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Solution of Homogeneous and Non-Homogeneous Simultaneous Linear Equations

5.0.020

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SOLUTION OF HOMOGENEOUS AND NON-HOMOGENEOUS SIMULTANEOUS LINEAR EQUATIONS

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Program Abstract

Title:

"Solution of Homogeneous and Non-Homogeneous Simultaneous Linear Equations"

Subject Classification:

5.0

Author; Organization:

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Purpose/Description:

The program solves homogeneous and non-homogeneous systems of linear equations; at the same time, it provides a measure of the accuracy of the computed solution.

If the system has a unique solution, this solution is typed. If the system has an infinite number of solutions, a general solution can be obtained.

If the system is inconsistent, it has no solution and this is the answer typed.

Mathematical Method:

The system's augmented matrix formed by the system's coefficient matrix and the constant column matrix, is reduced by means of elementary linear operations to an equivalent matrix, in which the system's coefficient matrix appears as a diagonal matrix with only 1 and 0 in the main diagonal. If only 1's appear in the main diagonal, the system has a unique solution; if 1's and 0's appear in the main diagonal, the system may have an infinite number of solutions or no solution at all, depending on the values of the equivalent constant column matrix.

Restrictions, Range:

The program is restricted to "Non-ill-conditioned" systems, and to a maximum of 17 equations with 17 unknowns.

Storage Requirements:

19938 positions

Equipment Specifications:

Memory 20K X 40K ___ 60K ___ K ___ Automatic Divide: Yes X No___

Indirect Addressing: Yes X No___Other Special Features Required: Card system

Additional Remarks:

(a) Program is written in FORTRAN with FORMAT.

(b) The program has run successfully about 20 times; these include solution of a maximum system (17 equations with 17 unknowns).

(c) The program was used successfully in conjunction with the calculation of the gain, input impedance, and output impedance of a transistorized amplifier designed in our Laboratories.

(d) If source deck is compiled to obtain Program deck, the FORTRAN with FORMAT system used must contain the ABSOLUTE VALUE SUBROUTINE.

(e) If Program deck is used directly, it already contains the ABSOLUTE VALUE SUBROUTINE.

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Deck Labeling

Two decks are being submitted.

Deck #1 includes a Hash total card, three comment cards, and all the FORTRAN statements comprising the program. When the Hash total card is removed from this deck, the deck left is known as the SOURCE deck. This deck can be compiled by the FORTRAN with FORMAT processor in order to obtain what from now on will be called the PROGRAM deck.

Deck #2 includes a Hash total card, and 582 cards containing the machine language instructions comprising the program.

When the Hash total card is removed from this deck, the deck left is known as the PROGRAM deck.

Program Description

The aim of this program is to provide an approximate solution to a homogeneous, or non-homogeneous, system of linear equations. The program is restricted to a maximum system of 17 equations with 17 unknowns, and to "non-ill-conditioned"[1] systems. The question of extending the program's range to systems which contain a larger number of equations, or to "ill-conditioned" systems, is out of direct consideration. This restriction is imposed here because the program is set up for single precision arithmetic in a computer which carries only eight significant digits on any arithmetic operation, and has an available 20K memory. With a larger core storage, the program could be set up for multiple precision arithmetic, and this is at the present time the most common way to extend the accuracy and the size of the system that can be handled.

The program provides core storage for a rectangular matrix of 17 x 18 maximum size, but it will adjust itself for systems whose augmented matrix is smaller than this size.

For any N x N system ($N \leq 17$), the program can obtain solutions for as many as 18-N constant vectors which carry the name "VECTOR N+1, VECTOR N+2,....., VECTOR 18."

When a solution has been determined, the sentence SOLUTION FOR VECTOR L is typed; under this heading the appropriate solution is typed.

If the equations are independent of each other, there is a unique solution for the system, and the solution values are typed one per line. These values correspond to the unknowns $X_1, X_2, \ldots, X_n$.

If the equations are not all independent, and together with the constant vector they form a consistent system, there is an infinite number of solutions for the system, and a message is typed to indicate so. This message also provides two choices; to obtain a general solution, or to continue with the next problem.

If the equations are not all independent, and together with the constant vector they form an inconsistent system, there is no solution for the system, and a message is typed to indicate inconsistency.

[1] See "Determination of the Zero Value" section, for explanation of meaning.

The program is based on the theory described under Method of Calculation, and consists of reducing the system's augmented matrix to the row equivalent matrix described in Theorem 1. Analysis of the elements forming this equivalent matrix determines whether the system is consistent or not.

Two problems arise at this time. First, when checking for inconsistency or linear dependence, elements of the equivalent matrix in Theorem 1 must be compared against a computed zero value. Actual computations showed that a computed zero very seldom reaches the value zero; this is due to the round-off and truncation errors introduced by arithmetic computations. Second, when a solution has been reached, how accurately do the calculated unknowns satisfy the system under solution.

In order to solve these problems, a tolerable zero value is introduced in the program, and a rule to determine this value is explained under Determination of the Zero Value. This tolerable zero value worked out for all examples analyzed by the program, and it seems to be a good approximation; however, the author does not imply that this particular approximation is absolutely correct.

This zero value is also a measure of the accuracy of the solution obtained. In other words, once the unknowns are typed out, an evaluation of the left side of an equation in the system will show that it differs from its equality constant by this zero value, at most.

If the calculated unknowns had not satisfy the equations within the accuracy specified by the zero value, they would not have been typed. Instead, a message suggesting to increase the value of zero would have been typed.

This accuracy self-adjustment provided for in the program and the program's ability to determine whether the system has a unique solution, an infinite number of solutions, or no solutions at all, are features not provided for by any of the existing programs that the author has knowledge of.

Another extra feature is the program's ability to generate the general solution to a system containing an infinite number of solutions. Such is the case in a homogeneous system of two equations with three unknowns, for example.

Once the object deck has been obtained and loaded, any number of systems can be solved one after the other without reloading the Program Deck.

Additional information about the characteristics of the program is contained in the sections to follow.

Method of Computation

The computations carried out by this program are based on the theorems appearing below. These theorems are fully explained and proved by S. Perlis[1], or F. B. Hildebrand[2], and any one interested in a deeper mathematical background of the material presented here should refer to their books. The theorems are listed here for a better understanding of the program.

Theorem 1 - Every n x m rectangular matrix C can be reduced to a matrix D with the following properties:

(1) For some $r \geq 0$, all elements d_{ij} $\binom{j = 1, \ldots\ldots, m}{i = r+1, r+2, \ldots\ldots, n}$ are equal to zero.

(2) The first non-zero element appearing in the K^{th} row $(1 \leq K \leq r)$ is equal to 1; or $d_{kj} = 1$ for some $j = L$ $(1 \leq L \leq m)$, and $d_{kj} = 0$ $(j < L, L \neq 1)$. The column L in which this d_{kj} occurs being numbered CL.

(3) All elements on column CL above are equal to zero, except the unity element in row K.

(4) There exists exactly r columns of the type described in 3, or $1 \leq L \leq r$, and $C_1 < C_2 < \ldots\ldots < C_r$.

D is said to be "row equivalent to C." An example of a matrix with properties 1-4 above is:

[1] Sam Perlis, Theory of Matrices, (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1958), pp. 36-48.

[2] F. B. Hildebrand, Methods of Applied Mathematics, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961), pp. 18-23.

$$D = \begin{bmatrix} 1 & 5 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \approx \begin{bmatrix} 2 & 10 & 0 & 0 & 0 & 10 \\ 3 & 15 & 1 & 2 & 0 & 19 \\ 4 & 20 & 3 & 6 & 1 & 35 \\ 5 & 25 & 4 & 8 & 3 & 50 \\ 6 & 30 & 5 & 10 & 4 & 62 \end{bmatrix} = C$$

for which $r = 3$, $1 \leq L \leq 3$, and C_1, C_2, and C_3 correspond to columns number one, three, and five, respectively.

The steps followed in the reduction of C to D are called "elementary operations."

Theorem 2 - If A and A_1 are the matrices resulting from deleting the extreme righthand column of the matrices C and D of Theorem 1, respectively, then A_1 is row equivalent to A and A_1 has the properties 1-4.

By definition, the "row rank" of a matrix C is the maximum number of linearly independent rows in C. With this definition, the proof of the following theorem is obvious.

Theorem 3 - The row rank of a matrix C is the number "r" of non-zero rows appearing in its equivalent matrix D of Theorem 1.

Now, if we consider a system of n linear equations in unknowns $X_1, X_2, \ldots\ldots, X_n$ of the form

$$\begin{matrix} a_{11} X_1 & + a_{12} X_2 & + \cdots\cdots + & a_{1n} X_n & = k_1 \\ a_{21} X_1 & + a_{22} X_2 & + \cdots\cdots + & a_{2n} X_n & = k_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1} X_1 & a_{n2} X_2 & + \cdots\cdots + & a_{nn} X_n & = k_n \end{matrix}$$

and let

$A = (a_{ij})$ (n x n) matrix

$X = \text{col} (X_1, X_2, \cdots\cdots, X_n)$ (n x 1) matrix

$K = \text{col} (k_1, k_2, \cdots\cdots, k_n)$ (n x 1) matrix the system becomes

$$AX = K$$

which is the common matrix notation of a system of linear equations, and in which A is called the "coefficient matrix" of the system.

Going one step further, we let

$$C = (A, K) \qquad (n \times (n + 1)) \text{ matrix}$$

be the "augmented matrix" of the system. Then, Theorem 1 indicates that C can be reduced to an equivalent matrix

$$D = (A_1, K_1)$$

Using Theorem 2, we conclude that $A \approx A_1$, which indicates that $AX = K$ is equivalent to $A_1X = K_1$.

Let us assume now that once D is found, by Theorem 3 we determine that A, the coefficient matrix is of rank L, and C, the augmented matrix, is of rank r. Then,

<u>Theorem 4</u> - The system $AX = K$ has a solution if and only if $L = r$.

Two possibilities exist:

(1) $L = r = n$. In this case, D would be of the form

$$D = \left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & r_1 \\ 0 & 1 & \cdots & 0 & r_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & r_n \end{array}\right], \quad A_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \quad K_1 = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

and

$$A_1X = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{matrix} X_1 + 0.X_2 + \cdots + 0.X_n = r_1 \\ 0.X_2 + X_2 + \cdots + 0.X_n = r_2 \\ \vdots \\ 0.X_1 + 0.X_2 + \cdots + X_n = r_n \end{matrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = K_1$$

from which we obtain $X_1 = r_1, \cdots\cdots, X_n = r_n$. The system is consistent and has a unique solution.

(2) $L = r < n$. An example of such a system is the following:

$$\begin{aligned} X_1 + 2X_2 - X_3 - 2X_4 &= -1 \\ 2X_1 + X_2 + X_3 - X_4 &= 4 \\ X_1 - X_2 + 2X_3 + X_4 &= 5 \\ X_1 + 3X_2 - 2X_3 - 3X_4 &= -3 \end{aligned}$$

for which

$$C = \begin{bmatrix} 1 & 2 & -1 & -2 & -1 \\ 2 & 1 & 1 & -1 & 4 \\ 1 & -1 & 2 & 1 & 5 \\ 1 & 3 & -2 & -3 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = D$$

and

$$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 2 & 1 & 1 & -1 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & -2 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A_1$$

By Theorem 3, $r = 2$ and $L = 2 \therefore L = r < n$

$$A_1X = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{array}{l} X_1 + 0.X_2 + X_3 + 0.X_4 = 3 \\ 0.X_1 + X_2 - X_3 - X_4 = -2 \\ 0.X_1 + 0.X_2 \quad 0.X_3 + 0.X_4 = 0 \\ 0.X_1 + 0.X_2 + 0.X_4 + 0.X_4 = 0 \end{array}$$

and

$$X_1 = -X_3 + 3$$
$$X_2 = +X_3 + X_4 - 2 \qquad (1)$$

We have found X_1 and X_2 in terms of the remaining unknowns which can be considered parameters.

The system is consistent and it is said to have a two-fold infinity of solutions; all solutions X_1, X_2, X_3, X_4 are formed by assigning arbitrary values to X_2 and X_4, and determining X_1 and X_2 from (1), known as the general solution.

On the other hand, if $L < r \leq n$, the system is inconsistent; that is, it has no solution. An example of such a system is the following:

$$- X_1 + X_2 + X_3 = 1$$
$$-5X_1 + 5X_2 + X_3 = 1$$
$$X_1 - X_2 + X_3 = 2$$

for which

$$C = \begin{bmatrix} -1 & 1 & 1 & 1 \\ -5 & 5 & 1 & 1 \\ 1 & -1 & 1 & 2 \end{bmatrix} \approx \begin{bmatrix} 1 & -1 & 0 & 1/6 \\ 0 & 0 & 1 & 11/6 \\ 0 & 0 & 0 & -2/3 \end{bmatrix} = D$$

and

$$A = \begin{bmatrix} -1 & 1 & 1 \\ -5 & 5 & 1 \\ 1 & -1 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = A_1$$

By Theorem 3, $r = 3$, $L = 2 \therefore L < r$

$$A_1X = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{array}{l} X_1 - X_2 + 0.X_3 = 1/6 \\ 0.X_1 + 0.X_2 + X_3 = 11/6 \\ 0.X_1 + 0.X_2 + 0.X_3 = -2/3 \end{array} \qquad (2)$$

But since the last equation in the set (2) cannot be satisfied by any (X_1, X_2, X_3), we conclude that the system is inconsistent.

Note that if the n x n system is homogeneous, it is consistent, and its solution depends on the value of L, the rank of the coefficient matrix. If $L = n$, the system has a unique solution; namely, the trivial solution. If $L < n$, the system has an infinite number of solutions.

The method of reducing the augmented matrix C to its equivalent matrix D is carried out by operations on C registered in the same storage locations where the original matrix C is placed at the beginning of the program.

