at ________ operations is ________.
2. Hazard rate at ________ operations is ________.
3. Hazard rate at ________ operations is ________.

Generally, this statement is true for the given point and NOT FOR ANY OTHER POINT.

Reliability, failure rate, and hazard rate should not be used for any purpose other than that for which each is defined, i.e., hazard rate or failure rate should not be used to find reliability.

Requested By: ____________________ Approved By: ____________________

Sales Project: ____________________

QUALITY CONTROL SAMPLING APPLICATION

Sample sizes and lot acceptance numbers derived through the use of the Weibull distribution in combination with the Military Standard Tables often lead to more certain conclusions regarding the population from which the sample was taken. The information generated by a Weibull analysis is useful in determining:

1. The probability of acceptance or rejection for a lot;
2. The level of customer protection and the level of producer protection afforded by a specific sampling plan; and
3. The cost of quality inherent in a specified sampling plan having a known probability of acceptance.

Sampling Procedure

A random sample is selected. The sample items are life-tested over a preassigned time period. The number of items failing during the time period is observed and the lot is accepted – provided that the number of items failing is less than or equal to a specified acceptance number.

For this assumed sampling procedure and for derivations based upon the "mean life criterion", the probability of an item failing is solely a function of mean life provided that the testing time period is given, and that the Weibull shape parameter, β, and the location parameter, γ, are known. Further derivations indicate that the sample sizes and the acceptance numbers for any specified sampling plan are dependent only upon the test truncation time and the mean item life.

As an example, consider the manufacturer who wishes to know the acceptance number which must be applied to ensure that a large proportion of the lots will be passed – so long as the known mean item life is maintained.

Exhibit 10

Known Information

Mean item life, μ	= 52,000 operations
Location parameter, γ	= 0.000
Shape parameter, β	= 0.50
Truncation test period, t	= 1000 operations
Sample size, n	= 150 items

Assume

Probability of acceptance, P(A), of 95 percent is desired for lots at the accepted quality level of mean item life, u, equal to 52,000 operations.

Exhibit 10 (cont)

Results

Utilizing the probability of failure at the truncation point, the desired probability of acceptance, and the sample size, the cumulative binominal distribution can be used to derive the acceptance number, c = 35. Therefore, this acceptance criterion indicates that a lot is passed as long as the number of failures encountered before the truncation test period, t, is less than or equal to 35.

Other questions such as the customer protection afforded by the above sampling plan can be answered. A measure of customer protection is the mean life value at which lots will likely be rejected.

Exhibit 11

Assume

Probability of acceptance, P(A), = 10 percent.

Results

Utilizing the sampling plan and the Standard Tables, mean life = 17,500 operations.

The proper interpretation of the above result is that, under the given sampling plan, on the average 90 percent of the lots passed to the customer will have a mean life of at least 17,500 operations. If the existing plan does not represent adequate customer protection, a plan with a larger sample size must be designed.

The relative merit of two sampling plans with respect to the level of customer protection afforded can easily be compared.

Exhibit 12

New Sampling Plan

Known

Truncation test period, t,	= 300 operations
Sample size, n,	= 500 items

Exhibit 12 (cont)

Assume

The desired probability of acceptance, P(A), = 95 percent.

Results

For the new sampling plan the acceptance number, c, = 62.

In order to compare the two alternative procedures, the level of customer protection afforded by the new sampling plan must be determined.

Assume

Probability of acceptance, P(A), = 10 percent.

Results

Utilizing the new sampling plan and the Standard Tables, mean life = 26, 100 operations.

Conclusion

The new sampling plan with sample size of 500 items and test time of 300 operations gives greater consumer protection since on the average 90 percent of the lots passed on to the consumer will have a mean life of at least 26, 100 operations.

Notice that the comparison of alternative procedures with respect to desirable characteristics could add the cost of quality as an element for study. Simulation techniques could be incorporated to assist management in determining the desirable level of quality based upon cost considerations.

PRODUCTION CONTROL AND PRICING APPLICATION

Certain products exhibit pricing patterns which are dependent upon measurable characteristics such as (for snap-action switches) operating point, release point, and contact resistance. Closer tolerances on these characteristics require greater costs which must be reflected in higher prices to achieve a reasonable profit. A knowledge of the percentage yield for each tolerance interval and of the accumulated costs gives an indication of the necessary prices.

