Linear Programming – Meat Blending

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Data Processing Application

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INTRODUCTION

Linear programming is a powerful analytical tool that can provide management with more detailed and precise decision-making information. In recent years, the introduction of linear programming (LP) has produced outstanding benefits in many industries — notably those that involve the blending of ingredients to manufacture a finished product. This manual demonstrates the application of linear programming in the meat-packing industry. The nature of the industry, the mechanics of blending, and the basic concepts of the LP model formulation are discussed.

By the use of linear programming, the meat packer can determine the specific allocation of ingredients required to produce a given blended product at minimum cost — subject to any stated restrictions on blend composition and ingredient availability. The advantages of linear programming pervade all areas of plant operation and can significantly improve profits in an industry noted for fierce competition and narrow margins. The immediate and more obvious LP results enable the meat packer to:

- Minimize the cost of blended products
- Maximize the use of available ingredients
- Reduce ingredients inventory and waste
- Minimize off-standard blends
- Supply accurate purchasing information
- Ensure a more uniform product

The basis of the LP technique is the formulation of a mathematical model of the allocation problem. For problems of any practical size, this model is entered into a computer, and the computer LP system rapidly calculates the optimal (least-cost) solution. The system may also produce reports which indicate the effect on the optimal solutions of possible changes in the given prices, availabilities, etc.

Contrary to popular belief, little mathematical knowledge or skill is required to formulate an LP model. Nor do the operation of the computer and analysis of computer results require any advanced technical skill. Linear programming requires nothing more than the expression of all elements in the process-blending and processing effects, ingredient compositions, blend specifications, etc. — in the form of simple linear equations or inequalities. The general principles of linear programming are discussed in the IBM data processing application manual An Introduction to Linear Programming (E20-8171), which should be read in conjunction with this manual.

To make use of a linear programming system, the meat packer must perform two basic tasks:

1. Define the problem. Essentially this consists of deciding what function is to be optimized (typically, as in this manual, total material cost of production is to be minimized) and what products and batch quantities are to be blended.

2. Collect data and formulate the model. Interrelations among ingredients, restrictions on the final composition of the blended product, and any other limitations (such as inventory, blending and storage capacities, etc.) must be expressed in the form of linear equations (or inequalities). The relevant data, including ingredient costs, can usually be obtained from sources within the plant, such as the quality control laboratory, the sausage maker, the plant supervisor, and the data processing department. It is emphasized that a good laboratory is a necessity to supply chemical composition of the meat ingredients, and the full support of the sausage maker should be solicited in the collection of data to formulate the model.

In the following sections, the meat blending problem will be analyzed in detail, and formulation of the LP model will be described. A complete sample problem will then be presented, including an analysis of the solution and output reports.

PROBLEM PROFILE AND ECONOMICS

Blended meat products are made from beef, pork, and other meat trimmings that are ground and chopped by machinery and then seasoned and spiced to suit the trade for which the product is intended. The finished preparation, in a semiliquid form, is automatically stuffed into a specially prepared casing and then cooked, smoked, and chilled. Blended products include frankfurters, various types of sausages, bologna, minced ham, and many varieties of sandwich meats. Each of these products is blended according to a specific recipe which is governed by the availability of ingredients, consumer tastes, and federal, state, and local restrictions. The basic problem of the meat-packing industry is to meet the requirements of these recipes and to do it at a minimum cost.

The costs of meat blending are affected by many areas of company operation and may vary from plant to plant. The concept of costs may differ between a company that has its own slaughterhouse and one that purchases all its raw ingredients. A company that sells ingredients, dressed or otherwise, must often weigh the profit of such sales against the cost of internal use and the profit from the sale of manufactured products.

Changes in market prices of livestock — due to variations in livestock production and shipments — may call for frequent revision of ingredient costs. The determination of costs can be a complex operation in which such major factors as the following may be considered:

- Purchase price
- Transportation cost
- Handling cost
- Inventory and refrigeration costs
- Sales and purchasing costs
- Spoilage
- Losses in the blending process

Usually, not all of these factors will be used to determine ingredient cost. One simple technique is to use current market price, either alone or modified by a fixed handling charge.

Once ingredient costs have been determined, the problem is to find the most economical blend using these costs. Historically, blend formulas have been calculated manually (using a desk calculator), based upon the standard blend recipes, a knowledge of the ingredients available in inventory, experience, and intuition. Using only these tools it is relatively easy to calculate a formula that meets all the restrictions on the blend and inventory, but it is another matter to be certain that this formula represents the least-cost solution.

Given enough time, a competent worker with a desk calculator could find the most economical blend. However, the calculation is tedious, and the time involved, even for small problems, is prohibitive; therefore, it is usually impossible from a practical standpoint to explore manually all (or even a large number) of the possible blends and their associated costs. Thus, when the sausage maker is developing a formula, time permits him to compute cost on only a few formulas, and he cannot be sure of finding the one with minimum cost. Also, to simplify the computation, he tends to take mathematical shortcuts which may introduce uncontrolled error into the final blend.

To illustrate the problems of manual calculation, assume the need to blend 100 lbs. of a meat product whose recipe calls for four ingredients in any combination, limited by a specific set of restrictions. The ingredients are bull meat, cow meat, pork trimmings, and fat pork. The restrictions are on fat and protein. Figure 1 lists the composition and price of each of the ingredients, and Figure 2 lists the restrictions.

Cost/lb. ($)	Ingredient	Fat %	Protein	Moisture	Ash
0.470	Bull meat	8	20	71	1
0.450	Cow meat	15	18	66	1
0.30	Pork trimmings	50	10	39	1
0.11	Fat pork	70	5	24	1

Figure 1. Ingredient composition and price

Fat minimum	24.0%
Fat maximum	28.0%
Protein minimum	14.0%

Figure 2. Formula restrictions

A first intuitive solution might be to use as many pounds as possible of the three cheapest ingredients. This might lead to the use of cow meat, pork trimmings, and fat pork. A mix consisting of 50 lbs. of cow meat, 45 lbs. of pork trimmings and 5 lbs. of fat pork will give 13.75% protein and 33.5% fat. This mix analysis is shown in Figure 3, and the calculations are as follows:

Ingredient	Weight (lbs.)	Fat (lbs.)	Protein (lbs.)	Cost ($)
Cow meat	50.0	7.5	9.0	22.50
Pork trimmings	45.0	22.5	4.5	13.50
Fat pork	5.0	3.5	0.25	0.55
	100.0	33.5	13.75	36.55

Figure 3. Mix 1 analysis

For fat:

50 lbs. cow meat X 15% fat = 7.5 lbs. fat

45 lbs. pork trimmings X 50% fat = 22.5 lbs. fat

5 lbs. fat pork X 70% fat = 3.5 lbs. fat

and, for protein:

50 lbs. cow meat X 18% protein = 9 lbs. protein

45 lbs. pork trimmings X 10% protein = 4.5 lbs. protein

5 lbs. fat pork X 5% protein = 0.25 lb. protein

Since the protein content was too low and the fat content too high in the first mix, it would appear logical to replace all of the fat pork and 5 lbs. of pork trimmings with cow meat. As the analysis in Figure 4 shows, the protein content increased to 14.8%, which is acceptable, but the fat content has decreased only to 29%, which is still not acceptable. In addition, the cost has greatly increased.