The total number of divisions and multiplications carried out for the reduction of a system containing a unique solution is given by the formula

$$\frac{1}{3}\left[n^3 + (2L + 1) n^2 + 2n \right]$$

where

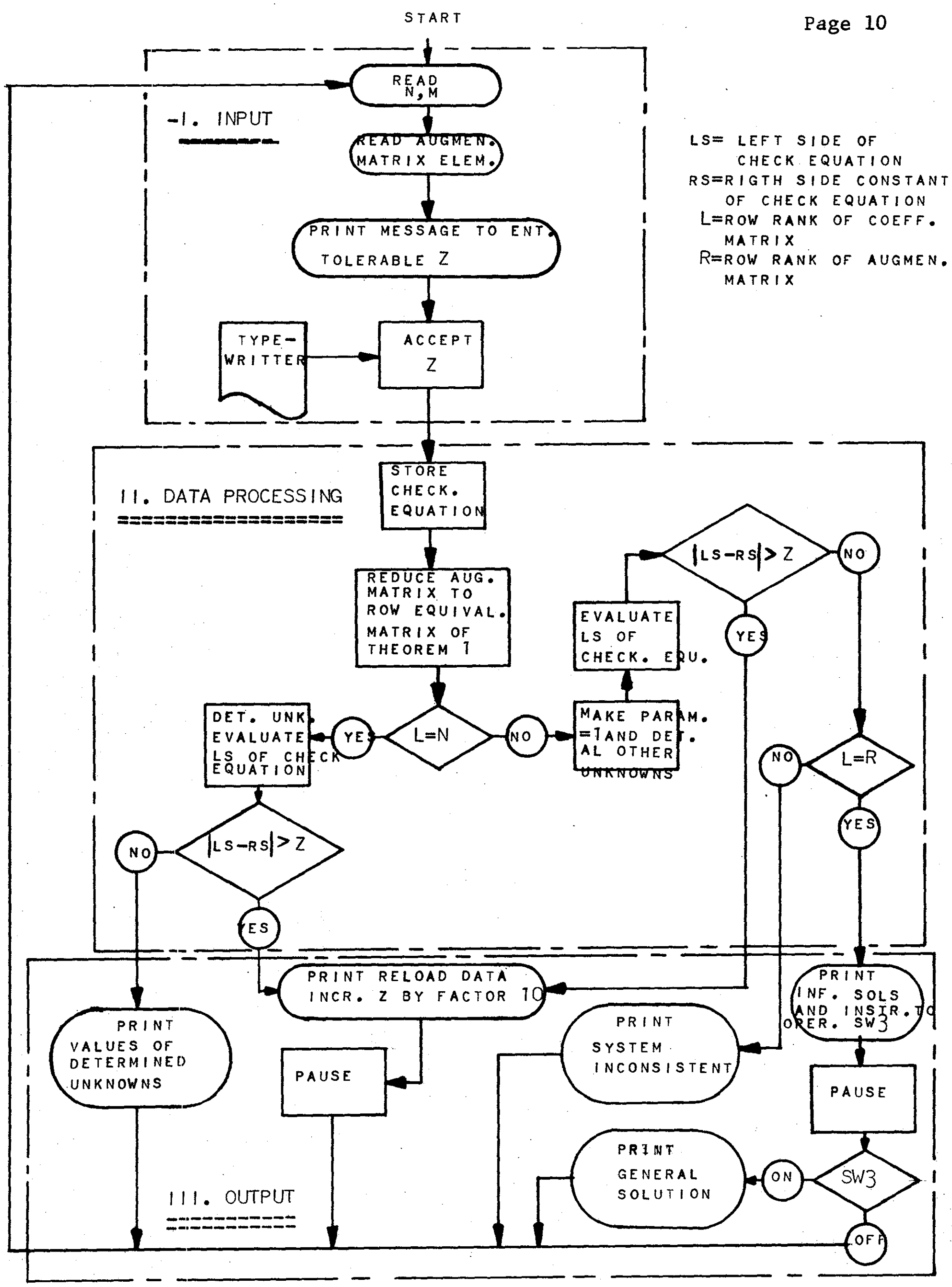
n = number of equations

L = total number of constant vectors

When the reduction is completed, the matrix D is in storage, and an analysis of its elements will determine the correct solution. Obviously, the problem of computed zero values mentioned in the Program Description arises in this part of the problem. This problem and its solution is the subject of the next section of this program write-up.

The time for solution of any system is approximated by

$$T \approx \frac{\alpha}{2} \left[n^3 + (2L + 1) n^2 + n \right] \beta$$



where β is the multiplication factor of the FORTRAN multiplication operation, and $1 \le \alpha \le 3$ is a factor which takes care of additions, bookkeeping, and special routines if an infinite number of solutions exist.

The procedure taken to produce the answers is shown schematically by the flow chart on page 10. This flow chart is a functional flow chart and, as such, does not show all the steps taken by the program. A detailed flow chart of the program is shown on pages 51 through 61.

Determination of the Zero Value

As mentioned before in the Program Description and suggested by the theory explained in the Method of Computation, when checking for inconsistency the elements of the equivalent augmented matrix D must be compared against a calculated zero value. Also, it is desirable to estimate how accurate the calculated unknowns satisfy the system under solution.

Unfortunately, the computer for which this program was set up consists of a 20K memory and uses a floating point arithmetic which carries eight significant digits only. This type of precision very seldon results in computed zero value and exact calculation of the unknowns in the system.

For example, the matrix

$$\begin{bmatrix} 3 & 3 & 4 & 8 & 5 & 15 & 9 & 12 \\ 2 & 2 & 3 & 6 & 7 & 21 & 6 & 8 \\ 1 & 1 & 5 & 10 & 8 & 24 & 3 & 4 \\ 4 & 4 & 10 & 20 & 11 & 33 & 12 & 16 \\ 6 & 6 & 11 & 22 & 13 & 39 & 1.8 & 24 \\ 5 & 5 & 12 & 24 & 12 & 36 & 15 & 20 \\ 9 & 9 & 31 & 62 & 33 & 99 & 27 & 36 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

when reduced by computer operations showed the elements enclosed in dashed lines as

-.30000000E-06	.40000000E-06
.20000001E+01	.30000000E-06
.60000000E-06	.29999997E+01
.14000000E-05	-.24400000E-05

In order to take care of the above restriction, it was necessary to introduce a lower bound for the value of computed zero; this value does not only reveal possible zero elements, but it is also a measure of the accuracy carried through computations. This lower bound is determined by the relative size of the elements present in the augmented matrix, and by the way floating point arithmetic is handled in the computer itself.

We have mentioned before that the reduction of the system's augmented matrix to its equivalent is carried out through a process of divisions, multiplications, and subtractions. In the course of the calculations, some elements become small with respect to others; and, in general, these elements are likely to contain larger percentage of errors; consequently, if division by any one of these elements takes place, the error is propagated to all elements in the matrix, at a fast rate. For example:

Let $X = .X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8 E{+}NM$ and $Y = .Y_1 Y_2 Y_3 Y_4 Y_5 Y_6 Y_7 Y_8 E{+}NL$

be two elements appearing during the calculations. Let us assume that NL = NM-1, X-Y < 10, and $Y_8 > 5$. If the element Z = X-Y is calculated, the following happens

$$\begin{array}{rl} & .X_1 X_2 X_3 X_4 X_5 X_6 X_7 X_8 \quad E+(NM) \\ - & .0\ Y_1 Y_2 Y_3 Y_4 Y_5 Y_6 Y_7 \quad E+(NL + 1 = NM) \\ \hline & .0\ Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 \quad E+NM \end{array}$$

since X_8 is considered inaccurate, and $Y_8 > 5$ has been neglected, the Z_8 digit is also inaccurate. Furthermore, since $Z_1 = 0$, the number Z will be taken as $Z = .Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 0E{+}(NL = MN-1)$, which carries the error in the seventh digit; division by this number will propagate this error.

The program, however, is set up so that when the operation division is to be carried out, it will be carried out in the row which contains the largest element (largest value indicates the largest number in absolute value); in this way, the "round-off" errors can be made as small as possible.

Even though the above precaution has been taken, the following possibility exists. Suppose $L = .L_1 L_2 L_3 L_4 L_5 L_6 L_7 L_8\ E{+}LM$ is the largest element in the augmented matrix; depending on its position in the matrix, this element may be converted into another whose exponent may reach the value of LM+1; let us call this element $L' = .L_1' L_2' L_3' L_4' L_5' L_6' L_7' L_8'\ E{+}(LM{+}1)$.

If, during the calculations, A and B are two elements such that Z = L' - AB = L' - C = 0, the following may happen

$$\begin{array}{rl} & .L_1' L_2' L_3' L_4' L_5' L_6' L_7' L_8' \quad E+(LM+1) \\ - & .C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8 \quad E+(LM+1) \\ \hline & .0\ 0\ 0\ 0\ 0\ 0\ 0\ Z_8 \quad E+(LM+1) \end{array}$$

the digit Z_8 appears in the answer, since L_8' and C_8 are considered inaccurate according to FORTRAN manuals.

Knowing that this is the worse case that can appear during calculation, the value $Z = .Z_8 0000000\ E{+}(LM{+}1{-}7) = .Z_8 0000000\ E{+}(LM{-}6)$ was assumed to be a good "lower bound" for the value of zero. Z_8 could be anywhere between one and nine, but actual calculations showed that $Z_8 = 1$ was accurate enough.

This value Z is considered to be the zero of the system, and any element whose absolute value becomes smaller than or equal to Z is considered a zero element by the program.

For any (n x n) system, Z = .1000000E(EX), where EX is the number resulting from the subtraction of six from the exponent of the largest element in the augmented matrix. Thus, if 350 is the largest element, Z = .1000000E-03.

This value of Z must be entered into the program from the typewriter, after the message

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.

is typed.

Z is a greatest lower bound and should not be selected any smaller. On the contrary, if it seems to be very small for the amount of operations carried during a given calculation, a message is typed indicating that Z should be increased in value.

This zero value also implies that the relative size of the elements forming the augmented matrix, shall not differ by more than a factor of 10^5 or shall not be any closer than a factor of 10^{-5}. Otherwise, the accuracy called for goes beyond the assumed zero value, which is a contradiction. Such systems shall be considered as "ill-conditioned" systems and are out of the scope of this program.

One may argue that the system

$$10^7 X_1 + 10^7 X_2 = 10^{-3}$$

$$2 \times 10^7 X_1 + 3 \times 10^7 X_2 = 10^{-2}$$

is very consistent, and its unknowns are $X_1 = -7 \times 10^{-10}$ and $X_2 = 8 \times 10^{-10}$. However, if analyzed by this program, Z = .10000000E+02, and the answers could be interpreted as zeroes.

But this is not quite an ill-conditioned system, since it could be written as:

$$X_1' + X_2' = 10^{-3} \qquad X_1' = 10^7 X_1,\ X_2' = 10^7 X_2$$

$$2X_1' + 3X_2' = 10^{-2}$$

for which Z = .1000000E-05 and $X_1' = -7 \times 10^{-3}$, $X_2' = 8 \times 10^{-3}$, which implies $X_1 = -7 \times 10^{-10}$, and $X_2 = 8 \times 10^{-10}$

The system

$$10^5 X_1 + 10^5 X_2 = 10^{-2}$$

$$.34 X_1 + 10^2 X_2 = 10^7$$

for which Z = .10000000E+02 is a true ill-conditioned system, and could not be handled by this program, not only because of the zero value chosen, but because the numbers are so far apart that single precision arithmetic could lead to very inaccurate answers.

Such problems could be solved if the program was set up for multiple precision arithmetic.

Input Card Format

This program is set up to solve systems of N equations with N unknowns only.

If the system contains L equations and N unknowns and L < N, a number of N-L equations should be added after the L equation, before attempting to use this program with such a system. These extra equations must consist of zero coefficients and zero constant equalities. For example, the system

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = 0$$

$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = 0$$

shall be transformed into

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = 0$$

$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = 0$$

$$0.\ X1 + 0.\ X_1 + 0.\ X_3 = 0$$

For any N x N system (smaller than or equal to 17 x 17) one can obtain solutions for as many as 18-N constant vectors, which will carry the names "Vector N+1, Vector N+2,····, Vector 18."

The first card in the input data must contain two numbers, N and M. N is the number of unknowns (or the number of equations) forming the system, and M is the number of unknowns plus the number of constant vectors. These two numbers must be punched in the FORMAT (I3, I3) specification form; that is, the units position of number N must be column 3, and the units position of number M must be column 6.

The rest of the cards must contain the elements of a rectangular matrix formed by the system coefficient matrix and all the constant vectors added as columns to the right of it. These elements must be punched row wise starting with the element in the first column and ending with the element in the last column. Each card must contain six elements, and they must be punched in the E12.6 FORMAT specification form. Depending on the number of elements per row, M to be exact, a minimum of one card, or a maximum of three cards per row will exist. If M is not a multiple of six, zero values must be introduced in order to complete six numbers in the last card of a row. A number of N rows must be present in the input.[1] Thus, depending on the value of M, the following table gives the total number of cards present in the input:

Value of M	Number of Cards
M ≤ 6	N + 1
6 < M ≤ 12	2N + 1
12 < M ≤ 18	3N + 1

For those unfamiliar with the FORMAT specifications, the specification E12.6 means the following: The number under this specification must take a total of 12 spaces. Space one is for the sign of the number, space two for the decimal point, spaces three through eight for the significant digits of the number and zeros if required, and spaces nine through twelve for a minus sign if the exponent is negative or a plus or blank if it is positive, and two spaces for the exponent. The following examples show how the numbers on the left must be punched on the card:

Number	Punched
35	+.350000E+02 or b.350000E+02
785.25	b.785250E+03
-.00032	-.320000E-03

b indicates a blank space

Output Format

The output from this program is printed by means of the console typewriter, and consists of four types:

[1] Note: If the first column of the coefficient matrix consists of zero elements only, the system should be rearranged so that this column appears in a different position.

(1) The message "RELOAD DATA INCR. Z BY FACTOR OF 10." This message is typed to indicate that the computed unknowns do not satisfy the system within the accuracy of the tolerable Z value.

After this message is typed, the operator should reload the data and push the start button. Once the program requests the entering of the tolerable Z value, the operator shall enter a new Z value according to the corresponding instructions found under Operating Instructions.

This new Z value is determined from the equation

$$Z = 10 \times Z_1$$

where Z_1 is the tolerable Z value used previously.

(2) If the system is consistent, and the answers computed satisfy the accuracy assigned by Z, the message

SOLUTION FOR VECTOR N+1

is typed first, to indicate that the column of numbers immediately below it are the answers corresponding to the first constant vector. The answers are next typed one per line, in the E14.8 FORMAT specification, and they correspond to $X_1, X_2 \ldots, X_n$.

If more than one constant vector has been included in the input data, the program continues to type SOLUTION FOR VECTOR N+2, and will continue as above until the solution for the last vector (vector M) has been typed.

(3) If together with any one of the constant vectors, let us say vector N+1, the system is inconsistent, the message

SOLUTION FOR VECTOR N+1
SYSTEM INCONSISTENT

is typed.

(4) If together with any one of the constant vectors, let us say vector M, the system has an infinite number of solutions, the message

SOLUTION FOR VECTOR M
INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

is typed, and the program halts.

This message tells the operator that the system has an infinite number of solutions, and it gives him the choice of obtaining a general solution or continuing with the solution for the next vector, or for the next system, whichever happens to be next.

If the operator does not want to obtain a general solution, he should set SW3 OFF and PUSH START.

If the operator wishes to obtain a general solution, he should set SW3 ON and PUSH START.

The output will then consist of (N-L+1) columns, each containing L numbers in the E14.8 FORMAT specification; L is the row rank of the coefficient matrix. The first (N-L) columns are headed by the heading X_i $(1 < i \leq n)$, and the last column appears without any heading, but reasonably spaced from the column before.

The X_i's indicate those variables that can be considered as parameters, and the numbers immediately below them indicate their coefficients in the general solution. The last column consists of the constants appearing in the general solution.

How to form a general solution as (1) in the Method of Computation section is better explained through an example.

The system

$$3X_1 + 3X_2 + 4X_3 + 8X_4 + 5X_5 + 15X_6 + 9X_7 = 12$$

$$2X_1 + 2X_2 + 3X_3 + 6X_4 + 7X_5 + 21X_6 + 6X_7 = 8$$

$$X_1 + X_2 + 5X_3 + 10X_4 + 8X_5 + 24X_6 + 3X_7 = 4$$

$$4X_1 + 4X_2 + 10X_3 + 20X_4 + 11X_5 + 33X_6 + 12X_7 = 16$$

$$6X_1 + 6X_2 + 11X_3 + 22X_4 + 13X_5 + 39X_6 + 1.8X_7 = 24$$

$$5X_1 + 5X_2 + 12X_3 + 24X_4 + 12X_5 + 36X_6 + 15X_7 = 20$$

$$9X_1 + 9X_2 + 31X_3 + 62X_4 + 33X_5 + 99X_6 + 27X_7 = 36$$

for which Z = .10000000E-04 produces the following answers

RELOAD DATA INCR. Z BY FACTOR OF 10

Once the data is reloaded and Z is made .0000000E-03, the following answer is obtained:

SOLUTION FOR VECTOR 8
INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

If SW3 is ON, the following is obtained:

X_2

-.10000000E+01
.00000000E-99
.00000000E-99
.00000000E-99

X_4

.30000000E-06
-.20000001E+01
-.60000000E-06
-.14000000E-05

X_6

-.40000000E-06
-.30000000E-06
-.29999997E+01
.24400000E-05

.40000000E+01
-.00000000E-99
-.00000000E-99
.00000000E-99

Since Z was originally estimated as .10000000E-04, all of those numbers whose exponent is smaller than E-04 are considered zero.

