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Subject: TRANSISTOR CIRCUITS COURSE. NUMBER 3. CHARACTERISTIC CURVES.

To: Distribution List

From: D. J. Eckl

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Approved: DRB
D. R. Brown

Abstract: The equivalent-T parameters can be obtained from the characteristic curves of the transistor. The collector or r_{22} characteristics provide values of r_c and α. The base characteristics give r_b. The grounded emitter curves are in general more useful since they are modified by the factor $(1-\alpha)$ in such a way as to bring out the properties of the transistor in magnified form. Punch-through and avalanche voltages are readily illustrated. In general, for switching applications these curves are more useful in specifying the over-all quality of the transistor than they are for circuit design. This would not necessarily be the case in amplifier design, however.

1.0 Grounded-Base Collector (r_{22}) Characteristics.

In Fig. 1 a typical pnp junction collector-characteristic family is shown. This is a plot of collector (or output) voltage against collector current for different values of I_e. There are three general regions into which the characteristics are divided as shown: Cutoff, Active, Saturation. These are also frequently referred to as Regions I, II, and III. In the Off Region, $I_e < 0$ and both emitter and collector are high impedances. In the active region $I_e > 0$ and r_c is high. This is the normal operating region for the transistor. The emitter is a low impedance. In the saturation region both r_c and r_e are low impedances.

Figure 2 shows the complete set of characteristics with the important parameters indicated. The slope of the curves,

$$\left|\frac{\partial v_c}{\partial I_c}\right|_{I_e} = r_c + r_b \approx r_c.$$

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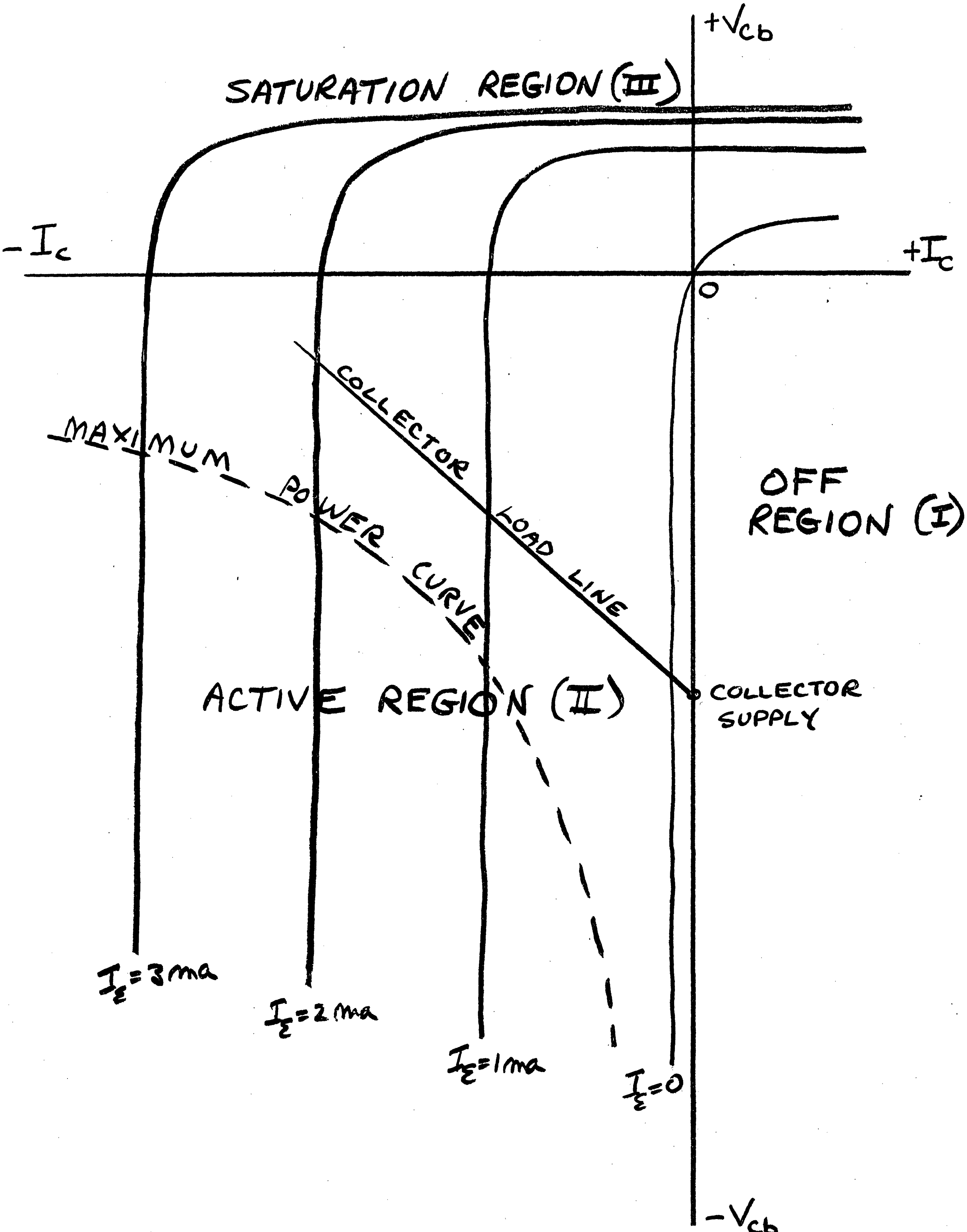