Ingredient	Weight (lbs.)	Fat (lbs.)	Protein (lbs.)	Cost ($)
Cow meat	60.0	9.0	10.8	27.00
Pork trimmings	40.0	20.0	4.0	12.00
	100.00	29.0	14.8	39.00

Figure 4. Mix 2 analysis

Another alternative might be to increase the cow meat to 70 lbs. and to use 15 lbs. of pork trimmings and 15 lbs. of fat pork. The analysis in Figure 5 shows that the protein restriction is satisfied but the fat content is 28.50%, which is still not acceptable. (In practice, this last blend might be used, since it is "close" to meeting the restrictions.)

Ingredient	Weight (lbs.)	Fat (lbs.)	Protein (lbs.)	Cost ($)
Cow meat	70.0	10.50	12.60	31.50
Pork trimmings	15.0	7.50	1.50	4.50
Fat pork	15.0	10.50	.75	1.65
	100.0	28.50	14.85	37.65

Figure 5. Mix 3 analysis

This simple blending problem has involved only four ingredients and three restrictions; yet, after three calculations a completely feasible solution has not been found, to say nothing of the least-cost feasible solution. If the number of ingredients or restrictions or both were increased, the number of alternative solutions to be evaluated would also increase, and at an alarming rate. It is not uncommon in the meat-packing industry to have 25 to 45 different ingredients to select from and to impose 10 to 15 restrictions on a blend for one product. This problem becomes even more complex when limited availability of ingredients and price changes are introduced.

Solution of this problem by a computer LP system produced an optimal formula which called for 76.4 lbs. of cow meat and 23.6 lbs. of fat pork at a total cost of $36.98. The resulting mix analysis is shown in Figure 6.

The computer-prepared formula satisfies all of the imposed restrictions. Further, the linear programming technique guarantees that there is no other feasible formula with a lower cost.

Ingredient	Weight (lbs.)	Fat (lbs.)	Protein (lbs.)	Cost ($)
Cow meat	76.4	11.46	13.75	34.38
Fat pork	23.6	16.52	1.18	2.60
	100.0	27.98	14.93	36.98

Figure 6. Analysis of LP solution mix

This simple problem also illustrates the importance of having accurate data on the composition of the various ingredients. A small inaccuracy in the given percentages of fat and protein in the ingredients would be multiplied in the final solution, which might then violate some restriction. Thus, the importance of testing the product ingredients cannot be overemphasized. A fully equipped laboratory which can analyze the ingredients on a continuing basis is absolutely necessary.

MODEL FORMULATION - SINGLE PRODUCT

A linear programming model is a mathematical representation of all known and estimated factors that define the problem to be solved. The mathematics involved is limited to expressing these factors as a set of linear equations and inequalities. The input data required to construct the model consist of the following:

- All restrictions on the blend recipe composition (percentage fat, protein, etc.)
- All restrictions on ingredients in the recipe (permissible combinations, etc.)
- Cost of each possible ingredient
- Composition of each possible ingredient
- Blending capacities
- Availability of ingredients

These data are obtained from the laboratory, from accounting, and from purchasing. Where exact information is not available, educated estimates should be made. These estimates can be based on experience, historical data, or industry surveys. As a simple example, the sausage maker knows that the amount of beef and pork cheek meat must be limited to a relatively small portion of the total meat in a frankfurter formulation, usually 20% or less. At the present time there is no precise research data to indicate the maximum amount of cheek meat which can be used under specific conditions, nor exactly why there must be a limit imposed. Thus, the estimate of the sausage maker must be used.

The quality and appearance of the blended meat product depend on the recipe and the restrictions imposed upon it. These restrictions define the composition of the manufactured product in terms of fat, protein, and water.

The use and proportions of individual ingredients also may be controlled, for various reasons:

- Oversupply or unavailability
- Contribution of ingredient to color and taste
- Special company policies or conditions (for example, the use of plant by-products)
- Government regulations
- Consumer requirements
- Cost

The per-pound cost of each ingredient in the recipe is multiplied by the quantity used, to evaluate the total cost. This is the figure that will be minimized in selecting the optimal formula. The manner in which ingredient cost data is obtained has already been discussed under "Problem Profile and Economics".

The fat, protein, and water composition of each ingredient must be known in order to calculate the composition of the optimal recipe and ensure its agreement with the restrictions. We have already discussed the importance of having accurate data on ingredient compositions.

The blending capacity is the amount of product to be blended. It can be limited by manpower or machinery factors in the plant. It can also be limited by a sales forecast, in-house orders, or the finished product inventory capacity of the plant.

CONSTRAINTS

The basic elements of a linear programming model are the equations or inequalities that express the restrictions or limitations on the problem solution and relate the variables in the problem. These equations or inequalities are called constraints. The four major categories of constraints in a meat blending model are:

- Cost constraint (objective function)
- Ingredient constraints
- Composition constraints
- Capacity constraints

A model will invariably contain at least one of each constraint type. It is imperative that every separate relation and restriction be incorporated in the LP model as an individual constraint. The formulation of these constraints is discussed in the following sections.

Cost Constraint (Objective Function)

The cost constraint, or objective function, is an equation which expresses the total cost of the recipe. The objective function has the following form (assuming that there are n different ingredients which may be used in the recipe):

C1 X M1 + C2 X M2 + C3 X M3... + Cn X Mn = Minimum Cost

where M1 is the weight used of ingredient 1, M2 is the weight used of ingredient 2, etc., and C1, ..., Cn are the respective per-pound costs of each ingredient. The per-pound costs (C1, ..., Cn) are constant coefficients supplied as input data in the model. The ingredient weights (M1, ..., Mn) are unknowns to be computed. (As we have already seen, an optimal recipe might not use all of the possible ingredients.)

The LP system computes the optimal recipe by solving for a set of ingredient weights which, while satisfying all other constraints in the model, will also yield the lowest possible total value for the objective function.

To illustrate, let us take a problem in which the model includes four possible ingredients (n = 4) with the following costs:

C1 = $0.10/lb. C3 = $0.11/lb.
C2 = $0.22/lb. C4 = $0.40/lb.

Assume that the LP system computes the following optimal formula (which satisfies all constraints in the model):

M1 = 10 lbs. M3 = 0 lbs.
M2 = 88 lbs. M4 = 2 lbs.

The total cost is calculated by substituting the above values in the equation, as follows:

(10) (0.10) + (88) (0.22) + (0) (0.11) +
(2) (0.40) = $21.16

Thus, $21.16 would be the lowest possible cost for a formula that satisfies the constraints of this problem.

Ingredient Constraints

Ingredient constraints are used to control the amount of an ingredient (or combinations of ingredients) in the mix. Such constraints can establish a maximum, a minimum, a fixed value, or a specific range for the ingredients. An ingredient might be limited by some maximum because of its poor texture characteristic, or an ingredient such as "rework" (product which was physically damaged in processing) may be forced into the formula because of no other method of using the product. It might also be necessary to control ingredients because of taste, color, or binding restrictions. Ingredients can be constrained singly, in pairs, or in any other possible combination.