From above, form a table consisting of four columns (N-L+1 in the general case), and four rows (L in the general case). The first column in this table must consist of the four numbers appearing under X_2, and X_2 is its heading; the second column consists of the numbers under X_4, and X_4 is its heading; the third column consists of the four numbers under X_6, and X_6 is its heading; the fourth column consists of the four numbers which appear in the column without heading. Thus so far, we have

X_2	X_4	X_6	
-1	0	0	4
0	-2	0	0
0	0	-3	0
0	0	0	0

Now, on the left side, assign names to the rows, by means of the remaining unknowns X_1, X_3, X_5, X_7, and in the same order. Thus, at the end, the table looks like

	X_2	X_4	X_6	
X_1	-1	0	0	4
X_3	0	-2	0	0
X_5	0	0	-3	0
X_7	0	0	0	0

and the general solution is obtained as

$$X_1 = -1.X_2 + 0.X_4 + 0.X_6 + 4$$

$$X_3 = 0.X_2 - 2.X_4 + 0.X_6 + 0$$

$$X_5 = 0.X_2 + 0.X_4 - 3.X_6 + 0$$

$$X_7 = 0.X_2 + 0.X_4 + 0.X_6 + 0$$

or

$$X_1 = -X_2 + 4$$

$$X_3 = -2X_4 \qquad X_2,\ X_4,\ X_6 \text{ take on any value}$$

$$X_5 = -3X_6$$

$$X_7 = 0$$

Operating Instructions

I. A. Initial Console Setting

	Program	Stop
Parity		X
I/O		X
Overflow	X	

B. Sense Switch Settings

Sense Switch 1	OFF	Not used
Sense Switch 2	OFF	Not used
Sense Switch 3	ON OFF	If inf. Sols. exist, general solution is typed General solution is not typed
Sense Switch 4	ON OFF	To correct error in typing tolerable zero value Tolerable zero value entered correctly

See section V for further comment on switches #3 and #4.

II. Input-Output

Card Reader

No. of Cards	Description
582	Program Deck
1	N&M Card - N - number of equations in first set of equations M - number of equations plus number of constant vectors - first set of equations
2-51	Coefficient cards for first set of equations
1	N&M Card - N - number of equations in second set of equations M - number of equations plus number of constant vectors - second set of equations
2-51	Coefficient cards for second set of equations
	ETC.

Typewriter Output

Control	Description
Margins	Set left margin as desired
Tab Stops	None
Forms	Standard paper

III. Normal Loading Procedure

(1) Clear Storage

(2) Depress RESET

(3) Place the PROGRAM DECK[1] in the reader hopper and depress LOAD button. When the READER NO FEED light comes on, depress the READER START so that the last two program cards are read in. At this time, the message LOAD DATA is typed.

(4) Place DATA cards in the reader hopper and depress COMPUTER START; when the READER NO FEED light comes on, depress the READER START so that the data cards are read into memory.

If the data consists of one system of equations only, the READER NO FEED light will come on, at which time the READER START must be depressed to read the last two data cards. If the data consists of two or more systems, the READER NO FEED light will come on when the computer is attempting to read the data corresponding to the last system; again, the READER START must be depressed to read the last two data cards for this system.

IV. Special Loading Instruction

After the program deck has been read into memory and it has been started, it can be stopped at any time by depressing COMPUTER INSTANT STOP.

The program can then be initialized and started by depressing the RESET button and inserting the instruction 4908000 by means of the console typewriter.

V. Special Instructions and Remarks

Tolerable Zero Value: When the program begins, it first reads the data for the first system, and then the message

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.

is typed. The operator must then type the Z value that applies for the system under analysis, and in accordance with the E14.8 FORMAT specification. That is, the Z value must be entered as +.10000000E(EX), where EX is the exponent determined as explained before in the "Determination of Zero Value" section.

[1] Recall the definition appearing on page 1 where the Program Deck is defined as Deck #2 without the hash total card; or, it can be obtained by compiling the Source Program (Deck #1 without hash total card) by means of a FORTRAN with FORMAT compiler containing the Absolute value subroutine.

If the Z value is typed correctly, set SW4 OFF and depress RELEASE and COMPUTER START. The program will then continue to solve the system under analysis.

If the Z value is not typed correctly, set SW4 ON and depress RELEASE and COMPUTER START. The program will immediately return control to the typewriter so that the Z value may be entered again. If the Z value is re-entered correctly, follow the procedure for a correct entry.

If the Z value is typed incorrectly, and SW4 is OFF, depressing the RELEASE and START buttons will produce the message ERROR F7 and the program will continue. Continuation of the program under these circumstances is useless; consequently, the programmer must press the COMPUTER INSTANT STOP.

After the program is stopped, the programmer must place the non-processed system data cards back in the reader hopper. He does this in the following way:

(a) Remove data cards, if any, from the reader hopper.

(b) Depress NON-PROCESS RUN-OUT key on the card reader.

(c) Remove cards, if any, from the error select stacker.

(d) Place these cards in front of the cards removed from the hopper.

(e) Remove non-processed system data cards from the non-select stacker.

(f) Place these cards in front of the cards removed from the error-select stacker and replace deck in reader hopper.

With the non-processed system data cards ready for processing, the programmer must depress the COMPUTER RESET and initialize the program as explained in section IV. When the READER NO FEED light comes on, the READER START is depressed and the program will continue as explained in previous sections.

Accuracy Warning: As explained under Output Format section of this write-up, when the computed unknowns do not satisfy the system under analysis within the accuracy of the tolerable Z value, the message

RELOAD DATA INCR. Z BY FACTOR OF 10

is typed; after this the program halts waiting for the programmer's action.

If the programmer does not want to continue with this system, but he wants to continue with the next system for solution, he does so by simply depressing the COMPUTER START.

If the programmer wants to continue with the solution of the system for which the warning message was typed, he does so by reloading the system's data cards as explained in this section under the title Tolerable Zero Value.

Once the data cards are in the hopper, pushing the COMPUTER START and the READER START will initialize the program. When the request for a new zero value is typed, the programmer follows the instructions insinuated in the "Output Format" section of this write-up.

General Solution: When the system under analysis has an infinite number of solutions, the message

INF SOLS SW3 ON FOR GEN SOL-SW3 TO CONT.

is typed; after this the program halts waiting for the programmer's action.

If the programmer does not want to obtain a general solution, he sets SW3 OFF and depresses START. The program will continue with the solution for the next constant vector or the next system, if any. If the program tries to continue with the next system, but there is no data cards for a next system, the program stops on a READER NO FEED warning.

If the programmer wants to obtain a general solution, he does so by setting SW3 ON and depressing START.

VI. Programmed Stops and Required Action

All programmed stops are accompanied by typewriter messages which are self-explanatory and which indicate the required action.

As the program is written in FORTRAN, all the error stops and messages of this system apply.

Sample Problems

The following seven sample problems were used one after the other and they illustrate the program reliability:

Problem No. 1:

$$
\begin{aligned}
X_1 + 2X_2 - X_3 - 2X_4 &= -1 \\
2X_1 + X_2 + X_3 - X_4 &= 4 \\
X_1 - X_2 + 2X_3 + X_4 &= 5 \\
X_1 + 3X_2 - 2X_3 - 3X_4 &= -3
\end{aligned}
\qquad Z = .10000000E-05
$$

SOLUTION FOR VECTOR 5

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X_3

-.10000000E+01
.10000000E+01

X_4

.00000000E-99
.10000000E+01

.30000000E+01
-.20000000E+01

	X_3	X_4	
X_1	-1	0	3
X_2	1	1	-2

$X_1 = -X_3 + 3$

$X_2 = X_3 + X_4 - 2$

X_3, X_4 could take on any real value

Problem No. 2:

$$
\begin{aligned}
3X_1 + 8X_2 + 6X_3 + 10X_4 + 42X_5 &= 0 \\
2X_2 + X_4 + 5X_5 &= 0 \\
.01X_3 + 4X_4 + 6X_5 &= 0 \\
2X_1 + 4X_2 + 7X_3 + 9X_5 &= 0
\end{aligned}
\qquad Z = .10000000E-04
$$

Add one more equation, as explained in the Input Card Format section, so that the system becomes

$$
\begin{aligned}
3X_1 + 8X_2 + 6X_3 + 10X_4 + 42X_5 &= 0 \\
2X_2 + \quad + X_4 + 5X_5 &= 0 \\
.01X_3 + 4X_4 + 6X_5 &= 0 \\
2X_1 + 4X_2 + 7X_3 + 9X_5 &= 0 \\
0.X_1 + 0.X_2 + 0.X_3 + 0.X_4 + 0.X_5 &= 0
\end{aligned}
\qquad Z = .10000000E-04
$$

SOLUTION FOR VECTOR 6

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X_5

-.87446120E+01
-.17472361E+01
.22111665E+01
-.15055279E+01

-.00000000E-99
-.00000000E-99
-.00000000E-99
.00000000E-99

	X_5	
X_1	-8.74461	0
X_2	-1.74724	0
X_3	2.21117	0
X_4	-1.50553	0

$X_1 = -8.74461\ X_5$

$X_2 = -1.74724\ X_5$

$X_3 = 2.21117\ X_5$

$X_4 = -1.50553\ X_5$

X_5 = any real constant

Problem No. 3:

$$
\begin{aligned}
X_1 + 2X_2 + 3X_3 - 4X_4 + 5X_5 - 6X_6 + 7X_7 - 8X_8 &= -28 \quad -14 \quad -84 \quad 60 \\
7X_2 - 2X_5 + X_7 + X_8 &= 19 \quad 9.5 \quad 57 \quad 30 \\
2X_1 + 3X_2 - 4X_3 - 5X_4 \quad 6X_6 &= 12 \quad 6 \quad 36 \quad 90 \\
X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 &= 36 \quad 18 \quad 108 \quad 45 \\
10X_1 + 3X_2 - 4X_3 + X_5 + 2X_6 - 9X_7 &= -42 \quad -21 \quad -126 \quad 135 \\
3X_1 - 3X_2 - 2X_3 + 2X_4 + X_6 &= 5 \quad 2.5 \quad 15 \quad 67.5 \\
-8X_1 - 9X_3 + 7X_5 + 6X_7 - 3X_8 &= 18 \quad 9 \quad 54 \quad 99 \\
3X_7 - 5X_8 &= -19 \quad -9.5 \quad -57 \quad 0
\end{aligned}
$$

Z = .10000000E-03

SOLUTION FOR VECTOR 9

.99999970E-00	$X_1 = 1$
.19999998E+01	$X_2 = 2$
.29999999E+01	$X_3 = 3$
.40000002E+01	$X_4 = 4$
.49999990E+01	$X_5 = 5$
.60000010E+01	$X_6 = 6$
.69999996E+01	$X_7 = 7$
.79999996E+01	$X_8 = 8$

SOLUTION FOR VECTOR 10

$X_1 = .5$ $X_2 = 1.0$ $X_3 = 1.5$ $X_4 = 2.0$

$X_5 = 2.5$ $X_6 = 3$ $X_7 = 3.5$ $X_8 = 4$

SOLUTION FOR VECTOR 11

$X_1 = 3$ $X_2 = 6$ $X_3 = 9$ $X_4 = 12$

$X_5 = 15$ $X_6 = 18$ $X_7 = 21$ $X_8 = 24$

SOLUTION FOR VECTOR 12

X_1 = .17373927E+02 X_4 = .23638003E+01 X_7 = .12568545E+02

X_2 = .46417256E+01 X_5 = .11300876E+02 X_8 = .75411273E+01

X_3 = -.11788599E+02 X_6 = .99859570E-00

Problem No. 4:

The system whose coefficient matrix is shown on page 29 will have an infinite number of solutions or will be inconsistent, depending on the values of the constant vectors. This is so because the determinant of the system is zero.

This coefficient matrix, together with the constant vectors shown to the right and Z = .10000000E-03, gives the following answer:

SOLUTION FOR VECTOR 14

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 15

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 16

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 17

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 18

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

0	35	10	40	10	30	150	115	40	41	53	55	67
-35	0	11	15	17	13	12	61	64	63	72	75	47
-10	-11	0	1	2	3	4	5	6	7	8	9	10
-40	-15	-1	0	11	15	32	23	17	11	15	16	17
-10	-17	-2	-11	0	150	149	148	147	146	145	144	143
-30	-13	-3	-15	-150	0	-2	-3	-4	-5	-6	-7	-8
-150	-12	-4	-32	-149	2	0	21	22	23	24	25	26
-115	-61	-5	-23	-148	3	-21	0	31	35	39	43	47
-40	-64	-6	-17	-147	4	-22	-31	0	160	250	349	471
-41	-63	-7	-11	-146	5	-23	-35	-160	0	-30	-40	-50
-53	-72	-8	-15	-145	6	-24	-39	-250	30	0	-13	-15
-55	-75	-9	-16	-144	7	-25	-43	-349	40	13	0	111
-67	-47	-10	-17	-143	8	-26	-47	-471	50	15	-111	0

COEFFICIENT MATRIX

141	17	20	38	0
48	49	52	15	-70
11	12	13	14	-20
18	19	20	21	-80
142	141	140	139	-20
-9	-10	-11	-12	-60
27	28	29	30	-300
147	148	149	150	-230
582	651	781	811	-80
-60	-70	-80	-90	-82
-14	-16	-15	-17	-106
121	133	144	155	-110
11	9	7	5	-134

CONSTANT VECTORS

EXAMPLE 4

$X_1 = -.89912030 \times X_{13} + 2.0000$

$X_2 = -1.1049142 \times X_{13}$

$X_3 = 57.970323 \times X_{13}$

$X_4 = -9.417142 \times X_{13}$

$X_5 = .14746840 \times X_{13}$

$X_6 = 1.7722926 \times X_{13}$

$X_7 = 7.5299710 \times X_{13}$

$X_8 = -12.344710 \times X_{13}$

$X_9 = 1.176188 \times X_{13}$

$X_{10} = 6.271787\ X_{13}$

$X_{11} = -3.5682707\ X_{13}$

$X_{12} = -2.0166340\ X_{13}$

X_{13} could take on any value

Problem No. 5:

The system whose coefficient matrix and constant vectors are shown on page 31 and for which Z = .10000000E-03 produces the following answers:

RELOAD DATA INCR. Z BY FACTOR OF 10

with Z = .10000000E-02

SOLUTION FOR VECTOR 14	SOLUTION FOR VECTOR 15
-.89017740E+01	-.77705800E+01
-.48386103E+01	-.49968307E+01
.41425661E+03	.37400470E+03
-.55480391E+02	-.51928873E+02
.14276311E-00	.33781753E-00
.15499623E+02	.15138997E+02
.56783579E+02	.49755229E+02
-.99629510E+02	-.88164190E+02
.10310057E+02	.88469110E+01

0	35	10	40	10	30	150	115	40	41	53	55	67
-35	0	11	15	17	13	12	61	64	63	72	75	47
-10	-11	0	1	2	3	4	5	6	7	8	9	10
-40	-15	-1	0	11	15	32	23	17	11	15	16	17
-10	-17	-2	-11	0	150	149	148	147	146	145	144	143
-30	-13	-3	-15	-150	0	-2	-3	-4	-5	-6	-7	-8
-150	-12	-4	-32	-149	2	0	21	22	23	24	25	26
-115	-61	-5	-23	-148	3	-21	0	31	35	39	43	47
-40	-64	-6	-17	-147	4	-22	-31	0	160	250	349	471
-41	-63	-7	-11	-146	5	-23	-35	-160	0	-30	-40	-50
-53	-72	-8	-15	-145	6	-24	-39	-250	30	0	-13	-15
-55	-75	-9	-16	-144	7	-25	-43	-349	40	13	0	111
67	-47	-10	-17	-143	8	-26	-47	-471	50	15	-111	0

COEFFICIENT MATRIX

141	17	20	38	0
48	49	52	15	-70
11	12	13	14	-20
18	19	20	21	-80
142	141	140	139	-20
-9	-10	-11	-12	-60
27	28	29	30	-300
147	148	149	150	-230
582	651	781	811	-80
-60	-70	-80	-90	-82
-14	-16	-15	-17	-106
121	133	144	155	-110
11	9	7	5	-134

CONSTANT VECTORS

EXAMPLE 5

.47450099E+02	.39042339E+02
-.12369882E+02	-.89736869E+01
-.26474642E+02	-.23229999E+02
.91686745E+02	.81447152E+01

SOLUTION FOR VECTOR 16	SOLUTION FOR VECTOR 17	SOLUTION FOR VECTOR 18
-.70374340E+01	-.68729740E+01	.96000000E-05
-.44552765E+01	-.50068592E+01	-.24577692E+01
.33750718E+03	.34873410E+03	.12894879E+03
-.46798547E+02	-.50506716E+02	-.20947452E+02
.29713861E-00	.45286507E-00	.32802864E-00
.13666631E+02	.12175005E+02	.39422583E+01
.45032153E+02	.47732087E+02	.16749622E+02
-.79983800E+02	-.82814740E+02	-.27457473E+02
.79584935E+01	.84018230E+01	.26163002E+01
.34742553E+02	.38005589E+02	.13950905E+02
-.74102383E+01	-.95950557E+01	-.79372437E+01
-.20863395E+02	-.21404485E+02	-.44857705E+01
.74716143E+01	.79367011E+01	.22243907E+01

Note that the value of X_1 in the solution for vector 18 is smaller than Z = .10000000E-03 and consequently X_1 = 0; this is the true value of X_1 in the system.