FIG. 1 - COLLECTOR (r_{22}) CHARACTERISTICS

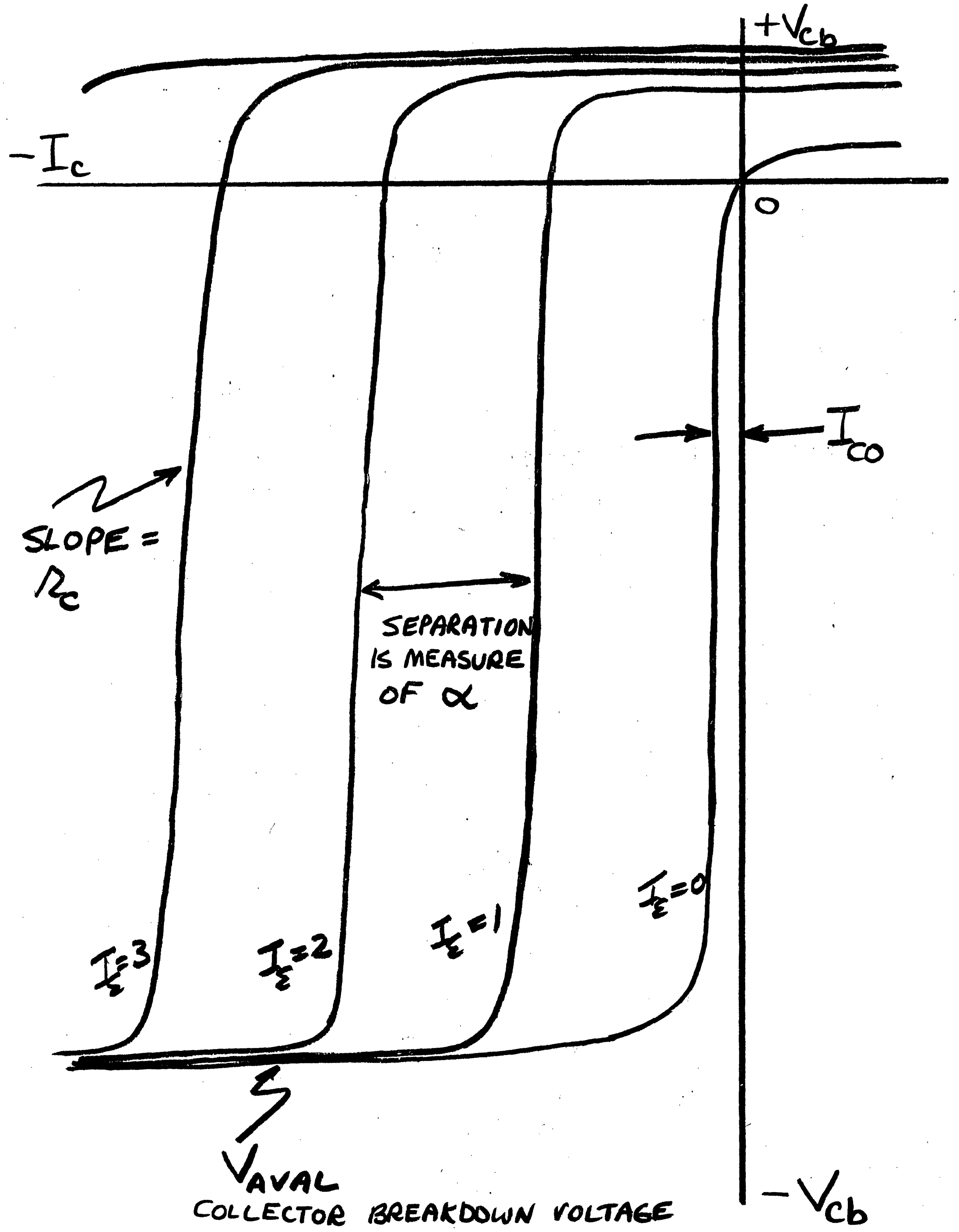

FIG. 2 COLLECTOR CHARACTERISTICS. (GROUNDED-BASE.)

In both the saturation region and the avalanche region the value of r_c is low. The avalanche voltage V_{aval} which is the maximum collector-to-base voltage will be discussed later. The spacing of the curves is a measure of α since

$$\left|\frac{\partial I_c}{\partial I_e}\right|_{V_c} = \alpha.$$

I_{co} is the cutoff leakage collector current. Note also that the collector voltage must go positive to saturate the transistor.

2.0 Grounded-Base r_{12} Characteristics.

The base or r_{12} characteristics are shown in Fig. 3.