The various forms of ingredient constraints are illustrated below.

1. Single equality:
   Dry ingredients = 6

or, 6 lbs. of dry ingredients (salt, corn syrup, cure, seasoning, etc.) must be used regardless of the meats which are used.

2. Multiple equality:
   Beef hearts + pork hearts = 10

or, a total of 10 lbs. of beef hearts and/or pork hearts must be used, in any proportion - for example, if beef hearts = 2 lbs., pork hearts = 8 lbs.; if beef hearts = 0 lbs., pork hearts = 10 lbs.

3. Inequalities:
   Bull meat + cow meat + beef navels ≥ 40 or,

at least 40 lbs. of bull meat, cow meat, and/or beef navels must be used.

50-50 pork trim + skinned pork jowls ≥ 40 or, at least 40 lbs. of 50-50 pork trim and/or skinned pork jowls must be used.

The above example of constraints would be used if bull meat, cow meat, beef navels, 50-50 pork trim and skinned jowls were the only meat available. In this case the label of the product should read "beef and pork", since meat inspection regulations require that at least 40% of the meat must be beef and 40% of the meat must be pork.

Another example: Beef cheek meat + pork cheek meat ≤ 20 or, no more than 20 lbs. of beef cheek meat and/or pork cheek meat may be used.

4. Bounding inequalities:
   Mutton ≥ 5
   Mutton ≤ 30

or, mutton must be no less than 5 lbs. and no greater than 30 lbs. This type of bounding inequality might be used if a label listed mutton; thus, some mutton must be used and the policy of the plant is that no more than 30% of the product can be composed of mutton.

Composition Constraints

Composition constraints are used to specify the required final composition of the mix. The right-hand side of such constraints reflects the minimum or maximum on a given component such as fat or protein. The left-hand side is a series of terms representing the percentage of this component in each ingredient (known from laboratory analysis) and the weight of each ingredient used in the optimal formula (to be computed).

Assume, for example, that a recipe may contain cow meat, 50-50 pork trim, skinned jowls, and 80% lean pork trim. The percentage of fat content in these ingredients is 10%, 50%, 70% and 20%, respectively, and the fat restriction on the final mix is that it must contain no more than 30% fat. The following constraint inequalities then express this restriction:

0.10 (lbs. of cow meat used) + 0.50 (lbs. of 50-50 pork trim used) + 0.70 (lbs. jowls used) + 0.20 (lbs. lean pork trim used) ≤ 30.

Next, assume that the computed optimal formula contains ingredients in the following amounts:

Cow meat = 40 lbs.
50-50 pork trim = 13 lbs.
Skinned jowls = 14 lbs.
80% lean pork trim = 27 lbs.

Substituting these amounts into the left-hand side of the constraints yields: 0.10(40) + 0.50(13) + 0.70(14) + 0.20(27) = 25.7, which is less than 30 lbs. and, therefore, satisfies the fat restriction. Constraints similar to the above must be formulated for all composition restrictions such as protein, moisture, color, bind, etc.

Capacity Constraint

A capacity constraint is incorporated in the model to limit the quantity of mix to the capacity of the manufacturing system. The constraint has the following form:

M1 + M2 + M3 + M4 ... Mn = Maximum Capacity where M1, M2 ..., Mn are the actual weights of the n possible ingredients used in the mix.

For example, assume that a system capacity of 650 lbs. is specified (by the right-hand side of the equation). The individual weights of the ingredients in the optimal solution will then be computed so that their total equals 650 lbs. However, there are many advantages to specifying the system capacity at 100 lbs. of finished product. The total finished product weight can then be read as 100%, and the ingredient weights in the solution can be read as percentages. The calculated total pounds of fat, protein, moisture, etc., of the mix will equal the chemical analysis, which is routinely expressed in percent. In addition, the cost of the mix is given as dollars per hundredweight of finished product, which usually saves additional manual calculation in other departments of the plant.

The weight or percentage of each ingredient used in the mix can then be multiplied by a constant factor to obtain the actual ingredient weights needed for the desired total quantity of blend.

Moisture Constraint and Shrinkage Evaluation

The difficulty encountered in setting up moisture and shrinkage constraints for meat blending is partly due to the fact that although management, the sausage maker, and laboratory and computer personnel are using the same terms, each has defined them differently. There are basically three methods of determining the constraints to be used; only one of these will be discussed in this manual. It is the simplest to understand and use and has many advantages over the other methods, especially for a model formulated for a single product.

In determining the constraints to be used, it will be assumed that the capacity constraint is equal to 100 lbs. Most important is the fact that this 100 lbs. is based on the actual pounds of finished product (after cooking and chilling, ready to be packaged). A simple method of adjusting for shrinkage (loss of moisture in cooking, chilling, etc.) is discussed below.

One of the various government restrictions on cooked blended meat products is that the moisture in the finished product shall not exceed four times the percentage of meat protein plus 10% of the finished weight. However, in actual practice most meat ingredients have protein to moisture ratios of less than four times percent protein, while a few meat ingredients have protein to moisture ratios greater than four times percent protein. For example, the protein to moisture ratio for bull meat listed in Figure 15 is 3.48 (percent moisture divided by percent protein, 70.00/20.1 = 3.48), while beef navels (ingredient 4 of Figure 15) has a protein ratio of 4.03, or 39.1/9.7 = 4.03.

In setting up the moisture constraint, it is necessary to calculate a moisture coefficient for each ingredient because of differences among the various ingredients in their protein to moisture ratios. This moisture coefficient can be easily calculated for each ingredient by the following expression: % actual moisture of the meat - 4 times the % actual protein of the product. In the case of the bull meat given above, the moisture coefficient would be -.104, or (0.70-4(0.201) = -.104). For beef navels the moisture coefficient would be +.003, or (0.391-4(.097) = +.003). If bull meat and beef navels were the only ingredients available, the moisture constraint inequality would be: -.104 (lbs. of bull meat) + .003 (lbs. of beef navels) $\leq$ 10. This expression states that the maximum moisture permitted is 10 + .104 lbs. of water for each pound of bull meat used (to be computed) -.003 lbs. of water for each pound of beef navels used (to be computed).

For example, assume that the computed optimal formula contained bull meat, beef navels, and water in the amounts shown in Figure 7. The calculations (determined to the third decimal) given below prove that neither the maximum moisture restriction nor the governmental restriction that the maximum moisture cannot exceed four times percent protein plus 10% of the finished weight of the product were violated.

Ingredient	Weight (lbs.)	Protein (lbs.)	Moisture (lbs.)
Bull meat	45.40	9.125	31.780
Beef navels	40.00	3.880	15.640
Water	14.60	0.000	14.600
	100.00	13.005	62.020

Figure 7. Mix analysis of computed formula using moisture constraint

The maximum amount of water that could be added is:

.104(45.5 lbs. of bull meat) + -.003(40.0 lbs. of beef navels) + 10 = 14.60, which is the amount used.

The maximum amount of moisture that governmental restrictions permit in the finished product is:

4(13.005% protein) + 10% = 62.02%

Thus, the maximum amount of water was used (14.60 lbs.). However, the federal regulation was not violated.