Problem No. 6:

The system shown on page 33 whose determinant is clearly zero, and for which Z = .10000000E-03, produces the following answers:

RELOAD DATA INCR. Z BY FACTOR OF 10

with Z = .10000000E-02, we obtain

$X_1 = -.48874995\ X_{17} + 2.000$

$X_2 = 2.3976300\ X_{17}$

$X_3 = -28.744234\ X_{17}$

$X_4 = 9.7720987\ X_{17}$

$X_5 = -.61967767\ X_{17}$

$X_6 = -1.7527315\ X_{17}$

0	35	10	40	10	30	150	115	40	41	53	55	67	141	17	20	38		0
-35	0	11	15	17	13	12	61	64	63	72	75	47	48	49	52	15		-70
-10	-11	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14		-20
-40	-15	-1	0	11	15	32	23	17	11	15	16	17	18	19	20	21		-80
-10	-17	-2	-11	0	150	149	148	147	146	145	144	143	142	141	140	139		-20
-30	-13	-3	-15	-150	0	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12		-60
-150	-12	-4	-32	-149	2	0	21	22	23	24	25	26	27	28	29	30		-300
-115	-61	-5	-23	-148	3	-21	0	31	35	39	43	47	147	148	149	150		-230
-40	-64	-6	-17	-147	4	-22	-31	0	160	250	349	471	582	651	781	811		-80
-41	-63	-7	-11	-146	5	-23	-35	-160	0	-30	-40	-50	-60	-70	-80	-90		-82
-53	-72	-8	-15	-145	6	-24	-39	-250	30	0	-13	-15	-14	-16	-15	-17		-106
-55	-75	-9	-16	-144	7	-25	-43	-349	40	13	0	111	121	133	144	155		-110
-67	-47	-10	-17	-143	8	-26	-47	-471	50	15	-111	0	11	9	7	5		-134
-141	-48	-11	-18	-142	9	-27	-147	-582	60	14	-121	-11	0	15	20	30		-282
-17	-49	-12	-19	-141	10	-28	-148	-651	70	16	-133	-9	-15	0	1	20		-34
-20	-52	-13	-20	-140	11	-29	-149	-781	80	15	-144	-7	-20	-1	0	10		-40
-38	-15	-14	-21	-139	12	-30	-150	-811	90	17	-155	-5	-30	-20	-10	0		-76

COEFFICIENT MATRIX   C.V. 18

EXAMPLE 6

$X_7 = -.82128261\ X_{17}$

$X_8 = .02129996\ X_{17}$

$X_9 = -.25086719\ X_{17}$

$X_{10} = -1.8757052\ X_{17}$

$X_{11} = 3.5436282\ X_{17}$

$X_{12} = 1.9503036\ X_{17}$

$X_{13} = .43309011\ X_{17}$

$X_{14} = -1.8223183\ X_{17}$

$X_{15} = 10.547266\ X_{17}$

$X_{16} = -10.317607\ X_{17}$

X_{17} could take on any value

Problem No. 7:

The system shown on page 35 , for which Z = .10000000E-03, gives

RELOAD DATA INCR. Z BY FACTOR OF 10

Z = .10000000E-02

RELOAD DATA INCR. Z BY FACTOR OF 10

Z = .10000000E-01

SOLUTION FOR VECTOR 18

X_1 = .91800000E-04
X_2 = .11183719E+02
X_3 = -.13360065E+03
X_4 = .45581915E+02
X_5 = -.28904901E+01
X_6 = -.81754060E+01
X_7 = -.38309755E+01
X_8 = .99337500E-01
X_9 = -.11702277E+01
X_{10} = -.87497130E+01
X_{11} = .16529699E+02
X_{12} = .90972961E+01
X_{13} = .20201331E+01
X_{14} = -.85001620E+01
X_{15} = .49197987E+02
X_{16} = -.48126945E+02
X_{17} = .46646303E+01

Note that $X_1 < .10000000E-03$, and consequently $X_1 = 0$; this is the true value of X_1 in the system

0	35	10	40	10	30	150	115	40	41	53	55	67	141	17	20	38	0
-35	0	11	15	17	13	12	61	64	63	72	75	47	48	49	52	15	-70
-10	-11	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	-20
-40	-15	-1	0	11	15	32	23	17	11	15	16	17	18	19	20	21	-80
-10	-17	-2	-11	0	150	149	148	147	146	145	144	143	142	141	140	139	-20
-30	-13	-3	-15	-150	0	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-60
-150	-12	-4	-32	-149	2	0	21	22	23	24	25	26	27	28	29	30	-300
-115	-61	-5	-23	-148	3	-21	0	31	35	39	43	47	147	148	149	150	-230
-40	-64	-6	-17	-147	4	-22	-31	0	160	250	349	471	582	651	781	811	-80
-41	-63	-7	-11	-146	5	-23	-35	-160	0	-30	-40	-50	-60	-70	-80	-90	-82
-53	-72	-8	-15	-145	6	-24	-39	-250	30	0	-13	-15	-14	-16	-15	-17	-106
-55	-75	-9	-16	-144	7	-25	-43	-349	40	13	0	111	121	133	144	155	-110
-67	-47	-10	-17	-143	8	-26	-47	-471	50	15	-111	0	11	9	7	5	-134
-141	-48	-11	-18	-142	9	-27	-147	-582	60	14	-121	-11	0	15	20	30	-282
-17	-49	-12	-19	-141	10	-28	-148	-651	70	16	-133	-9	-15	0	1	20	-34
-20	-52	-13	-20	-140	11	-29	-149	-781	80	15	-144	-7	-20	-1	0	10	-40
38	-15	-14	-21	-139	12	-30	-150	-811	90	17	-155	-5	-30	-20	-10	0	-76

COEFFICIENT MATRIX C.V. 18

EXAMPLE 7

SAMPLE PROBLEMS-INPUT

Problem No. 1

```
 4  5
.100000E+01 .200000E+01-.100000E+01-.200000E+01-.100000E+01 .000000E-99
.200000E+01 .100000E+01 .100000E+01-.100000E+01 .400000E+01 .000000E-99
.100000E+01-.100000E+01 .200000E+01 .100000E+01 .500000E+01 .000000E-99
.100000E+01 .300000E+01-.200000E+01-.300000E+01-.300000E+01 .000000E-99
```

Problem No. 2

```
 5  6
.300000E+01 .800000E+01 .600000E+01 .100000E+02 .420000E+02 .000000E-99
.000000E-99 .200000E+01 .000000E-99 .100000E+01 .500000E+01 .000000E-99
.000000E-99 .000000E-99 .100000E-01 .400000E+01 .600000E+01 .000000E-99
.200000E+01 .400000E+01 .700000E+01 .000000E-99 .900000E+01 .000000E-99
.000000E-99 .000000E-99 .000000E-99 .000000E-99 .000000E-99 .000000E-99
```

Problem No. 3

```
 8 12
 .100000E+01 .200000E+01 .300000E+01-.400000E+01 .500000E+01-.600000E+01
 .700000E+01-.800000E+01-.280000E+02-.140000E+02-.840000E+02 .600000E+02
 .000000E-99 .700000E+01 .000000E-99 .000000E-99-.200000E+01 .000000E-99
 .100000E+01 .100000E+01 .190000E+02 .950000E+01 .570000E+02 .300000E+02
 .200000E+01 .300000E+01-.400000E+01-.500000E+01 .000000E-99 .600000E+01
 .000000E-99 .000000E-99 .120000E+02 .600000E+01 .360000E+02 .900000E+02
 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01
 .100000E+01 .100000E+01 .360000E+02 .180000E+02 .108000E+03 .450000E+02
 .100000E+02 .300000E+01-.400000E+01 .000000E-99 .100000E+01 .200000E+01
-.900000E+01 .000000E-99-.420000E+02-.210000E+02-.126000E+03 .135000E+03
 .300000E+01-.300000E+01-.200000E+01 .200000E+01 .000000E-99 .100000E+01
 .000000E-99 .000000E-99 .500000E+01 .250000E+01 .150000E+02 .675000E+02
-.800000E+01 .000000E-99-.900000E+01 .000000E-99 .700000E+01 .000000E-99
 .600000E+01-.300000E+01 .180000E+02 .900000E+01 .540000E+02 .990000E+02
 .000000E-99 .000000E-99 .000000E-99 .000000E-99 .000000E-99 .000000E-99
 .300000E+01-.500000E+01-.190000E+02-.950000E+01-.570000E+02 .000000E-99
```


Problem No. 4

```
 13 18
 .000000E-99 .350000E+02 .100000E+02 .400000E+02 .100000E+02 .300000E+02
 .150000E+03 .115000E+03 .400000E+02 .410000E+02 .530000E+02 .550000E+02
 .670000E+02 .141000E+03 .170000E+02 .200000E+02 .380000E+02 .000000E-99
-.350000E+02 .000000E-99 .110000E+02 .150000E+02 .170000E+02 .130000E+02
 .120000E+02 .610000E+02 .640000E+02 .630000E+02 .720000E+02 .750000E+02
 .470000E+02 .480000E+02 .490000E+02 .520000E+02 .150000E+02-.700000E+02
-.100000E+02-.110000E+02 .000000E-99 .100000E+01 .200000E+01 .300000E+01
 .400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01 .900000E+01
 .100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02-.200000E+02
-.400000E+02-.150000E+02-.100000E+01 .000000E-99 .110000E+02 .150000E+02
 .320000E+02 .230000E+02 .170000E+02 .110000E+02 .150000E+02 .160000E+02
 .170000E+02 .180000E+02 .190000E+02 .200000E+02 .210000E+02-.800000E+02
-.100000E+02-.170000E+02-.200000E+01-.110000E+02 .000000E-99 .150000E+03
 .149000E+03 .148000E+03 .147000E+03 .146000E+03 .145000E+03 .144000E+03
 .143000E+03 .142000E+03 .141000E+03 .140000E+03 .139000E+03-.200000E+02
-.300000E+02-.130000E+02-.300000E+01-.150000E+02-.150000E+03 .000000E-99
-.200000E+01-.300000E+01-.400000E+01-.500000E+01-.600000E+01-.700000E+01
-.800000E+01-.900000E+01-.100000E+02-.110000E+02-.120000E+02-.600000E+02
-.150000E+03-.120000E+02-.400000E+01-.320000E+02-.149000E+03 .200000E+01
 .000000E-99 .210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02
 .260000E+02 .270000E+02 .280000E+02 .290000E+02 .300000E+02-.300000E+03
-.115000E+03-.610000E+02-.500000E+01-.230000E+02-.148000E+03 .300000E+01
-.210000E+02 .000000E-99 .310000E+02 .350000E+02 .390000E+02 .430000E+02
 .470000E+02 .147000E+03 .148000E+03 .149000E+03 .150000E+03-.230000E+03
-.400000E+02-.640000E+02-.600000E+01-.170000E+02-.147000E+03 .400000E+01
-.220000E+02-.310000E+02 .000000E-99 .160000E+03 .250000E+03 .349000E+03
 .471000E+03 .582000E+03 .651000E+03 .781000E+03 .811000E+03-.800000E+02
-.410000E+02-.630000E+02-.700000E+01-.110000E+02-.146000E+03 .500000E+01
-.230000E+02-.350000E+02-.160000E+03 .000000E-99-.300000E+02-.400000E+02
-.500000E+02-.600000E+02-.700000E+02-.800000E+02-.900000E+02-.820000E+02
-.530000E+02-.720000E+02-.800000E+01-.150000E+02-.145000E+03 .600000E+01
-.240000E+02-.390000E+02-.250000E+03 .300000E+02 .000000E-99-.130000E+02
-.150000E+02-.140000E+02-.160000E+02-.150000E+02-.170000E+02-.106000E+03
-.550000E+02-.750000E+02-.900000E+01-.160000E+02-.144000E+03 .700000E+01
-.250000E+02-.430000E+02-.349000E+03 .400000E+02 .130000E+02 .000000E-99
 .111000E+03 .121000E+03 .133000E+03 .144000E+03 .155000E+03-.110000E+03
-.670000E+02-.470000E+02-.100000E+02-.170000E+02-.143000E+03 .800000E+01
-.260000E+02-.470000E+02-.471000E+03 .500000E+02 .150000E+02-.111000E+03
 .000000E-99 .110000E+02 .900000E+01 .700000E+01 .500000E+01-.134000E+03
```