Exhibit 13

Number of sample values	= 80.0
Location parameter, γ ,	= 0.0
Shape parameter, β ,	= 9.88
Scale parameter, η ,	= 40.77
Mean	= 38.77
Median	= 39.29
Standard deviation	= 4.71

Design Tolerance Influences Costs

Operating point	= 42 ± 12

Sample Information

Nominal value = Mean	= 38.77

31.7%	of the	values are	in the	interval	42 ± 2
57.0%	"	"	"	"	42 ± 4
74.0%	"	"	"	"	42 ± 6
84.6%	"	"	"	"	42 ± 8
91.2%	"	"	"	"	42 ± 10
95.2%	"	"	"	"	42 ± 12

Since the assembly machinery, which is set at 42, cannot be adjusted beyond the design tolerance, it turns out parts with operating point of 42 ± 12. A production lot must be run and switches inspected until the required number having the customer-requested tolerance is accepted. The above Weibull analysis pinpoints the inspection loss factor incorporated in the price at various tolerance levels. For example, a customer requiring an operating point within 42 ± 2 increases the cost to three times that of a product with the generally accepted operating point, 42 ± 12, i.e., 31.7 percent yield versus 95.2 percent.

The major advantage of the Weibull distribution involves the reproduction of many types of skewed frequency functions which give better estimates of the parent distribution. Such estimates yield more certain loss factors and more reasonable prices for customers than were previously possible.

MANUFACTURING ENGINEERING APPLICATION

The Weibull technique is used to conduct running evaluations of part dimensions and to note any changes in the characteristics of these dimensions. The major advantages of the Weibull in this application are that it provides more realistic estimates of the parent distribution than had previously been possible and that it pinpoints any change in parameters as new sample data becomes available. Production control methods or inspection tolerances can then be adjusted according to the results of the evaluations.

A specific example of a problem solved by Weibull analysis involved a precision snap-action switch. Manufacturing engineers were having difficulty holding the operating force within the design engineering prescribed tolerance. Knowledge of the product indicated that tightening the tolerances on two critical dimensions would solve the problem. However, knowledge of the tooling and the setup involved in the manufacturing process pointed out that the tighter tolerances were not possible under existing conditions.

A study was initiated to determine if new tooling was needed or if setup procedures were inadequate. The ability of a tool to repeatedly turn out samples of parts within the required tolerances on the critical dimensions was checked with the Weibull distribution using special measuring devices and supersensitive gauges for machine setup. The Weibull information is illustrated in the sample calculations given below.

Exhibit 14

First Critical Dimension	=	.057 ± .002
Weibull Analysis		
Location Parameter, γ,	=	0.00000
Shape Parameter, β,	=	220.40089
Scale Parameter, η,	=	.05760
Mean	=	.05745
Median	=	.05750
Percentage of expected values between .055 and .059 is .997		
Second Critical Dimension	=	.094 ± .002
Weibull Analysis		
Location Parameter, γ,	=	0.00000
Shape Parameter, β,	=	196.74486
Scale Parameter, η,	=	.09408
Mean	=	.09381
Median	=	.09390
Percentage of expected values between .092 and .096 is .988		

Successive samples after changing setups indicated that the required tolerances could be maintained by adjusting the tool to nominal with finely calibrated tool gauges. In other words, the setup man had inadvertently been creating the tolerance problem since the tool gauges were not sensitive enough to result in a true nominal height. A setting of .057 on the old gauges could result in actual nominal heights from .055 to .060: therefore, the job change-over procedures were altered; the critical dimensions were maintained within the required tolerances; and, the operating force problem was eliminated.

SUMMARY

The preceding examples merely serve to illustrate the flexibility of the Weibull distribution as a technique to aid in scientific problem solving. The surface of possible uses has merely been scratched; further research will undoubtedly expose more profitable applications along with more refined methods for drawing conclusions based upon this mathematical tool. As its development continues, this technique can prove to be one of the most worthwhile tools recently added to the repertoire of the business scientist.

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