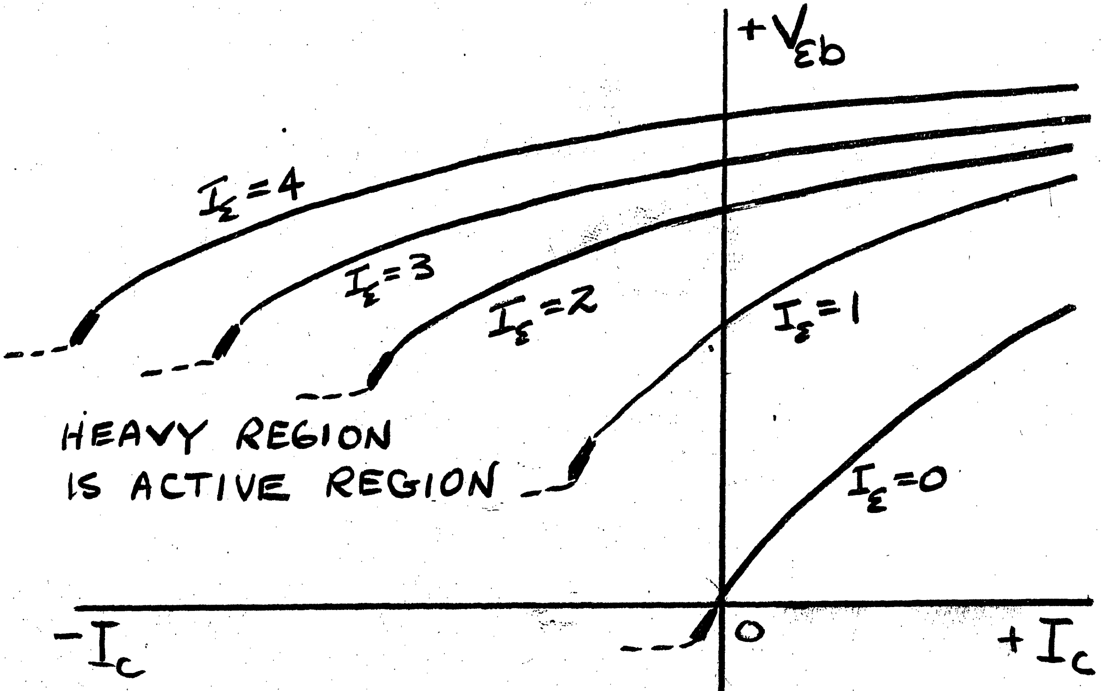

FIG. 3 - BASE CHARACTERISTICS.

The slope of these curves is

$$\left|\frac{\partial V_e}{\partial I_c}\right|_{I_e} = r_b = r_b' + h_{12}r_c.$$

Thus in the saturation region where r_c becomes small the value of r_b becomes smaller than the active r_b. In the avalanche region where r_c becomes low the base resistance will be low as shown by the dotted lines.

3.0 The Grounded-Emitter Circuit.

Suppose we consider the equivalent-T circuit with the emitter grounded as shown in Fig. 4.

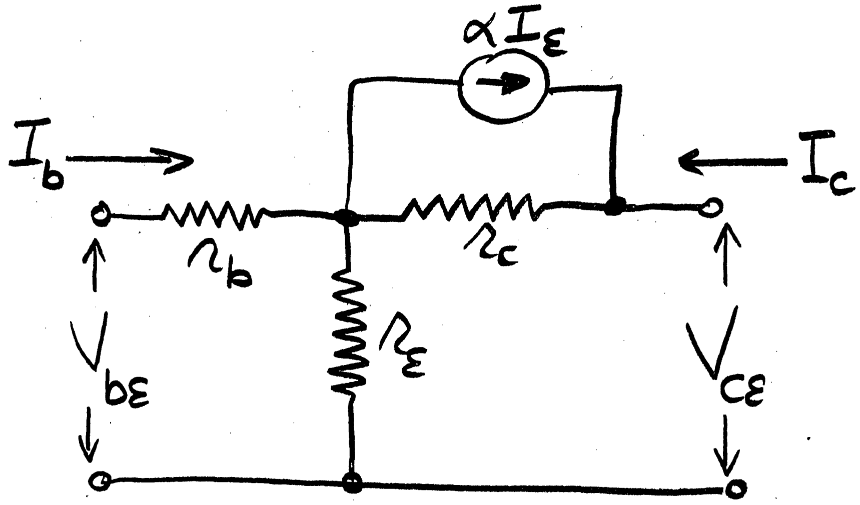

FIG. 4 - GND-EMITTER EQUIVALENT-T.

The current-voltage relations are:

$$V_{be} = (r_b + r_e)I_b + r_e I_c.$$

$$V_{ce} = r_e I_b + \alpha I_e r_c + (r_c + r_e)I_c.$$

But,

$$I_e = -I_b - I_c.$$

$$\therefore \quad V_{ce} = (r_e - \alpha r_c)I_b + \left[r_e + (1-\alpha)r_c\right] I_c.$$

The grounded-emitter parameters are the following:

$$r_{11_e} = r_b + r_e.$$

$$r_{12_e} = r_e.$$

$$r_{21_e} = r_e - \alpha r_c.$$

$$r_{22_e} = r_e + r_c(1-\alpha).$$

The grounded-emitter short-circuit current gain is,

$$\alpha_{ce} = \frac{r_{21_e}}{r_{22_e}} = \frac{\alpha r_c - r_e}{(1-\alpha)r_c + r_e} \approx -\left(\frac{\alpha}{1-\alpha}\right) = -\beta.$$

We can now draw an equivalent circuit for the grounded emitter configuration in terms of I_b, the input current. This is shown in Fig. 5.