Problem No. 5

```
 13 18
 .000000E-99 .350000E+02 .100000E+02 .400000E+02 .100000E+02 .300000E+02
 .150000E+03 .1 5000E+03 .400000E+02 .410000E+02 .530000E+02 .550000E+02
 .670000E+02 .141000E+03 .170000E+02 .200000E+02 .380000E+02 .000000E-99
-.350000E+02 .000000E-99 .110000E+02 .150000E+02 .170000E+02 .130000E+02
 .120000E+02 .610000E+02 .640000E+02 .630000E+02 .720000E+02 .750000E+02
 .470000E+02 .480000E+02 .490000E+02 .520000E+02 .150000E+02-.700000E+02
-.100000E+02-.1 0000E+02 .000000E-99 .100000E+01 .200000E+01 .300000E+01
 .400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01 .900000E+01
 .100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02-.200000E+02
-.400000E+02-.150000E+02-.100000E+01 .000000E-99 .1 0000E+02 .150000E+02
 .320000E+02 .230000E+02 .170000E+02 .1 0000E+02 .150000E+02 .160000E+02
 .170000E+02 .180000E+02 .190000E+02 .200000E+02 .210000E+02-.800000E+02
-.100000E+02-.170000E+02-.200000E+01-.1 0000E+02 .000000E-99 .150000E+03
 .149000E+03 .148000E+03 .147000E+03 .146000E+03 .145000E+03 .144000E+03
 .143000E+03 .142000E+03 .141000E+03 .140000E+03 .139000E+03-.200000E+02
-.300000E+02-.130000E+02-.300000E+01-.150000E+02-.150000E+03 .000000E-99
-.200000E+01-.300000E+01-.400000E+01-.500000E+01-.600000E+01-.700000E+01
-.800000E+01-.900000E+01-.100000E+02-.1 0000E+02-.120000E+02-.600000E+02
-.150000E+03-.120000E+02-.400000E+01-.320000E+02-.149000E+03 .200000E+01
 .000000E-99 .210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02
 .260000E+02 .270000E+02 .280000E+02 .290000E+02 .300000E+02-.300000E+03
-.115000E+03-.610000E+02-.500000E+01-.230000E+02-.148000E+03 .300000E+01
-.210000E+02 .000000E-99 .310000E+02 .350000E+02 .390000E+02 .430000E+02
 .470000E+02 .147000E+03 .148000E+03 .149000E+03 .150000E+03-.230000E+03
-.400000E+02-.640000E+02-.600000E+01-.170000E+02-.147000E+03 .400000E+01
-.220000E+02-.310000E+02 .000000E-99 .160000E+03 .250000E+03 .349000E+03
 .471000E+03 .582000E+03 .651000E+03 .781000E+03 .811000E+03-.800000E+02
-.410000E+02-.630000E+02-.700000E+01-.1 0000E+02-.146000E+03 .500000E+01
-.230000E+02-.350000E+02-.160000E+03 .000000E-99-.300000E+02-.400000E+02
-.500000E+02-.600000E+02-.700000E+02-.800000E+02-.900000E+02-.820000E+02
-.530000E+02-.720000E+02-.800000E+01-.150000E+02-.145000E+03 .600000E+01
-.240000E+02-.390000E+02-.250000E+03 .300000E+02 .000000E-99-.130000E+02
-.150000E+02-.140000E+02-.160000E+02-.150000E+02-.170000E+02-.106000E+03
-.550000E+02-.750000E+02-.900000E+01-.160000E+02-.144000E+03 .700000E+01
-.250000E+02-.430000E+02-.349000E+03 .400000E+02 .130000E+02 .000000E-99
 .1 1000E+03 .121000E+03 .133000E+03 .144000E+03 .155000E+03-.1 0000E+03
 .670000E+02-.470000E+02-.100000E+02-.170000E+02-.143000E+03 .800000E+01
-.260000E+02-.470000E+02-.471000E+03 .500000E+02 .150000E+02-.111000E+03
 .000000E-99 .1 0000E+02 .900000E+01 .700000E+01 .500000E+01-.134000E+03
```


Problem No. 6

```
 17 18
 .000000E-99 .350000E+02 .100000E+02 .400000E+02 .100000E+02 .300000E+02
 .150000E+03 .1 5000E+03 .400000E+02 .410000E+02 .530000E+02 .550000E+02
 .670000E+02 .141000E+03 .170000E+02 .200000E+02 .380000E+02 .000000E-99
-.350000E+02 .000000E-99 .1 0000E+02 .150000E+02 .170000E+02 .130000E+02
 .120000E+02 .610000E+02 .640000E+02 .630000E+02 .720000E+02 .750000E+02
 .470000E+02 .480000E+02 .490000E+02 .520000E+02 .150000E+02-.700000E+02
-.100000E+02-.110000E+02 .000000E-99 .100000E+01 .200000E+01 .300000E+01
 .400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01 .900000E+01
 .100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02-.200000E+02
-.400000E+02-.150000E+02-.100000E+01 .000000E-99 .110000E+02 .150000E+02
 .320000E+02 .230000E+02 .170000E+02 .110000E+02 .150000E+02 .160000E+02
 .170000E+02 .180000E+02 .190000E+02 .200000E+02 .210000E+02-.800000E+02
-.100000E+02-.170000E+02-.200000E+01-.110000E+02 .000000E-99 .150000E+03
 .149000E+03 .148000E+03 .147000E+03 .146000E+03 .145000E+03 .144000E+03
 .143000E+03 .142000E+03 .141000E+03 .140000E+03 .139000E+03-.200000E+02
-.300000E+02-.130000E+02-.300000E+01-.150000E+02-.150000E+03 .000000E-99
-.200000E+01-.300000E+01-.400000E+01-.500000E+01-.600000E+01-.700000E+01
-.800000E+01-.900000E+01-.100000E+02-.110000E+02-.120000E+02-.600000E+02
-.150000E+03-.120000E+02-.400000E+01-.320000E+02-.149000E+03 .200000E+01
 .000000E-99 .210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02
 .260000E+02 .270000E+02 .280000E+02 .290000E+02 .300000E+02-.300000E+03
-.115000E+03-.610000E+02-.500000E+01-.230000E+02-.148000E+03 .300000E+01
-.210000E+02 .000000E-99 .310000E+02 .350000E+02 .390000E+02 .430000E+02
 .470000E+02 .147000E+03 .148000E+03 .149000E+03 .150000E+03-.230000E+03
-.400000E+02-.640000E+02-.600000E+01-.170000E+02-.147000E+03 .400000E+01
-.220000E+02-.310000E+02 .000000E-99 .160000E+03 .250000E+03 .349000E+03
 .471000E+03 .582000E+03 .651000E+03 .781000E+03 .811000E+03-.800000E+02
-.410000E+02-.630000E+02-.700000E+01-.110000E+02-.146000E+03 .500000E+01
-.230000E+02-.350000E+02-.160000E+03 .000000E-99-.300000E+02-.400000E+02
-.500000E+02-.600000E+02-.700000E+02-.800000E+02-.900000E+02-.820000E+02
-.530000E+02-.720000E+02-.800000E+01-.150000E+02-.145000E+03 .600000E+01
-.240000E+02-.390000E+02-.250000E+03 .300000E+02 .000000E 99-.130000E+02
-.150000E+02-.140000E+02-.160000E+02-.150000E+02-.170000E+02-.106000E+03
-.550000E+02-.750000E+02-.900000E+01-.160000E+02-.144000E+03 .700000E+01
-.250000E+02-.430000E+02-.349000E+03 .400000E+02 .130000E+02 .000000E-99
 .1 1000E+03 .121000E+03 .133000E+03 .144000E+03 .155000E+03-.110000E+03
-.670000E+02-.470000E+02-.100000E+02-.170000E+02-.143000E+03 .800000E+01
-.260000E+02-.470000E+02-.471000E+03 .500000E+02 .150000E+02-.111000E+03
 .000000E-99 .1 0000E+02 .900000E+01 .700000E+01 .500000E+01-.134000E+03
-.141000E+03-.480000E+02-.110000E+02-.180000E+02-.142000E+03 .900000E+01
-.270000E+02-.147000E+03-.582000E+03 .600000E+02 .140000E+02-.121000E+03
-.110000E+02 .000000E-99 .150000E+02 .200000E+02 .300000E+02-.282000E+03
-.170000E+02-.490000E+02-.120000E+02-.190000E+02-.141000E+03 .100000E+02
-.280000E+02-.148000E+03-.651000E+03 .700000E+02 .160000E+02-.133000E+03
-.900000E+01-.150000E+02 .000000E-99 .100000E+01 .200000E+02-.340000E+02
-.200000E+02-.520000E+02-.130000E+02-.200000E+02-.140000E+03 .110000E+02
-.290000E+02-.149000E+03-.781000E+03 .800000E+02 .150000E+02-.144000E+03
-.700000E+01-.200000E+02-.100000E+01 .000000E-99 .100000E+02-.400000E+02
-.380000E+02-.150000E+02-.140000E+02-.210000E+02-.139000E+03 .120000E+02
-.300000E+02-.150000E+03-.81 000E+03 .900000E+02 .170000E+02-.155000E+03
-.500000E+01-.300000E+02-.200000E+02-.100000E+02 .000000E-99-.760000E+02
```

<u>Problem No. 7</u>

```
17 18
.000000E-99 .350000E+02 .100000E+02 .400000E+02 .100000E+02 .300000E+02
.150000E+03 .115000E+03 .400000E+02 .410000E+02 .530000E+02 .550000E+02
.670000E+02 .141000E+03 .170000E+02 .200000E+02 .380000E+02 .000000E-99
-.350000E+02 .000000E-99 .110000E+02 .150000E+02 .170000E+02 .130000E+02
.120000E+02 .610000E+02 .640000E+02 .630000E+02 .720000E+02 .750000E+02
.470000E+02 .480000E+02 .490000E+02 .520000E+02 .150000E+02-.700000E+02
-.100000E+02-.110000E+02 .000000E-99 .100000E+01 .200000E+01 .300000E+01
.400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01 .900000E+01
.100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02-.200000E+02
-.400000E+02-.150000E+02-.100000E+01 .000000E-99 .110000E+02 .150000E+02
.320000E+02 .230000E+02 .170000E+02 .110000E+02 .150000E+02 .160000E+02
.170000E+02 .180000E+02 .190000E+02 .200000E+02 .210000E+02-.800000E+02
-.100000E+02-.170000E+02-.200000E+01-.110000E+02 .000000E-99 .150000E+03
.149000E+03 .148000E+03 .147000E+03 .146000E+03 .145000E+03 .144000E+03
.143000E+03 .142000E+03 .141000E+03 .140000E+03 .139000E+03-.200000E+02
-.300000E+02-.130000E+02-.300000E+01-.150000E+02-.150000E+03 .000000E-99
-.200000E+01-.300000E+01-.400000E+01-.500000E+01-.600000E+01-.700000E+01
-.800000E+01-.900000E+01-.100000E+02-.110000E+02-.120000E+02-.600000E+02
-.150000E+03-.120000E+02-.400000E+01-.320000E+02-.149000E+03 .200000E+01
.000000E-99 .210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02
.260000E+02 .270000E+02 .280000E+02 .290000E+02 .300000E+02-.300000E+03
-.115000E+03-.610000E+02-.500000E+01-.230000E+02-.148000E+03 .300000E+01
-.210000E+02 .000000E-99 .310000E+02 .350000E+02 .390000E+02 .430000E+02
.470000E+02 .147000E+03 .148000E+03 .149000E+03 .150000E+03-.230000E+03
-.400000E+02-.640000E+02-.600000E+01-.170000E+02-.147000E+03 .400000E+01
-.220000E+02-.310000E+02 .000000E-99 .160000E+03 .250000E+03 .349000E+03
.471000E+03 .582000E+03 .651000E+03 .781000E+03 .811000E+03-.800000E+02
-.410000E+02-.630000E+02-.700000E+01-.110000E+02-.146000E+03 .500000E+01
-.230000E+02-.350000E+02-.160000E+03 .000000E-99-.300000E+02-.400000E+02
-.500000E+02-.600000E+02-.700000E+02-.800000E+02-.900000E+02-.820000E+02
-.530000E+02-.720000E+02-.800000E+01-.150000E+02-.145000E+03 .600000E+01
-.240000E+02-.390000E+02-.250000E+03 .300000E+02 .000000E-99-.130000E+02
-.150000E+02-.140000E+02-.160000E+02-.150000E+02-.170000E+02-.106000E+03
-.550000E+02-.750000E+02-.900000E+01-.160000E+02-.144000E+03 .700000E+01
-.250000E+02-.430000E+02-.349000E+03 .400000E+02 .130000E+02 .000000E-99
.111000E+03 .121000E+03 .133000E+03 .144000E+03 .155000E+03-.110000E+03
-.670000E+02-.470000E+02-.100000E+02-.170000E+02-.143000E+03 .800000E+01
-.260000E+02-.470000E+02-.471000E+03 .500000E+02 .150000E+02-.111000E+03
.000000E-99 .110000E+02 .900000E+01 .700000E+01 .500000E+01-.134000E+03
-.141000E+03-.480000E+02-.110000E+02-.180000E+02-.142000E+03 .900000E+01
-.270000E+02-.147000E+03-.582000E+03 .600000E+02 .140000E+02-.121000E+03
-.110000E+02 .000000E-99 .150000E+02 .200000E+02 .300000E+02-.282000E+03
-.170000E+02-.490000E+02-.120000E+02-.190000E+02-.141000E+03 .100000E+02
-.280000E+02-.148000E+03-.651000E+03 .700000E+02 .160000E+02-.133000E+03
-.900000E+01-.150000E+02 .000000E-99 .100000E+01 .200000E+02-.340000E+02
-.200000E+02-.520000E+02-.130000E+02-.200000E+02-.140000E+03 .110000E+02
-.290000E+02-.149000E+03-.781000E+03 .800000E+02 .150000E+02-.144000E+03
-.700000E+01-.200000E+02-.100000E+01 .000000E-99 .100000E+02-.400000E+02
.380000E+02-.150000E+02-.140000E+02-.210000E+02-.139000E+03 .120000E+02
-.300000E+02-.150000E+03-.811000E+03 .900000E+02 .170000E+02-.155000E+03
-.500000E+01-.300000E+02-.200000E+02-.100000E+02 .000000E-99-.760000E+02
```


<u>Comments on the Typewriter Log for Sample Problems</u>

Pages 42-50 include the typewriter log obtained when the program was used to solve the sample problems. The following should be noted:

(a) A tolerable zero value was entered for each one of the problems.

(b) Problems No. 1, 2, 4, and 6 each has an infinite number of solutions; SW3 was set to the ON position in order to obtain the general solution for each one of them.

(c) The tolerable Z value for Problem No. 2 was typed incorrectly; SW4 was set to the ON position, and the Z value was re-entered in the proper way.

(d) The tolerable Z value for Problem No. 4 was typed incorrectly and SW4 was left in the OFF position. The message ERROR F7 was typed and the program continued. According to the OPERATING INSTRUCTIONS, the program was stopped and re-initialized.

(e) Problems No. 5, 6, and 7 each required readjustment of the tolerable Z value. This was done according to instructions

Typewriter Log for Sample Problems

160001000000
LOAD DATA

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-05

SOLUTION FOR VECTOR 5

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X 3
-.10000000E+01
.10000000E+01

X 4
.00000000E-99
.10000000E+01

.30000000E+01
-.20000000E+01

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-0J
+.10000000E-04

SOLUTION FOR VECTOR 6

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X 5
-.87446120E+01
-.17472361E+01
.22111665E+01
-.15055279E+01

-.00000000E-99
-.00000000E-99
-.00000000E-99
.00000000E-99

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-03

SOLUTION FOR VECTOR 9
.99999970E-00
.19999998E+01
.29999999E+01
.40000002E+01
.49999990E+01
.60000010E+01
.69999996E+01
.79999996E+01

SOLUTION FOR VECTOR 10
.49999970E-00
.10000003E+01
.15000001E+01
.20000000E+01
.24999999E+01
.30000005E+01
.34999996E+01
.39999995E+01

SOLUTION FOR VECTOR 11
.29999987E+01
.59999994E+01
.89999980E+01
.11999999E+02
.14999995E+02
.18000000E+02

.20999998E+02
.23999996E+02

SOLUTION FOR VECTOR 12
.17373927E+02
.46417256E+01
-.11788599E+02
.23638003E+01
.11300876E+02
.99859570E-00
.12568545E+02
.75411273E+01

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-00
ERROR F7
4908000

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-03

SOLUTION FOR VECTOR 14

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 15

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 16

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 17

SYSTEM INCONSISTENT

SOLUTION FOR VECTOR 18

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X 13
-.89912030E-00
-.11049142E+01
.57970323E+02
-.94171420E+01
.14746840E-00
.17722926E+01
.75299710E+01
-.12344710E+02
.11761880E+01
.62717870E+01
-.35682707E+01
-.20166340E+01

.20000000E+01
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
.00000000E-99

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-03
RELOAD DATA INCR. Z BY FACTOR OF 10

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-02

SOLUTION FOR VECTOR 14
-.89017740E+01
-.48386103E+01
.41425661E+03
-.55480391E+02
.14276311E-00
.15499623E+02
.56783579E+02
-.99629510E+02
.10310057E+02
.47450099E+02
-.12369882E+02
-.26474642E+02
.91686745E+01