It is emphasized that the above formula is for the actual content of the finished product ready for sale. In this case, each 100 lbs. of finished product contained 45.4 lbs. of bull meat, 40.0 lbs. of beef navels, and 14.6 lbs. of added water.

Shrinkage is the loss of moisture from the time of chopping until the product is ready to be packaged. A few of the many factors that affect the amount of shrinkage in processing are temperature, relative humidity, time in the smokehouse, the internal temperature at which the product is processed, and the length of time the product is held before packaging. The easiest method of adjusting for shrinkage, especially when the LP model is for a single product, is not to add a shrinkage factor in the LP model but to add manually to the formulation computed the amount of water that is lost in processing. For example, if results of shrinkage studies showed that 12 lbs. of water was lost in processing the product with the formula presented, then 12 lbs. of water would be added at the chopper. The formula would then be 45.4 lbs. bull meat, 40.0 lbs. beef navels, and 26.6 lbs. of water (14.60 lbs. from the computed formula + 12 lbs. of water that will be lost in processing). In addition to the ease of adjusting for shrinkage in this manner, the matrix for the LP model does not have to be changed if the amount of shrinkage changes due to different processing factors. For example, if the amount of water lost was 8 lbs. per 100 lbs. of finished product in one smokehouse and 12 lbs. of water was lost in a second smokehouse, a last minute decision can be made as to which smokehouse to use and what specific amount of additional water is to be included.

Binding Constraint

Sausages such as frankfurters and bologna are considered to be emulsions by the meat industry. The salt-soluble protein is the emulsifying agent and the main limiting factor as to the maximum amount of fat that can be emulsified or "bound". A number of terms are used to describe the condition of a broken sausage emulsion, such as "rendering", "greasing out", "capping", "fat caps", or "white capping". When emulsion breakdown occurs it is extremely expensive to the industry. Until recently "binding capacity" of the various meat ingredients could only be subjectively estimated by the sausage maker. Research work at the Food Science Department, University of Georgia, has objectively determined constant bind values for a number of meat ingredients. These constant binding values have been used daily with excellent results in actual plant operations for over a year and a half at the time of the writing of this manual. Specific details as to the method of determining the objective constant binding values can be found in publications (1, 3, and 4) listed under "References" at the end of this manual. The general rationale in developing these constant binding values was based on two factors: (1) The percent of the total protein which is salt-soluble remains a constant for each type of meat. The total amount of salt-soluble protein will vary directly with the change in the amount of total protein; however, when expressed as the percent of the total protein the value is a constant; (2) The amount of fat that can be emulsified by 1 gram or 1 lb. of salt-soluble protein is a constant for each type of meat; however, the efficiency or amount of fat emulsified by 1 gram or 1 lb. of salt-soluble protein from one type of meat may be considerably different than that from a different type of meat. For example, 1 lb. of salt-soluble protein from bull meat will emulsify or bind considerably more fat than 1 lb. of soluble protein extracted from beef cheek meat. However, the amount of fat emulsified by a given amount of salt-soluble protein for one type of meat is a constant for that one type of ingredient.

The two values, the percent of total protein that is salt-soluble, and the milliliters of fat emulsified by 100 milligrams of salt-soluble protein are multiplied together to obtain the constant bind value for each ingredient. Constant bind values for various ingredients may be found in Figure 15, and in publications (1 and 3) listed under "References".

Before the binding restraint inequality can be set up a binding coefficient must be calculated for each new batch of ingredients. This can be easily determined by multiplying the constant bind value (which never changes) X % protein for the same ingredient (as determined for each new batch by the laboratory). For example, bull meat has a constant bind value of 16.3, and if a batch of bull meat has a protein content of 20.1% the binding coefficient is 3.2763 or 16.3 X .201; the binding coefficient for 50-50 pork trim having a protein content of 9.1% and a constant bind value of 13.0 is 1.183 or 13.0 X .091. The binding coefficient is calculated for each ingredient in the same manner. If chemical analysis for percent protein is changed in a new batch of material, a new binding coefficient is calculated by multiplying the same constant bind value by the new value for percent protein. One last value must be determined before the binding constraint can be incorporated into the LP model. The minimum amount of total bind for each product must be determined for each plant, and will vary from plant to plant because of factors such as the type of machinery and the procedure used to make the product, temperature, and type of heat processing. Fortunately, this information may be obtained from past plant records. For example, if records show that a product had been produced with no emulsion breakdown by the formula (based on weight of finished product) in Figure 8, the minimum bind required for a maximum fat of 31% is calculated as follows:

Ingredients	Weight (lbs.)	% Protein	% Fat	Fat (lbs.)
Bull meat	40	19.0	10.0	4.0
50-50 pork trim	45	8.0	60.0	27.0
Dry ingredients plus water	15	0.0	0.0	0.0
	100			31.0

Figure 8. Formula and chemical data for product which had been produced according to past records

Total amount of bind = (Constant bind value for bull meat X % protein of bull meat X lbs. of bull meat used) + (Constant bind value for 50-50 pork trim X % protein of 50-50 pork trim X lbs. of 50-50 pork trim used). Thus,

Total amount of bind = (16.3 X .19 X 40) + (9.1 X .08 X 45)

Total amount of bind = 156.85

In this example, the amount of bind for a product that will have a maximum fat of 31% is ≥ 156.85. If the plant has only bull meat, cow meat, 50-50 pork trim, and skinned jowls having binding coefficients of 3.28, 2.49, 1.83, and 0.59 respectively, the binding inequality constraint for formulating the product by LP is:

3.28 (lbs. of bull meat - to be computed) + 2.49 (lbs. of cow meat - to be computed) + 1.83 (lbs. 50-50 pork trim - to be computed) + 0.59 (lbs. of skinned jowls - to be computed) ≥ 156.85

Color Constraint

Appearance has always been an important consumer requirement and is usually judged by the color and texture of the blended end product. Natural meat color is derived from the pigments (myoglobin and hemoglobin) in the lean tissue. Until recently, only subjectively determined color scales have been used. These color scales have been fairly successfully used, but they lacked precision primarily because of human error in subjectively ranking the various meats, and because of differences in the amounts of lean and fat in various batches of the same type of meat.

Objectively constant color values have been determined (refer to publications 3 and 5 in "References"). Basically, the procedure was to chemically determine the total amount of pigment in the various meat ingredients and to express the results as milligrams of pigment per 100 grams of protein. To use these constant color values it is only necessary to multiply the constant color value by the percent protein of the specific meat product (termed color coefficient). Specific values for certain meat ingredients are listed in Figures 9 and 15.

Ingredient	Constant Color Value	% Protein
Bull meat	23.5	20.1
Cow meat	20.7	17.8
50-50 pork trim	3.4	9.1
Pork cheek meat	11.3	17.2

Figure 9. Constant color values and percent protein

The color coefficients are: bull meat = 4.72 or 23.5 X 0.201; cow meat = 3.68 or 20.7 X 0.178; 50-50 pork trim = 0.31 or 3.4 X 0.091; and pork cheek meat = 1.94 or 11.3 X 0.172. The color constraint inequality can be formulated as part of the LP model. The equation will be in the form:

$$4.72\ M1 + 3.68\ M2 + 0.31\ M3 + 1.94\ M4 \geq 150$$

where: M1 = bull meat
M2 = cow meat
M3 = 50-50 pork trim
M4 = pork cheek meat

In this example, the composite color of ≥150 has been specified by the policy of the company to be the minimum color they will accept.