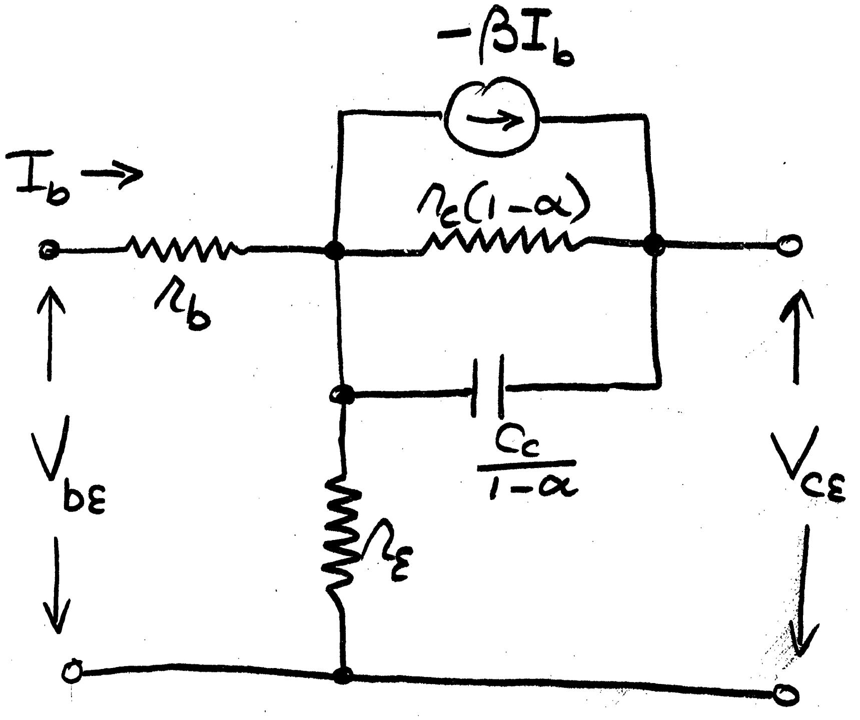

FIG. 5 - GROUNDED-EMITTER EQUIVALENT-T.

The grounded-emitter collector capacity can be evaluated by considering the collector impedance

$$1/Z_c = g_c + j\omega C_c.$$

Z_c would enter into all expressions above in the same way as r_c and would therefore be multiplied by $(1-\alpha)$ in the grounded-emitter circuit. This would give

$$\frac{1}{Z_c(1-\alpha)} \doteq \frac{g_c}{1-\alpha} + j\omega\frac{C}{(1-\alpha)}.$$

Therefore the capacity is effectively multiplied by the factor $(1/_{1-\alpha})$ which means an increase of 9-50 times.

A more rigorous treatment can be made using the circuit of Fig. 6. The equations given are straight forward. Small letters denote internal currents.