SOLUTION FOR VECTOR 15
-.77705800E+01
-.49968307E+01
.37400470E+03
-.51928873E+02
.33781753E-00
.15138997E+02
.49755229E+02
-.88164190E+02
.88469110E+01
.39042339E+02
-.89736869E+01
-.23229999E+02
.81447152E+01

SOLUTION FOR VECTOR 16
-.70374340E+01

-.44552765E+01
.33750718E+03
-.46798547E+02
.29713861E-00
.13666631E+02
.45032153E+02
-.79983800E+02
.79584935E+01
.34742553E+02
-.74102383E+01
-.20863395E+02
.74716143E+01

SOLUTION FOR VECTOR 17
-.68729740E+01
-.50068592E+01
.34873410E+03
-.50506716E+02
.45286507E-00
.12175005E+02
.47732087E+02
-.82814740E+02
.84018230E+01
.38005589E+02
-.95950557E+01
-.21404485E+02
.79367011E+01

SOLUTION FOR VECTOR 18
.96000000E-05
-.24577692E+01
.12894879E+03
-.20947452E+02
.32802864E-00
.39422583E+01
.16749622E+02
-.27459473E+02

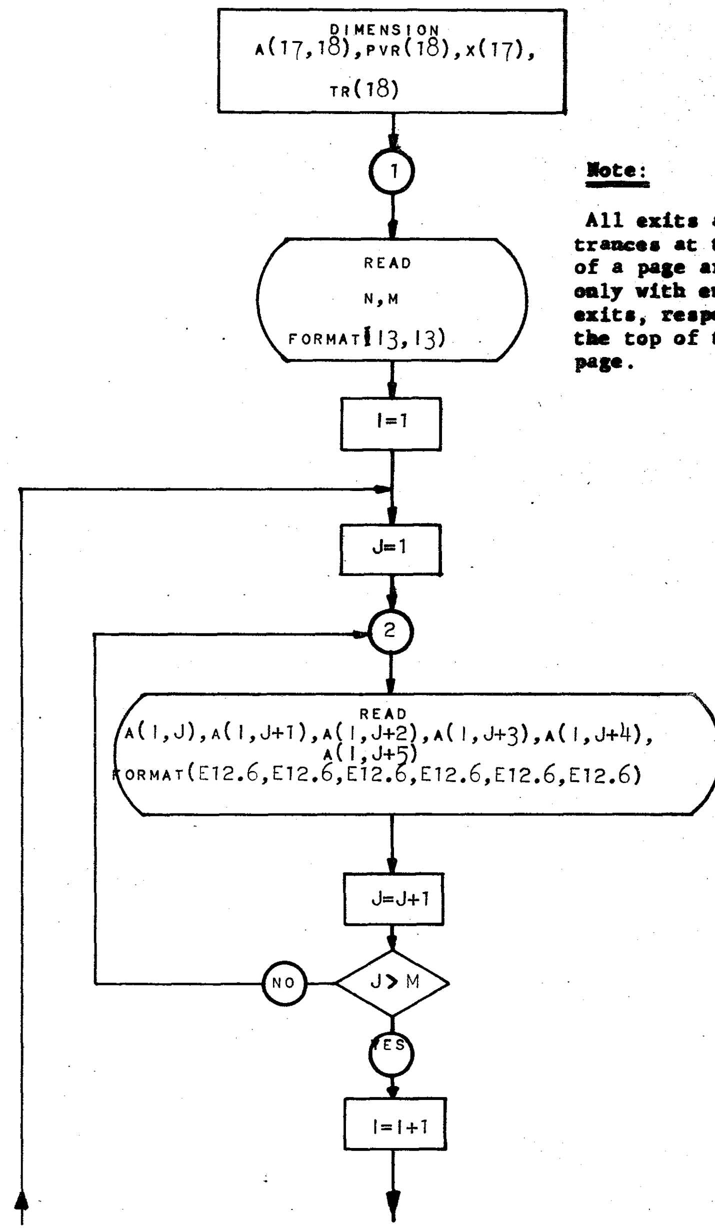
```
 .26163002E+01
 .13950905E+02
-.79372437E+01
-.44857705E+01
 .22243907E+01

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-03
RELOAD DATA INCR. Z BY FACTOR OF 10

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-02

SOLUTION FOR VECTOR 18

INF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.

X 17
-.42874995E-00
 .23976300E+01
-.28644234E+02
 .97720987E+01
-.61967767E-00
-.17527315E+01
-.82128261E-00
 .21299960E-01
-.25086718E-00
-.18757052E+01
 .35436282E+01
 .19503036E+01
 .43309011E-00
-.18223183E+01
 .10547266E+02
-.10317607E+02


 .20000000E+01
```



```
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
-.00000000E-99
 .00000000E-99

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-03
RELOAD DATA INCR. Z BY FACTOR OF 10

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-02
RELOAD DATA INCR. Z BY FACTOR OF 10

ENTER TOLERABLE Z IN E14.8 FORMAT SPEC.
+.10000000E-01

SOLUTION FOR VECTOR 18
 .91800000E-04
 .11183719E+02
-.13361065E+03
 .45581915E+02
-.28904901E+01
-.81754060E+01
-.38309755E+01
 .99337500E-01
-.11702277E+01
```

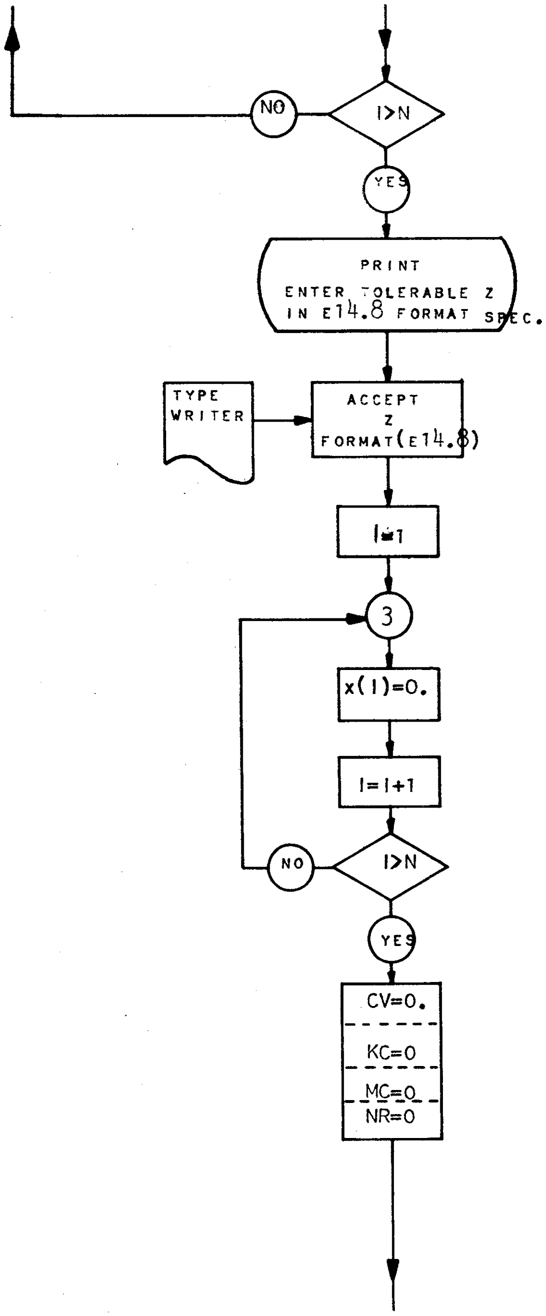
-.87497130E+01
.16529699E+02
.90972961E+01
.20201331E+01
-.85001620E+01
.49197987E+02
-.48126945E+02
.46646303E+01

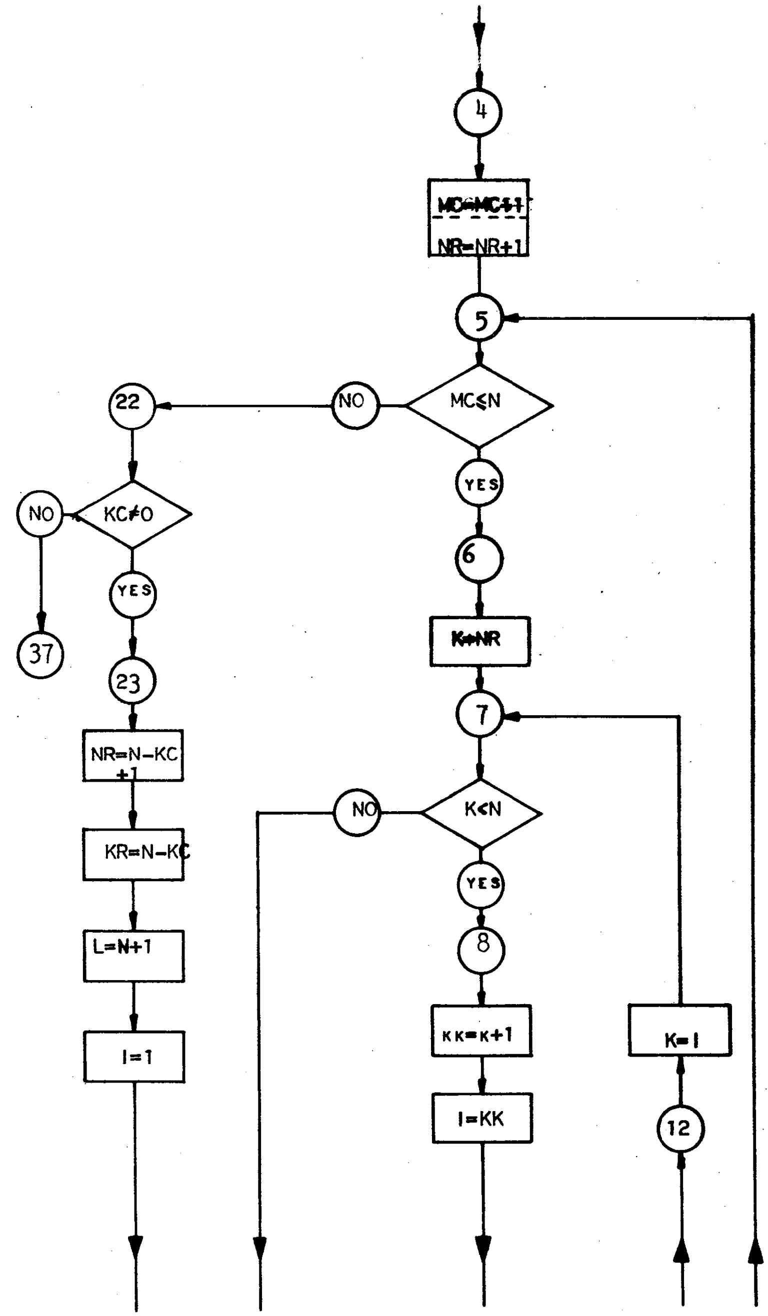
Detailed Flow Chart

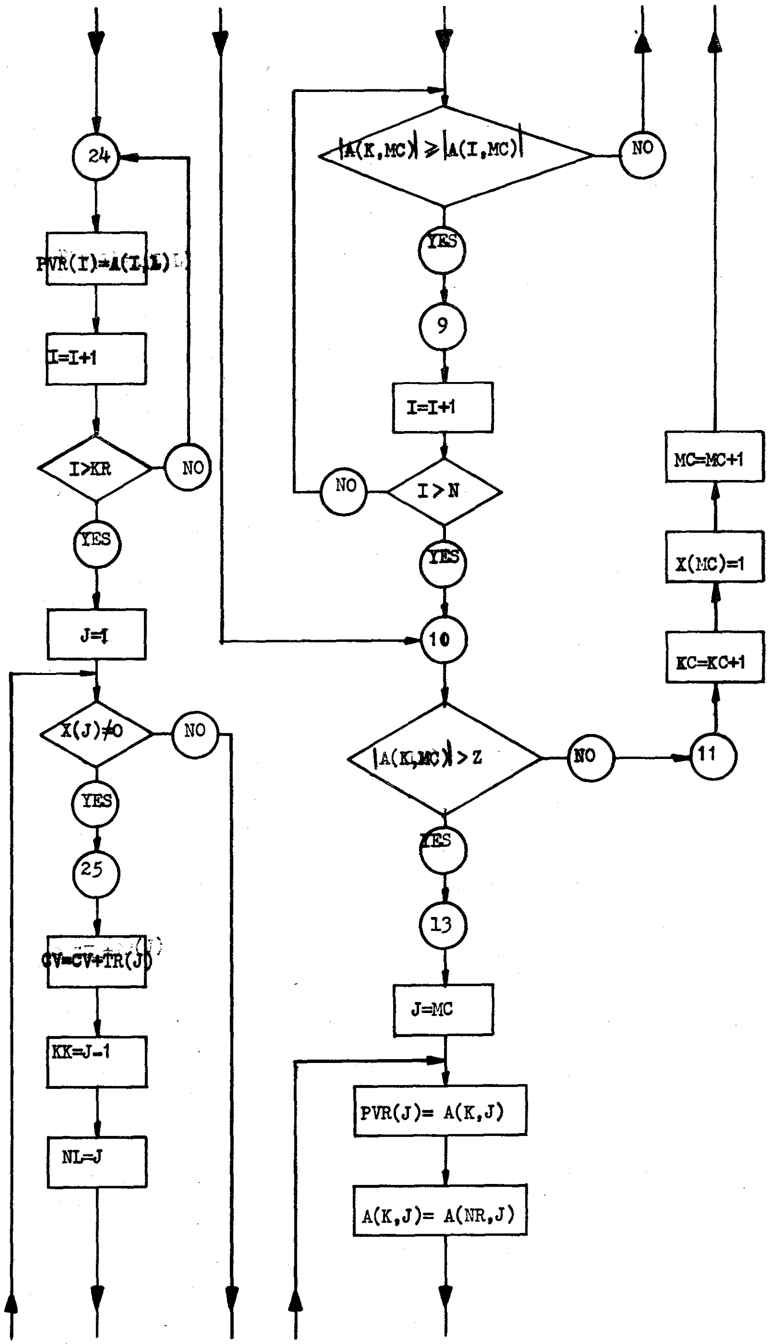


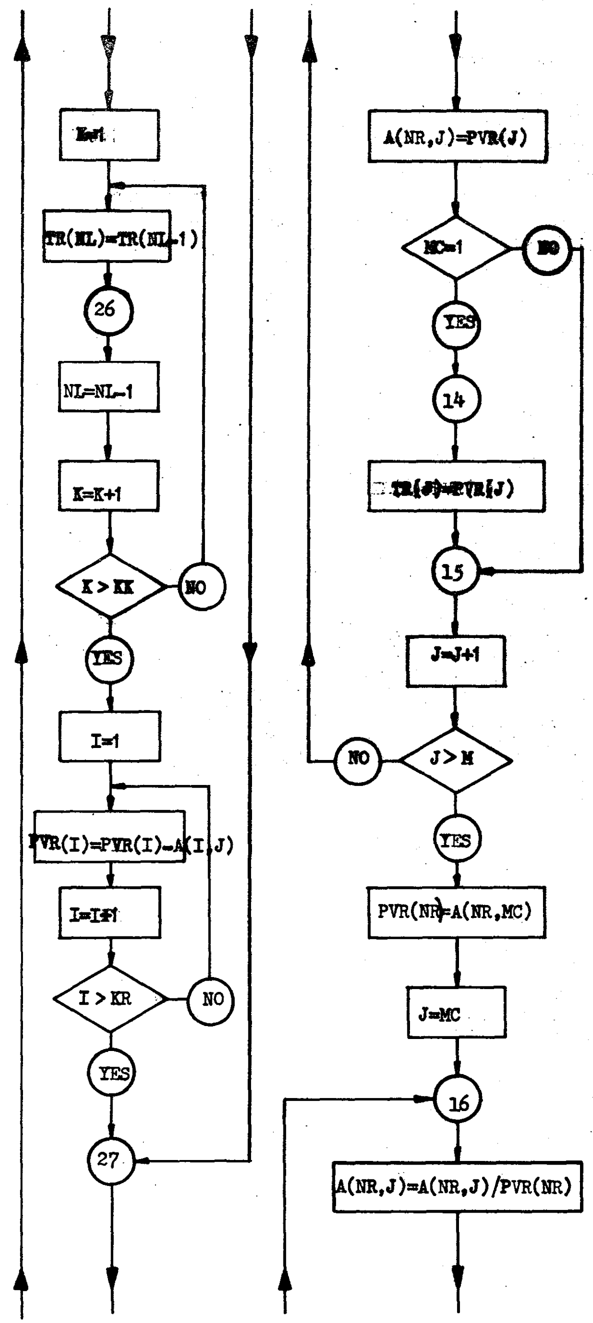
Note:

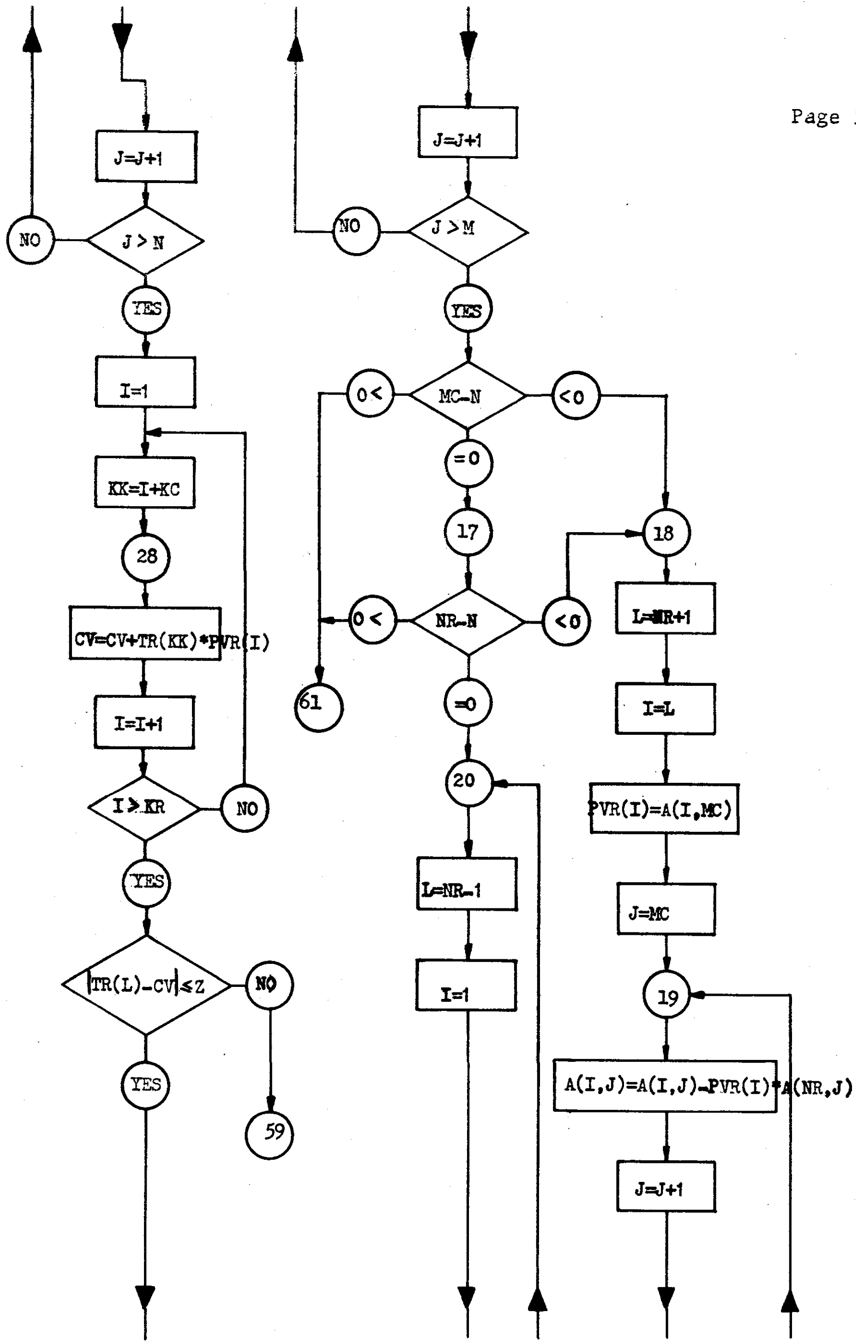
All exits and entrances at the bottom of a page are concerned only with entrances and exits, respectively, at the top of the following page.

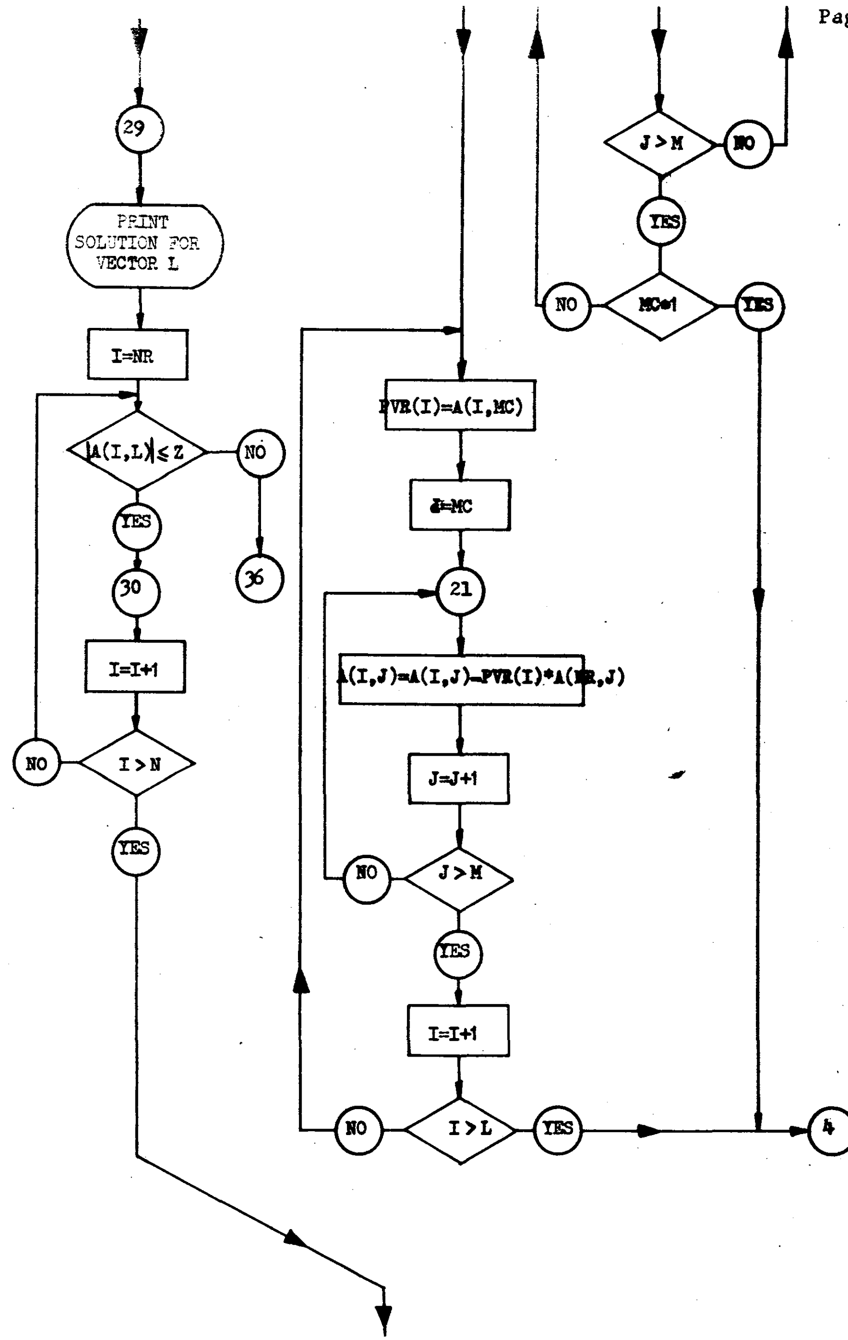

NO
I>N
YES
PRINT
ENTER TOLERABLE Z
IN E14.8 FORMAT SPEC.
TYPE
WRITER
ACCEPT
Z
FORMAT(E14.8)
I=1
3
x(I)=0.
I=I+1
NO
I>N
YES
CV=0.
KC=0
MC=0
NR=0

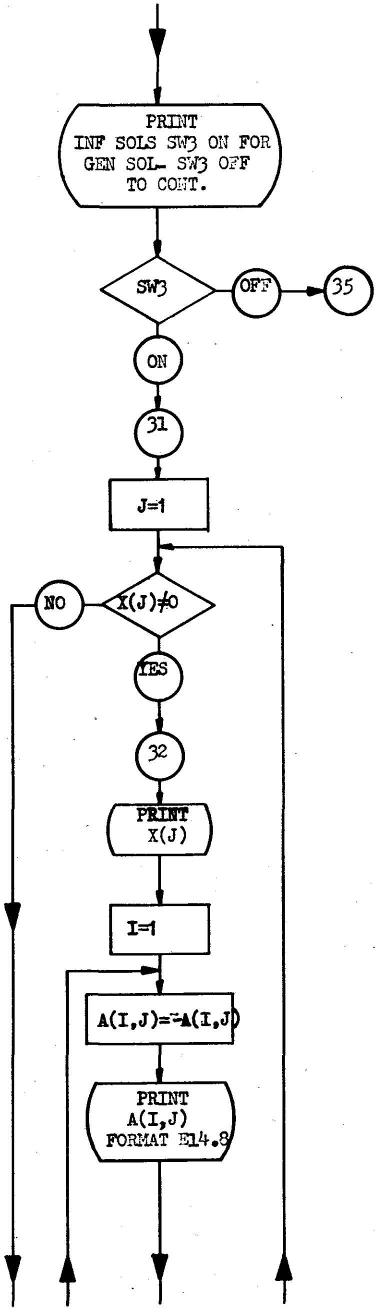

4
MC=MC+1
NR=NR+1
5
22
NO
MC≤N
NO
KC≠0
YES
6
37
YES
K=NR
23
7
NR=N-KC
+1
NO
K<N
KR=N-KC
YES
L=N+1
8
KK=K+1
K=I
I=1
I=KK
12

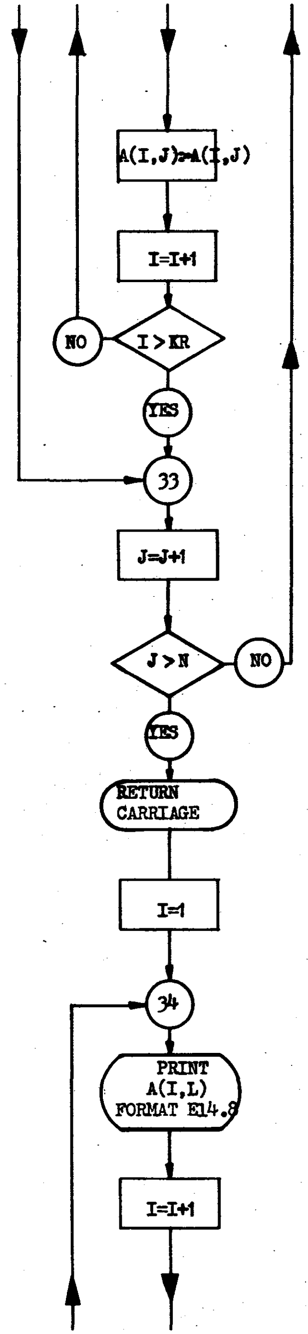

24
PVR(I)=A(I,L)
I=I+1
I>KR
NO
YES
J=1
X(J)≠0
NO
YES
25
CV=CV+TR(J)
KK=J-1
NL=J
|A(K,MC)| ≥ |A(I,MC)|
NO
YES
9
I=I+1
I > N
NO
YES
10
|A(K,MC)| > Z
NO
YES
13
J=MC
PVR(J)= A(K,J)
A(K,J)= A(NR,J)
11
KC=KC+1
X(MC)=1
MC=MC+1

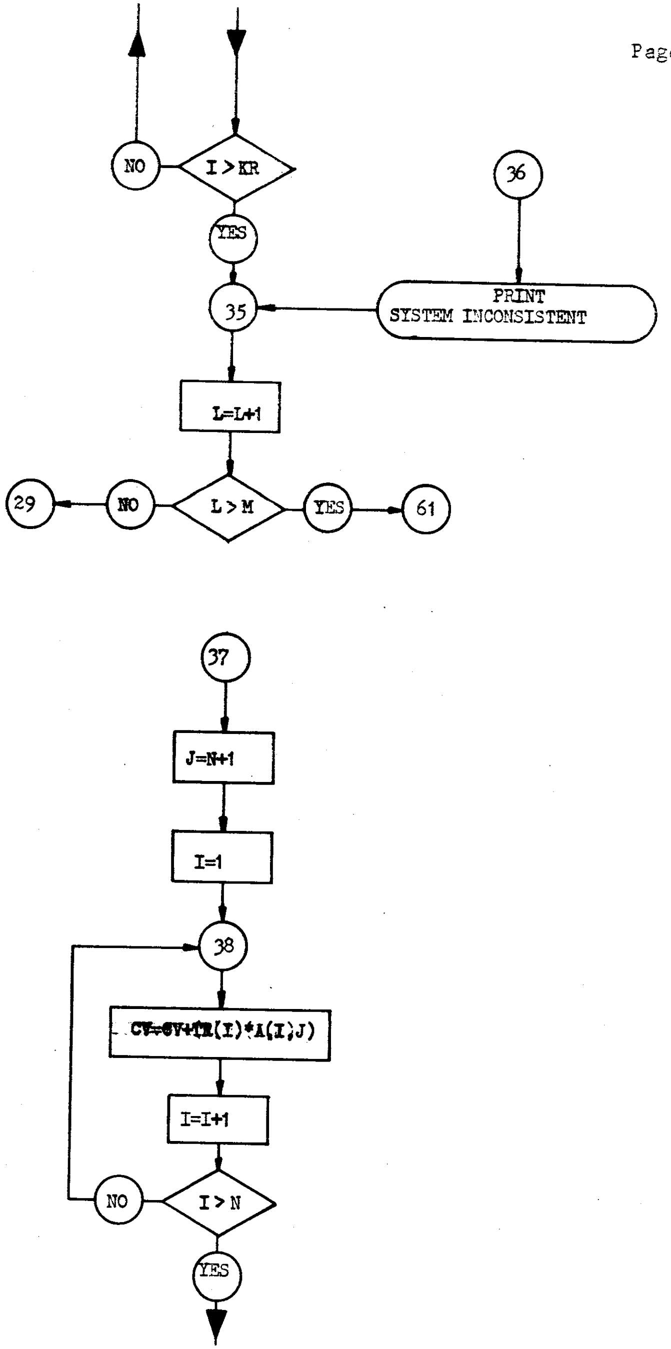

TR(NL)=TR(NL-1)
26
NL=NL-1
K=K+1
K > KK
NO
YES
I=1
PVR(I)=PVR(I)-A(I,J)
I=I+1
I > KR
NO
YES
27
A(NR,J)=PVR(J)
MC=1
NO
YES
14
15
J=J+1
J > M
NO
YES
PVR(NR)=A(NR,MC)
J=MC
16
A(NR,J)=A(NR,J)/PVR(NR)