SETTING UP MATRIX

The equations that define the blending model must be set up in matrix form to correspond to the linear programming system appropriate for the data processing installation. Usually, there is a matrix row (called a problem constraint) for each equation or inequality and a matrix column (called a problem activity) for each ingredient available to the blending process.

The values in the body of the matrix are the coefficients of the problem constraints and are called the matrix elements. The function of the LP system is to find a solution that simultaneously satisfies all constraints in the model and also minimizes the cost constraint (objective function). Such a solution is called an optimal solution. The solution consists of a set of values, called activity levels, for the variables in the constraint equations and inequalities.

Illustration

To illustrate these concepts, the ingredients and data given in Figure 1, as well as certain dry ingredients (salt, seasoning, cure, corn syrup, etc.), will be used to set up a simple LP model in matrix form. In addition, water will be included to demonstrate the use of the water constraint. The constraints to be included are as follows:

Capacity Constraint

$M1 + M2 + M3 + M4 + DI + W = 100$

where: M1 = bull meat
M2 = cow meat
M3 = pork trimmings
M4 = fat pork
DI = dry ingredients
W = water added

Composition Constraints

Fat:

$0.08M1 + 0.15M2 + 0.50M3 + 0.70M4 \geq 24$

$0.08M1 + 0.15M2 + 0.50M3 + 0.70M4 \leq 28$

Protein:

$0.20M1 + 0.18M2 + 0.10M3 + 0.05M4 \geq 11$

Water Constraint

$-0.08M1 - 0.01M2 - 0.01M3 + 0.04M4 + W \leq 10$

where: W = water added
Moisture 1 -4P1 = -0.08
Moisture 2 -4P2 = -0.01
Moisture 3 -4P3 = -0.01
Moisture 4 -4P4 = 0.04

Dry Ingredient (DI) Constraint

DI = 6

Cost Constraint (Objective Function)

$0.47M1 + 0.45M2 + 0.30M3 + 0.11M4 + 0.0W + 0.19DI$ = Minimum Cost

The basic meat blending LP matrix is shown in Figure 10.

M1	M2	M3	M4	W	DI	RHS/Cost
0.47	0.45	0.30	0.11	0.00	0.19	
1.00	1.00	1.00	1.00	1.00	1.00	= 100 (capacity)
0.08	0.15	0.50	0.70			≥ 24 (min. fat)
0.08	0.15	0.50	0.70			≤ 28 (max. fat)
0.20	0.18	0.10	0.05			≥ 11 (min. protein)
-0.08	-0.01	-0.01	0.04	1.00		≤ 10 (max. water)
					1.00	= 6 (dry ingredients)

Figure 10. Basic sausage LP matrix

If ingredient constraints are added to either force or limit the use of certain ingredients, care must be taken to avoid formulations that violate other constraints. For example, if 60 lbs. of fat pork were forced into the mix, two constraints would be violated. The first would be that the maximum limit on fat (28%) would be exceeded (60 X 70% = 42%); the second would be that there is no physical method of adding a sufficiently high protein ingredient to obtain the minimum protein (11%) constraint unless the capacity constraint (100 lbs.) was violated.

Visual inspection of the matrix will often uncover this kind of infeasibility. Construction of the initial linear programming model involves gathering data about the blending process and setting it up in a matrix format. This model needs to be constructed only once, because as changes occur they can be introduced into the original model. Such changes would include the addition and deletion of ingredients and constraints, and the changing of costs.

Elaboration of Capacity Constraints

In Figure 10, the capacity constraint has been set equal to 100 lbs., a very convenient total because it allows the expression of all right-hand sides and activity levels as percentages. Thus, the final minimum fat content can be read as 24% as well as 24 lbs., and so on for each of the constraint rows. If more than 100 lbs. of blend is required, one simply multiplies the activity levels in the solution by the appropriate constant. For example, if 575 lbs. of blend is required, we multiply the solution activity levels by 5.75.

Despite its convenience, however, such a formulation may lead to difficulties when inventory constraints are introduced: Limitations on the availability of one or more ingredients may make it impossible to reach a feasible solution for a blend capacity of 100 lbs. For example, if a high protein ingredient were severely limited, there might not be enough total protein available for a feasible solution at 100 lbs. of blend. Then again, suppose the constraint provided just enough protein for a feasible solution at 100 lbs. of blend. Multiplying the activity levels for a larger batch size would lead to an unrealistic formula in terms of actual inventory. This is especially true for a company having a number of plants at various locations.

It is possible, however, to preserve the convenience of percentage expression while at the same time including the desired batch size in the model. The new matrix formulation shown in Figure 11 yields an optimal solution which contains, still in percentage terms, the maximum batch size and corresponding activity levels achievable under the given ingredient constraints.

In the matrix of Figure 11, we have added an activity column for a new variable, SOL, which represents the quantity of blend produced; we have also added a constraint row (blend constraint), which sets the quantity of blend produced less than or equal to ($\leq$) the amount desired. The new variable must be provided with a negative cost — otherwise, the total cost of the blend could be minimized by the simple expedient of setting the quantity produced (SOL) to zero, which would be allowed by the "equal-to-or-less-than" blend constraint. Therefore, we associate the projected selling price (in this case, an arbitrary figure of $0.75) with the variable SOL as a negative cost coefficient. The LP system will try to minimize the total cost by making the negative cost variable SOL as large as possible (limited by the upper bound of the blend constraint). Such a formulation provides, in the output reports, not the cost of the optimal blend, but rather the profit (that is, the difference between the cost and the projected selling price).

The coefficients in the new activity column (SOL) are the specifications of the blend in terms of one unit of the final product. For example, the -0.24 coefficient in the min. fat row indicates that each pound of the product must contain at least 0.24 lb. of fat:

(Total fat) - 0.24 SOL $\geq$ 0,

(Total fat) $\geq$ 0.24 SOL,

since

(Total fat) = 0.08M1 + 0.15M2 + 0.50M3 + 0.70M4

The capacity row at the bottom ensures that no more than some batch size will be produced. Individual ingredient constraints can be easily added and are of the same form as the capacity constraint.

M1	M2	M3	M4	W	DI	SOL		RHS/Cost
0.47	0.45	0.30	0.11	0.00	0.10	-0.75		
1.0	1.0	1.0	1.0	1.0	1.0	-1.0	=	0 yield
0.08	0.15	0.50	0.70			-0.24	$\geq$	0 (min. fat)
0.08	0.15	0.50	0.70			-0.28	$\leq$	0 (max. fat)
0.20	0.18	0.10	0.05			-0.11	$\geq$	0 (min. protein)
-0.08	-0.01	-0.01	+0.04	+1.0		-0.10	$\leq$	0 (max. water)
					1.0	-0.06	=	0 (dry ingredients)
						1	$\leq$	100 (max capacity)

Figure 11. Flexible-yield matrix

A multiproduct model contains data for computing more than one product blend during the same computer run. The blends may be for consecutive production in one processing facility or simultaneous production in several facilities. In addition, the blends may be for the same product or different ones.