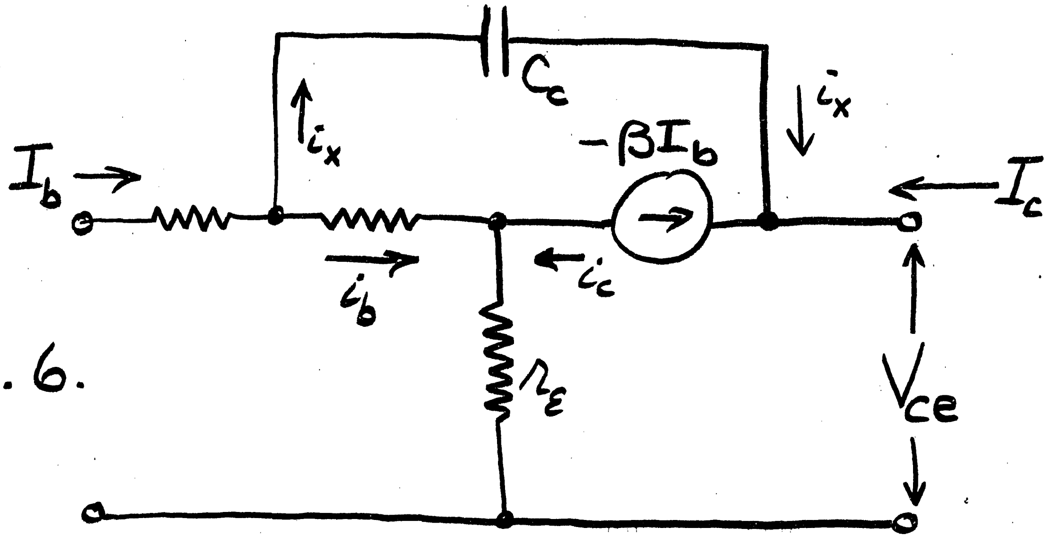

FIG. 6.

We would like to obtain a relationship for collector-voltage rise in terms of external currents and C_c.

$$i_c = \beta i_b.$$

$$I_b = i_b + i_x.$$

$$I_c = i_c - i_x.$$

$i_x \doteq -C_c \dfrac{dV_c}{dt}$, since the collector voltage appears largely across the collector junction and therefore across C_c.

$$\therefore I_c = \beta i_b + C_c \frac{dV_c}{dt}.$$

$$I_c = \beta(I_b - i_x) + C_c \frac{dV_c}{dt}.$$

$$I_c = \beta(I_b + C_c \frac{dV_c}{dt}) + C_c \frac{dV_c}{dt}.$$

$$I_c = \beta I_b + \beta C_c \frac{dV_c}{dt} + C_c \frac{dV_c}{dt}.$$

$$C_c(1+\beta)\frac{dV_c}{dt} = I_c - \beta I_b.$$

or,

$$\frac{dV_c}{dt} = \frac{I_c - \beta I_b}{C_c(1+\beta)} = \frac{I_c - \beta I_b}{C_c/(1-\alpha)}.$$

Thus the collector capacity is effectively multiplied by the factor $1/(1-\alpha)$ in the grounded-emitter configuration.

4.0 Grounded-Emitter Collector Characteristics.

The grounded-emitter output characteristics are shown in Fig. 7. These are curves of V_{ce} against I_c for various values of I_b. The slopes of the curves are

$$\left|\frac{\partial V_{ce}}{\partial I_c}\right|_{I_b} = r_c(1-\alpha).$$

The separation between the curves is a measure of the current gain, that is,

$$\left|\frac{\partial I_c}{\partial I_b}\right|_{V_{ce}} = \frac{\alpha}{1-\alpha} = \beta.$$