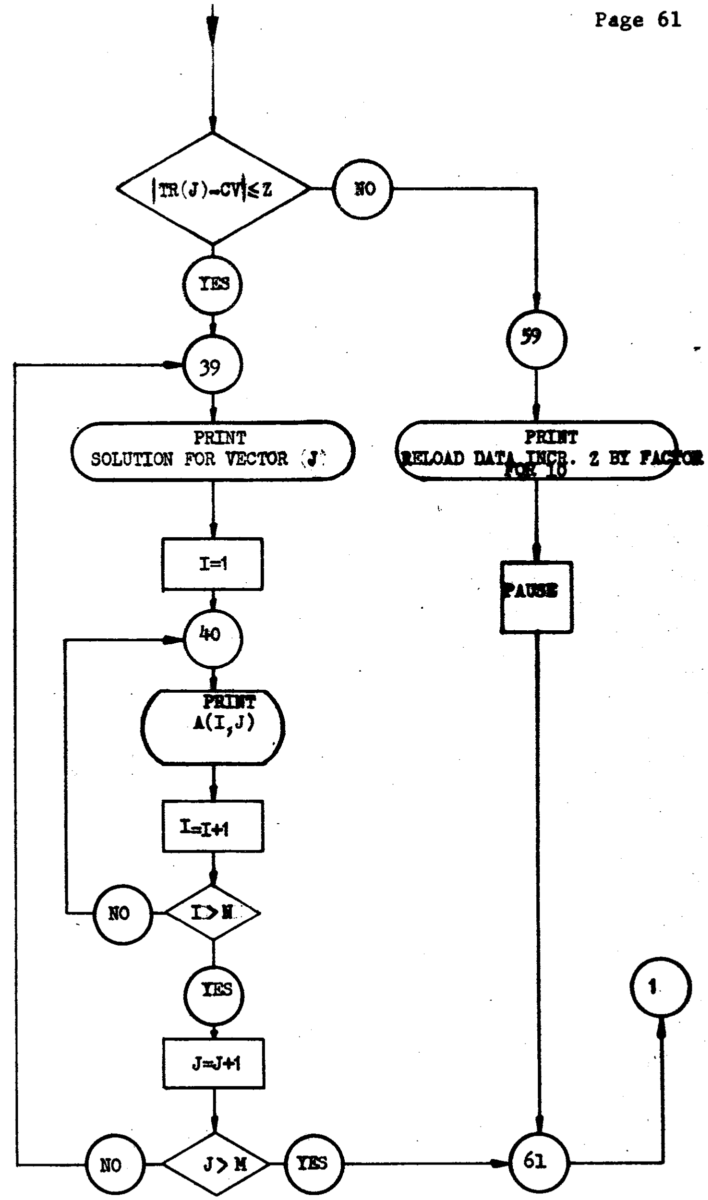

J=J+1
J > N
NO
YES
I=1
KK=I+KC
28
CV=CV+TR(KK)*PVR(I)
I=I+1
I > KR
NO
YES
|TR(L)-CV| ≤ Z
NO
YES
59
J=J+1
J > M
NO
YES
MC-N
0<
<0
=0
17
NR-N
0<
<0
61
=0
20
L=NR-1
I=1
18
L=NR+1
I=L
PVR(I)=A(I,MC)
J=MC
19
A(I,J)=A(I,J)-PVR(I)*A(NR,J)
J=J+1


29
PRINT
SOLUTION FOR
VECTOR L
I=NR
|A(I,L)| ≤ Z
NO
36
YES
30
I=I+1
I > N
NO
YES
J > M
NO
YES
MC=1
NO
YES
PVR(I)=A(I,MC)
J=MC
21
A(I,J)=A(I,J)-PVR(I)*A(NR,J)
J=J+1
J > M
NO
YES
I=I+1
I > L
NO
YES
4


PRINT
INF SOLS SW3 ON FOR
GEN SOL- SW3 OFF
TO CONT.
SW3
OFF
35
ON
31
J=1
NO
X(J)≠0
YES
32
PRINT
X(J)
I=1
A(I,J)=-A(I,J)
PRINT
A(I,J)
FORMAT E14.8


A(I,J)=-A(I,J)
I=I+1
NO
I > KR
YES
33
J=J+1
J > N
NO
YES
RETURN
CARRIAGE
I=1
34
PRINT
A(I,L)
FORMAT E14.8
I=I+1


NO
I > KR
36
YES
35
PRINT
SYSTEM INCONSISTENT
L=L+1
29
NO
L > M
YES
61
37
J=N+1
I=1
38
CV=CV+TR(I)*A(I,J)
I=I+1
NO
I > N
YES


|TR(J)-CV| ≤ Z
NO
YES
59
39
PRINT
SOLUTION FOR VECTOR (J)
PRINT
RELOAD DATA INCR. Z BY FACTOR FOR 10
I=1
PAUSE
40
PRINT
A(I,J)
I=I+1
NO
I > N
YES
1
J=J+1
NO
J > M
YES
61

Program Listing

```
08000 C  SOLUTION OF A SYSTEM OF N LINEAR EQUATIONS WITH N UNKNOWNS
08000 C  BY THE METHOD OF ELEMENTARY OPERATIONS.
08000 C  ENGINEER  *HEBERTO PACHON


08000       DIMENSION A(17,18),PVR(18),X(17),TR(18)
08000 1     READ 50,N,M
08036       DO 2 I=1,N
08048       DO 2 J=1,M,6
08060 2     READ 51,A(I,J),A(I,J+1),A(I,J+2),A(I,J+3),A(I,J+4),A(I,J+5)
08576       PRINT 52
08600       ACCEPT 53,Z
08624       DO 3 I=1,N
08636 3     X(I)=0.
08720       CV=0.
08744       KC=0
08768       MC=0
08792       NR=0
08816 4     MC=MC+1
08852       NR=NR+1
08888 5     IF(MC-N)6,6,22
08956 6     K=NR
08980 7     IF(K-N)8,10,10
09048 8     KK=K+1
09084       DO  9 I=KK,N
09096       IF(ABS(A(K,MC))-ABS(A(I,MC)))12,9,9
09320 9     CONTINUE
09356 10    IF(ABS(A(K,MC))-Z)11,11,13
09484 11    KC=KC+1
09520       X(MC)=1.
09568       MC=MC+1
```



```
09604       GO TO 5
09612 12    K=I
09636       GO TO 7
09644 13    DO 15 J=MC,M
09656       PVR(J)=A(K,J)
09764       A(K,J)=A(NR,J)
09908       A(NR,J)=PVR(J)
10016       IF(MC-1)15,14,15
10084 14    TR(J)=PVR(J)
10156 15    CONTINUE
10192       PVR(NR)=A(NR,MC)
10300       DO 16 J=MC,M
10312 16    A(NR,J)=A(NR,J)/PVR(NR)
10528       IF(MC-N)18,17,61
10596 17    IF(NR-N)18,20,61
10664 18    L=NR+1
10700       DO 19 I=L,N
10712       PVR(I)=A(I,MC)
10820       DO 19 J=MC,M
10832 19    A(I,J)=A(I,J)-PVR(I)*A(NR,J)
11168       IF(MC-1)20,4,20
11236 20    L=NR-1
11272       DO 21 I=1,L
11284       PVR(I)=A(I,MC)
11392       DO 21 J=MC,M
11404 21    A(I,J)=A(I,J)-PVR(I)*A(NR,J)
11740       GO TO 4
11748 22    IF(KC)23,37,23
11804 23    NR=N-KC+1
11852       KR=N-KC
11888       L=N+1
```

```
T1924       DO 24 I=1,KR
T1936 24    PVR(I)=A(I,L)
T2080       DO 27 J=1,N
T2092       IF(X(J))25,27,25
T2172 25    CV=CV+TR(J)
T2232       KK=J-1
T2268       NL=J
T2292       DO 26 K=1,KK
T2304       TR(NL)=TR(NL-1)
T2376 26    NL=NL-1
T2448       DO 27 I=1,KR
T2460       PVR(I)=PVR(I)-A(I,J)
T2604 27    CONTINUE
T2676       DO 28 I=1,KR
T2688       KK=I+KC
T2724 28    CV=CV+TR(KK)*PVR(I)
T2856       IF(ABS(TR(L)-CV)-Z)29,29,59
T2972 29    PRINT 54,L
T2996       DO 30 I=NR,N
T3008       IF(ABS(A(I,L))-Z)30,30,36
T3136 30    CONTINUE
T3172       PRINT 55
T3196       PAUSE
T3208       IF(SENSE SWITCH 3)31,35
T3228 31    DO 33 J=1,N
T3240       IF(X(J))32,33,32
T3320 32    PRINT 56,J
T3344       DO 33 I=1,KR
T3356       A(I,J)=-A(I,J)
T3512       PRINT 53,A(I,J)
T3596       A(I,J)=-A(I,J)
```



```
T3752 33    CONTINUE
T3824       PRINT 57
T3848       DO 34 I=1,KR
T3860 34    PRINT 53,A(I,L)
T3980 35    L=L+1
T4016       IF(L-M)29,29,61
T4084 36    PRINT 58
T4108       GO TO 35
T4116 37    J=N+1
T4152       DO 38 I=1,N
T4164 38    CV=CV+TR(I)*A(I,J)
T4332       IF(ABS(TR(J)-CV)-Z)39,39,59
T4448 39    PRINT 54,J
T4472       DO 40 I=1,N
T4484 40    PRINT 53,A(I,J)
T4604       J=J+1
T4640       IF(J-M)39,39,61
T4708 50    FORMAT(I3,I3)
T4736 51    FORMAT(E12.6,E12.6,E12.6,E12.6,E12.6,E12.6)
T4784 52    FORMAT(/39HENTER TOLERABLE Z IN E14.8 FORMAT SPEC.)
T4892 53    FORMAT(E14.8)
T4914 54    FORMAT(/19HSOLUTION FOR VECTOR,I3)
T4986 55    FORMAT(/44HINF SOLS SW3 ON FOR GEN SOL-SW3 OFF TO CONT.)
T5104 56    FORMAT(/1HX,I3)
T5140 57    FORMAT(/)
T5162 58    FORMAT(/19HSYSTEM INCONSISTENT)
T5230 59    PRINT 60
T5254 60    FORMAT(35HRELOAD DATA INCR. Z BY FACTOR OF 10)
T5348       PAUSE
T5360 61    GO TO 1
T5368       END
```

T̄9999 SIN
T̄9989 SINF
T̄9979 COS
T̄9969 COSF
T̄9959 ATAN
T̄9949 ATANF
T̄9939 EXP
T̄9929 EXPF
T̄9919 LOG
T̄9909 LOGF
T̄9899 SQRT
T̄9889 SQRTF
T̄9879 ABS
T̄9869 ABSF
T̄9859 RND
T̄9849 RNDF
T̄9839 A T̄6789
T̄6779 PVR T̄6609
T̄6599 X T̄6439
T̄6429 TR T̄6259
T̄6249 0̄001
T̄6239 0̄050
T̄6229 0̄050
T̄6219 N
T̄6209 M
T̄6199 0̄002
T̄6189 I
T̄6179 J
T̄6169 0̄051
T̄6159 0̄051
T̄6149 0̄052

T̄6139 0̄052
T̄6129 0̄053
T̄6119 0̄053
T̄6109 Z
T̄6099 0̄003
T̄6089 0̄00000009̄9̄
T̄6079 CV
T̄6069 KC
T̄6059 0̄000̄
T̄6049 MC
T̄6039 NR
T̄6029 0̄004
T̄6019 0̄00T̄
T̄6009 0̄00
T̄5999 0̄005
T̄5989 0̄006
T̄5979 0̄022
T̄5969 K
T̄5959 0̄007
T̄5949 0̄008
T̄5939 0̄010
T̄5929 KK
T̄5919 0̄009
T̄5909 0̄01
T̄5899 0̄012
T̄5889 0̄011
T̄5879 0̄013
T̄5869 T̄00000000̄1
T̄5859 0̄015
T̄5849 0̄014
T̄5839 0̄016

T̄5829 0̄018
T̄5819 0̄017
T̄5809 0̄061
T̄5799 0̄020
T̄5789 L
T̄5779 0̄019
T̄5769 0̄021
T̄5759 0̄023
T̄5749 0̄037
T̄5739 KR
T̄5729 0̄024
T̄5719 0̄027
T̄5709 0̄025
T̄5699 NL
T̄5689 0̄026
T̄5679 0̄028
T̄5669 0̄029
T̄5659 0̄059
T̄5649 0̄054
T̄5639 0̄054
T̄5629 0̄030
T̄5619 0̄036
T̄5609 0̄055
T̄5599 0̄055
T̄5589 0̄031
T̄5579 0̄035
T̄5569 0̄033
T̄5559 0̄032
T̄5549 0̄056
T̄5539 0̄056
T̄5529 0̄057

T̄5519 0̄057
T̄5509 0̄034
T̄5499 0̄058
T̄5489 0̄058
T̄5479 0̄038
T̄5469 0̄039
T̄5459 0̄040
T̄5449 0̄060
T̄5439 0̄060

Maintenance

I. Core Layout

The program listing and the symbol table show the storage addresses for the statements, variables, subroutines, and constants used in conjunction with the program. It is noted that

(a) Program starts at location 08000

(b) The five digit address on the left of each source statement is the starting address in core storage for instructions compiled for that statement.

(c) The address 15368 associated with the END statement is the first available location after the last program instruction, excluding subroutines.

(d) Sixteen ten digit fields are located in positions 19840 through 19999. The alphabetic representation of the names of the eight relocatable subroutines, in the two forms allowed--one with and one without the terminal F--are stored in the 16 fields.

(e) Unsubscripted variables used in the source program appear as

Field address   Variable name

(f) Subscripted variables appear as

Field address first element	Variable name	Field address last element

(g) Floating point constants appear as

Field address   constant (eight digit mantissa and two digit exponent)

(h) Fixed point constants appear as

Field address   constant (four digits right justified)

(i) Statement numbers appear as

Field address of compiled branch	Statement number

Fields are addressed from their units position (high core position).

The first 8000 positions include the multiply-add tables, and the arithmetic and input/output subroutines together with the work areas they require.

II. Method of Incorporating Changes

If the programmer wishes to use this program for a system of equations that do not require the maximum storage, he may economize core storage by reducing the dimensioned areas A(17,18), PVR(18), X(17), TR(18) appearing in the first source program statement.

The following rules should be observed:

(a) Determine the number N

N = number of equations = number of unknowns

(b) Determine the number M

M = number of equations plus number of constant vectors for which solutions are desired

(c) Adjust the DIMENSION statement according to the table below:

Value of M	DIMENSION STATEMENT
$M \le 6$	DIMENSION A(N,6), PVR(6), X(N), TR(6)
$6 < M \le 12$	DIMENSION A(N,12), PVR(12), X(N), TR(12)
$12 < M \le 18$	DIMENSION A(N,18), PVR(18), X(N), TR(18)

This method of reducing core storage was used in conjunction with a problem which required the solution of a system of five equations with five unknowns and one constant vector. The specifications further required a repeated solution of the system for each of the coefficient matrices resulting from the change of a variable parameter affecting the matrix elements.

To accomplish this problem, the DIMENSION statement was adjusted according to the above specifications, and the loop for reading the coefficient elements (DO 2 I = 1,N; DO 2 J = 1,M; 2 READ 51, A(I,J), etc., etc.,....) was replaced by a recursive routine which would generate the coefficient matrix elements, each time the variable parameter changed. The program would then evaluate the unknowns for each new system so generated.

III. Hash Totals

Two hash totals obtained by means of the CARD HASH TOTAL program written by Mr. William G. Weideman of the Marquette University Computer Center are listed below:

(1) Hash total for the SOURCE PROGRAM deck statements as listed in this write-up is

2838182384421190679$\bar{3}$

(2) Hash total for the PROGRAM DECK, consisting of 582 cards, is

5331348202955637644$\bar{2}$