Figure 12 is an outline of a multiproduct model containing the data for three products. This matrix is divided into three submatrices, each of which is similar to the matrix shown in Figure 11.

The ingredient activities (columns) may be the same for each product, or different, and are given names that distinguish their product application. For example, bull meat might be an ingredient of all three products in Figure 12, being named 1M1 for the first product, 2M1 for the second, and 3M1 for the third. Consequently, the meat packer knows the total amount used as well as the particular amount used in each product. A capacity row is included in the overall matrix to constrain the total amount of manufactured product to some maximum. This capacity may be determined by considerations such as processing facilities, customer orders, forecast sales, or the inventory capacity of the plant.

The multiproduct model makes it possible to allocate from the total ingredient inventory to many products simultaneously. Thus, the linear programming solution will determine the optimum overall distribution of ingredients among the blends. This is especially useful when ingredients are in short supply.

It is possible to develop multiproduct models that include — in addition to the overall blending operation — such variables and influences as labor and facilities utilization, purchasing and raw material procurement policies, and sales and distribution. The scope of the linear programming model is limited only by the size of the computer and the time required for solution. The model can represent the repeated production of a single product (or a number of products) during several time periods.

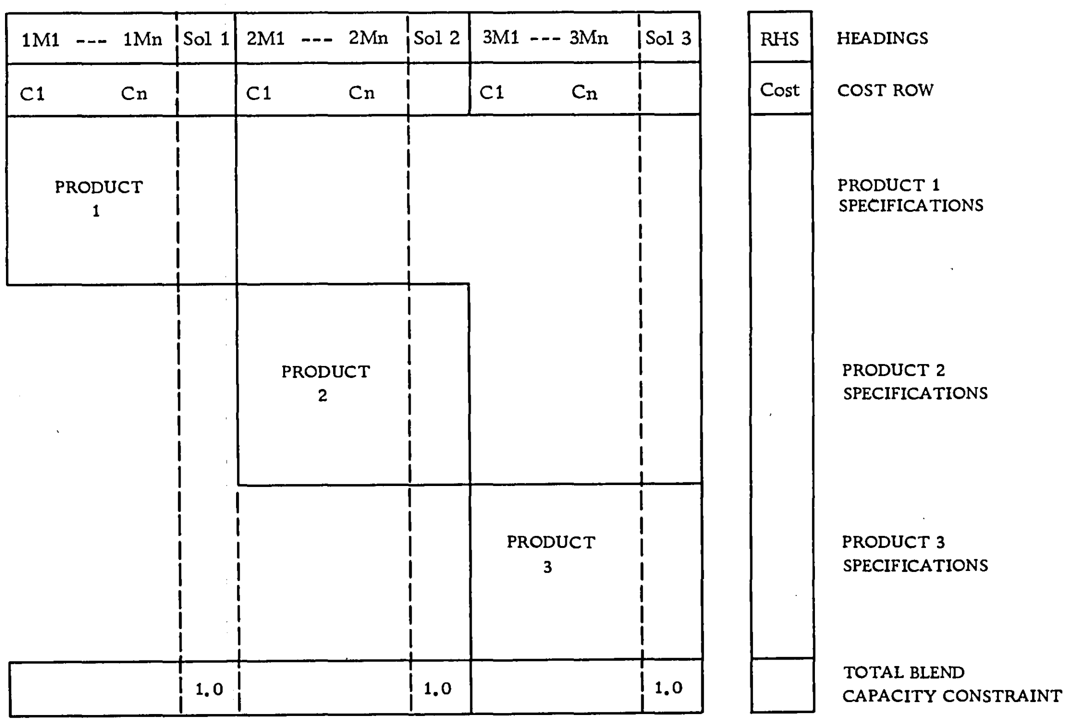

Figure 12. Multiproduct model

SAMPLE PROBLEM

In this section, we shall present a sample problem of more realistic dimensions than those discussed so far. The solution of this problem by a computer LP system serves as a basis for the discussion of output reports in the following section. The problem is one of frankfurter blending, using the ingredients listed in Figure 13.

Identification*	Description
M1	Fresh bull meat
M2	Fresh cow meat
M3	Beef cheek meat
M4	Navels
M5	50-50 Pork trim
M6	Boneless picnics
M7	Skinned jowls
M8	80% Lean pork trim
M9	Pork cheek meat
M10	Fat pork
M11	Blade meat
DI	Dry ingredients
W	Water

*As used in the LP matrix

Figure 13. Sample problem ingredients

The finished blend is subject to the restrictions listed in Figure 14.

Capacity	100 lbs.
Minimum protein	10 lbs.
Minimum fat	28 lbs.
Maximum fat	31 lbs.
Minimum beef	35 lbs.
Minimum pork	35 lbs.
Dry ingredients	6 lbs.
Maximum beef cheek meat plus pork cheek meat	20 lbs.
Minimum bind	155 units
Minimum color	150 units
Maximum water	4 X total protein plus 0.10 X total weight
Maximum blade meat	20 lbs.
Maximum boneless picnic	15 lbs.

Figure 14. Sample problem restrictions

The restrictions will vary from product to product and from one company to another. Very briefly, the reasons for imposing the restrictions used for this example are as follows:

Capacity is equal to 100 lbs. of finished product. By doing this, the fat, protein, and water may be expressed as either pounds or as percent and the cost will be given as dollars per hundredweight of finished product.

Minimum protein is equal to 10 lbs. It is generally accepted in the industry that 10 to 12% protein is necessary for an acceptable frankfurter. In addition, some states require a minimum for protein. No maximum has been set on protein because the frankfurter is very acceptable at higher levels, especially with a minimum restriction on fat. In addition, since protein is usually one of the most expensive constraints, LP will usually formulate at the lower level.

Minimum fat is equal to 28 lbs. and maximum fat is equal to 31 lbs. Fat is necessary for juiciness and flavor; however, a maximum must be used or the emulsion will break down or grease out.

Minimum beef and minimum pork are both equal to 35 lbs. It is assumed that the label reads "beef and pork". In this case, governmental regulations require that 40% of the meat must be pork and 40% must be beef. Since dry ingredients and water make up a portion of the 100 lb. capacity, it is safe to use 35 lbs. as minimum for each and yet have at least 40% of the meat to be pork and 40% to be beef.

Dry ingredients, which include seasoning, salt, cure, and corn syrup, is equal to exactly 6 lbs. and must remain at this level for taste, color, etc.

Maximum for beef cheeks plus pork cheeks is equal to 20 lbs. Most sausage makers agree that if too high a level of these two products is used, a "first line product" cannot be made.

Minimum bind is equal to 155 units (derived from past plant records of minimum for binding a product with 31% fat). It cannot be set lower because each time that the formulation resulted in a product with 31% fat, emulsion breakdown would occur.

Minimum color is equal to 150 units — specified by management as the minimum color they will accept in the finished product.

Maximum water has been set at 4 times protein plus 0.10 times the finished product. Water must be added or a frankfurter cannot be made. Also, water increases juiciness of the product. The maximum water restriction does not violate federal regulations.

Maximum blade meat is equal to 20 lbs. and maximum boneless picnic is equal to 15 lbs. These two restrictions are used to demonstrate how a single product can be restricted and will permit the demonstration of a method of modifying the LP matrix shown in Figure 16.

The ingredient compositions and costs are shown in Figure 15.

The information provided in Figures 13 through 15 was used to formulate the model matrix shown in Figure 16. In this matrix two rows are employed to express a range constraint (specifying a minimum other than zero and a maximum — such as one row for fat ≥ 28, and a second row for fat ≤ 31). In practice, however, most IBM LP systems permit the expression of range constraints as one row with double right-hand side, establishing both upper and lower bounds. This feature reduces the number of matrix rows and results in faster solution and greater machine capacity.

Similarly, though inventory availability constraints and ingredient constraints which forced an ingredient into the solution (such as dry ingredients) are expressed in Figure 16 by the use of constraint rows, most IBM LP systems permit their expression in the form of bounded variables. This technique also conserves rows and permits the solution of large multiproduct matrices in a single computer run.

In translating the problem matrix (Figure 16) we have actually used two devices so that rows 3 and 4 of Figure 16 become one row FAT and the following ingredient constraints (max. boneless picnic, max. blade meat, and dry ingredients) are transformed from rows into bounded variables (M6, M11 and DI). The new matrix appears in Figure 17.

Identification	Fat%	Protein %	Constant Bind Value	Constant Color Value	Moisture %	Cost lbs/$
M1	8.8	20.1	16.3	23.5	70.0	0.4450
M2	15.3	17.8	14.0	20.7	65.9	0.4300
M3	11.4	18.3	8.2	26.2	69.4	0.3900
M4	50.4	9.7	13.6	19.9	39.1	0.2300
M5	54.5	9.1	13.0	3.4	35.9	0.3525
M6	21.1	15.6	13.2	10.2	62.6	0.4900
M7	71.2	7.5	7.9	2.8	20.9	0.3125
M8	20.9	16.4	13.1	10.1	61.9	0.4500
M9	21.8	17.2	10.7	11.3	60.2	0.3900
M10	91.7	0.9	0.0	0.0	8.1	0.0700
M11	7.8	18.0	13.3	10.4	73.1	0.5200
DI	0.0	0.0	0.0	0.0	0.0	0.1900
Water	0.0	0.0	0.0	0.0	100.0	0.0000

Figure 15. Sample problem ingredient compositions and costs

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	DI	Water		RHS/Cost
Cost	0.4450	0.4433	0.3900	0.2300	0.3525	0.4900	0.3125	0.4500	0.3900	0.0700	0.5200	0.1900	0.0000		
Capacity	1	1	1	1	1	1	1	1	1	1	1	1	1	=	100
Min. protein	0.201	0.178	0.183	0.097	0.091	0.156	0.075	0.164	0.172	0.009	0.180	0.000	0.000	≥	10
Min. fat	0.088	0.153	0.114	0.504	0.545	0.211	0.712	0.209	0.218	0.917	0.078	0.000	0.000	≥	28
Max. fat	0.088	0.153	0.114	0.504	0.545	0.211	0.712	0.209	0.218	0.917	0.078	0.000	0.000	≤	31
Min. beef	1	1	1	1										≥	35
Min. pork					1	1	1	1	1	1	1			≥	35
Dry ingredients												1		=	6
Max. beef and pork cheek meat			1						1					≤	20
Min. bind	3.276	2.392	1.501	1.319	1.183	2.059	0.593	2.148	1.840	0.000	2.394	0.000	0.000	≥	155
Min. color	4.724	3.685	4.795	1.930	0.309	1.591	0.210	1.656	1.944	0.000	1.872	0.000	0.000	≥	150
Max. water	-0.104	-0.053	-0.038	+0.003	-0.005	+0.002	-0.091	-0.037	- .086	+0.045	+0.011	+0.000	0.000	≤	10
Max. blade meat											1			≤	20
Max. boneless picnic						1								≤	15

Figure 16. Sample problem LP matrix

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	DI	Water		
Costs	0.4450	0.4433	0.3900	0.2300	0.3525	0.4900	0.3125	0.4500	0.3900	0.0700	0.5200	0.1900	0.0000	RHS/Cost	
Capacity	1	1	1	1	1	1	1	1	1	1	1	1	1	= 100	
Protein	0.201	0.178	0.183	0.097	0.091	0.156	0.075	0.164	0.172	0.009	0.180	0.000	0.000	≥ 10	
Fat	0.088	0.153	0.114	0.504	0.545	0.211	0.712	0.209	0.218	0.917	0.078	0.000	0.000	≥ 28	≤ 31
Beef	1	1	1	1										≥ 35	
Pork					1	1	1	1	1	1	1			≥ 35	
Beef and Pork Cheek Meat			1						1						≤ 20
Bind	3.276	2.492	1.501	1.319	1.183	2.059	0.593	2.148	1.840	0.000	2.394	0.000	0.000	≥ 155	
Color	4.724	3.685	4.795	1.930	0.309	1.591	0.210	1.656	1.944	0.000	1.872	0.000	0.000	≥ 150	
Water	-0.104	-0.053	-0.038	+0.003	-0.005	+0.002	-0.091	-0.037	-0.086	+0.045	+0.011	0.000	0.000		≤ 10
Bounds						≥ 15					≥ 20	= 6			

Figure 17. Sample problem matrix with bounded variables

OUTPUT REPORTS

The linear programming system will employ the input data to compute four basic output reports:

- Basis variables report
- Check report
- Cost range report
- DO.D/J report

Each of these is discussed and illustrated below. (The IBM 1620/1311 Linear Programming System was used to solve the problem and produce the reports.)

BASIS VARIABLES REPORT

The basis variables (BASIS VARBLS.) report (Figure 18) lists each ingredient used in optimal blend together with the quantity required. This report, if it is to be implemented without change, can be used as a production order in the plant and as a record of ingredients expended by the inventory accounting department. Since the capacity constraints are set up in this example to equal 100 lbs. of finished product, the head of sausage production needs to make only two modifications. The first, to multiply each ingredient by a factor to comply with the capacity of his machinery (5.75 if the chopper's actual capacity was 575 lbs.). The second, to add the amount of water that would be lost in processing.

NAME	ACTIVITY LEVEL
M1	41.483
M3	3.671
M9	7.388
M10	27.612
DI	6.000
W	13.846

Figure 18. Basis variables report — optional variables and activity levels (solutions)

CHECK REPORT

The check report (Figure 19) indicates, for each row (or composition constraint) in the blend formula, how the constraint was met by the solution and the actual bounds that were imposed. This alerts the producer to a number of possibilities. For example, color was solved at a considerably higher level than the minimum imposed. This would indicate that a considerable variance in the intensity of color of the final product could occur. The producer may want to increase the lower limit to have a more intense color or he may wish to impose a top limit to have a more uniform color product.

The check report also gives the cost of the optimal blend (cost), in this case $25.90 for 100 lbs., or 25.9¢ per lb.

ROW NAME	UPPER LIMIT	SOLUTION VALUE	LOWER LIMIT
CAPACITY	100.0	100.0	----
PROTEIN	----	10.5	10.0
FAT	31.0	31.0	28.0
BEEF	----	45.2	35.0
PORK	----	35.0	35.0
BEEF AND PORK CHEEK MEAT	20.0	10.0	----
BIND	----	155.0	155.0
COLOR	----	227.0	150.0
WATER	13.8	13.8	----
COST	----	25.9	----

Figure 19. Check report

COST RANGE REPORT

The cost range (COST.R) report (Figure 20) indicates, for each ingredient used in the optimal solution, the following data: current cost, highest cost before the quantity in the optimal solution changes, what ingredient would enter the optimal solution at that highest cost, the lowest cost before the quantity in the optimal solution changes, what ingredient would leave the optimal solution at that lowest cost.

The quantity of each ingredient in the optimal solution (as given in the basis variables report) will remain unchanged within the cost range indicated by HIGHEST COST and LOWEST COST. For example, 41.483 lbs. of bull meat (M1) would be used in an optimal solution even if its cost rose to 44.87¢ per lb. Similarly, the same amount would be used if the cost dropped to 43.29¢ per lb. If the price of bull meat exceeded 44.87¢, however, some of the bull meat in the optimal solution would be replaced by beef navels (M4). If the price dropped below 43.29¢ per lb., some of the beef cheek meat (M3) would be replaced by bull meat.

The cost range report provides a good measure of sensitivity to ingredient price changes, since it indicates at what price the optimal solution will change and what ingredient may be used most appropriately to substitute for unavailable or overpriced stock.

DO.D/J REPORT

The DO.D/J report, often called the reduced costs report, consists of two parts. The first part (Figure 21) lists all the ingredients (column activities) which are solved at a bound or limit. Often the bound is zero — that is, the ingredient is not used at all in the optimal formula (or basis). In this case, the report indicates the ingredient's current cost and the amount it must drop before it reaches a level at which the ingredient may be introduced into the basis. When an upper bound restrains or limits the amount of an ingredient, the report indicates the highest price at which that material would remain in the basis at its bound.

Referring to Figure 21, the first line indicates that no cow meat (M2) is present in the optimal formula because its current price of 43.00¢ per lb. is too high. If the price were to drop 3.57¢ to 39.43¢ per lb., this ingredient would enter the optimal formula. Beef navels (M4) would only have to decrease from the current price of 23.50¢ per lb. to 23.32¢ per lb. to be used.

The reduced costs report is an excellent tool for management in determining what should be bought or sold, and its value if used in a product.

COST R

NAME	CURRENT COST	HIGHEST COST	HIGH VARIABLE	LOW VARIABLE	LOWEST COST
M1	.4450	.4487	M3	M4	.4329
M3	.3950	.3986	M4	WATER	.1965
M9	.3950	.4692	M8	M3	.3919

Figure 20. Cost range report

REDUCED COSTS

NAME	CURRENT COST	REDUCED COST	BASIS VALUE
M2	.4300	.03575	.39425
M4	.2350	.00181	.23319
M5	.3525	.11609	.23641
M6	.4900	.12686	.36314
M7	.3125	.10677	.20573
M8	.4500	.07278	.37722
M11	.5200	.10905	.41095

Figure 21. DO.D/J report — nonoptimal variables analysis

The second part of the DO. D/J report (Figure 22) provides a list of all constraint mnemonics and, for each constraint, indicates the cost of changing the right-hand side of the expression by one unit. This part of the report provides an analysis of the impact that specifications (constraints) have on the cost of the product. In Figure 22, the first two lines reveal that minimum protein and the minimum fat constraints do not affect the cost of the solution, since both were solved at above the minimum. The maximum fat restraint forces the cost up. In the neighborhood of the optimal solution, an increase of one pound of fat in the specification would result in a 36.828¢ saving in the total cost of the formula. Similarly, a decrease of one unit of bind value required would result in 0.752¢ saving in the total cost. These figures graphically reveal the cost of the constraints and suggest that resolutions with slightly relaxed specifications may result in significant cost reductions.

RHS ANALYSIS TYPE/NAME	INCREMENT VALUE	DECREMENT VALUE
MINIMUM PROTEIN	----	.00000
MINIMUM FAT	----	.00000
MAXIMUM FAT	.36828	----
MINIMUM BEEF	----	.00000
MINIMUM PORK	----	.01606
MAXIMUM BEEF AND PORK CHEEK MEAT	.00000	----
MAXIMUM WATER	.41010	----
MINIMUM BIND	----	.00752
MINIMUM COLOR	----	.00000

Figure 22. DO. D/J report — requirements (RHS) analysis

OUTPUT ANALYSIS

The data used in the sample problem is typical for the industry. Ultimately, the computer-prepared formula will be compared to standard formulas, and whatever additional ingredients or constraints are required to allow for a valid comparison of quality and cost should be incorporated into the model. As a basis for such comparison a blend analysis summary may be computed as in Figure 23.

SUMMARY

The solutions produced by linear programming affect virtually every major company operation. This data can provide the basis for improving purchasing, inventory control, quality control, pricing, cost analysis and, of course, mix computation.

In inventory control the LP solution can be used to update the inventory balances. The updated inventory figures then serve as the basis for replenishing inventory through purchase or internal

INGREDIENT	WEIGHT (LBS.)	FAT (LBS.)	PROTEIN (LBS.)	COST ($)
M1	41.483	3.651	8.338	18.46
M3	3.671	0.418	0.672	1.45
M9	7.388	1.611	1.271	2.92
M10	27.612	25.320	0.249	1.93
DI	6.000	0.000	0.000	1.14
WATER	13.846	0.000	0.000	0.00
	100.000	31.000	10.530	25.90

Figure 23. Blend analysis summary

production and for updating the LP models to reflect ingredient availability. The same information can be used to provide statistical estimates of ingredient usage, and rarely used items can be isolated for possible removal from the product line.

In purchasing and cost analysis, the cost ranges produced by LP can be used to:

- Determine how much the cost of an ingredient can fluctuate without affecting the cost of the mix
- Indicate the cost of substituting a nonoptimal ingredient for an optimal one
- Determine the maximum price at which an ingredient can economically be purchased or produced

Linear programming can provide information necessary to analyze these factors on a day-to-day basis.

Equally important, however, is the experimental analysis of the same factors in order to study the effect of a variety of possible conditions on the costs of blending a particular product. Linear programming is as applicable to hypothetical situations as it is to real ones, and this is one of its most powerful advantages. Experimental models may be originated, or existing ones modified, to show results of many different courses of action before a change is actually made. For instance, the cost of a meat blend normally goes down as the fat percentage increases. Through LP, this relationship may be studied in depth to determine a maximum fat level. Further, it has been shown that an increase in protein can result in a decrease in cost because protein absorbs water. This and other substitutions of a variety of "equivalent" or new ingredients may be similarly studied through LP with a minimum of "trial and error".

The extent to which linear programming can be used in the meat-packing industry will depend upon the ability of the individual company to merge the data produced by this technique into existing decision-making processes. The advantages of linear programming extend far beyond the blending problem.

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