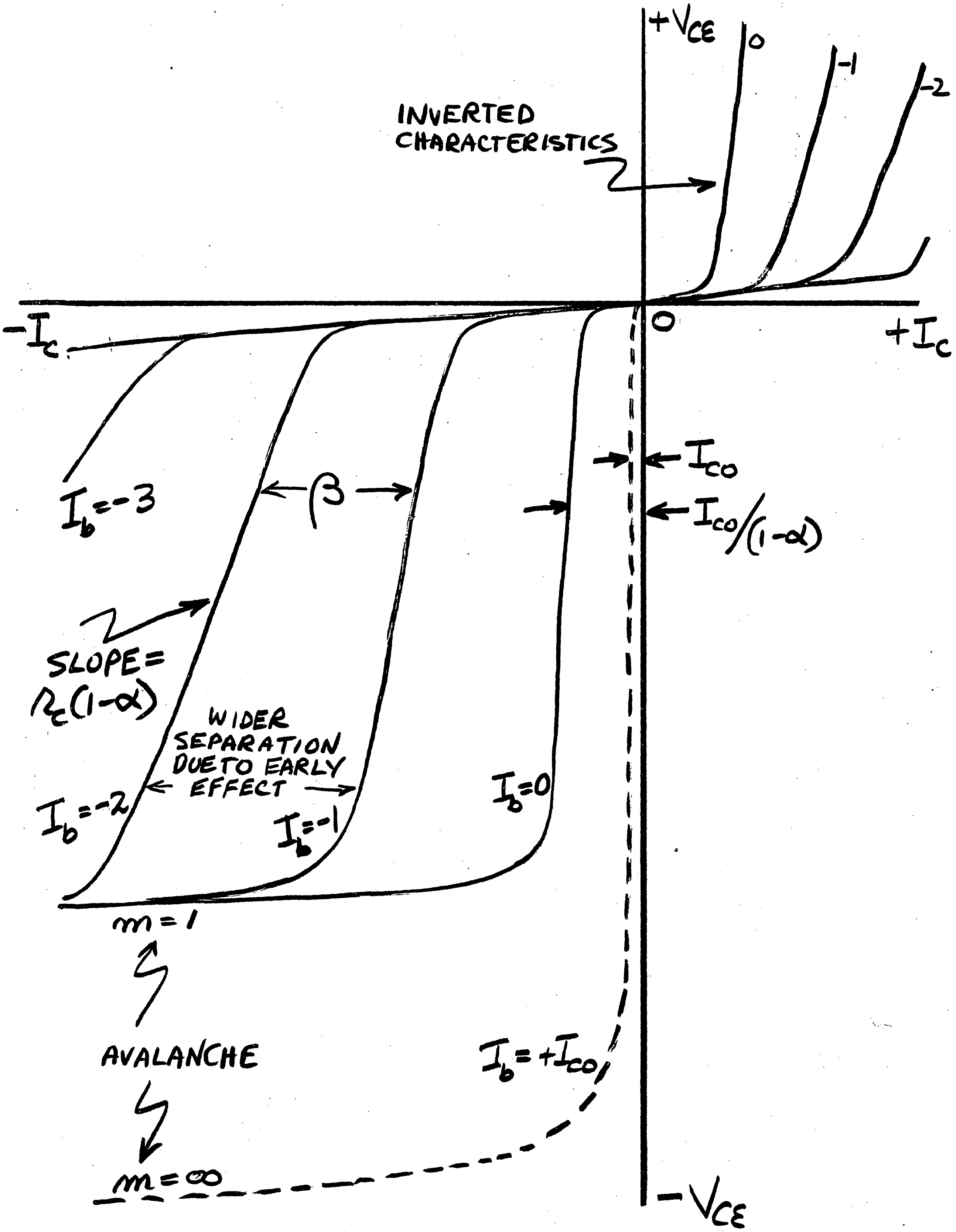

FIG 7. - GROUNDED-EMITTER OUTPUT CHARACTERISTICS

We have already seen that the Early effect causes α to increase with collector voltage. This appears in the grounded-emitter curves as a widening at higher voltages.

The leakage current is now $\frac{I_{co}}{1-\alpha}$. The grounded-base leakage current I_{co} will appear for a positive base current equal to I_{co}.

The inverse characteristics are obtained in the first quadrant by using a positive collector voltage. The collector acts as an emitter in this region.

5.0 Avalanche Breakdown.

In the collector space charge region there is an electric field which accelerates the minority carriers towards the collector. As the collector voltage is increased it is possible for a carrier to gain sufficient energy from the field to produce secondary carriers. A multiplication ratio m can be defined as the ratio of total carriers to primary carriers. We can then define the current gain α as

$$\alpha = m\alpha_0.$$

Where α_0 is the low voltage α. In the grounded-base configuration when $m = \infty$ the current gain becomes infinite and the collector breaks down. This is somewhat similar to an ionization breakdown in a gas. The breakdown voltage is shown as V_{aval} in Figure 2.

In the grounded-emitter configuration, however, the current gain is given by,

$$\beta = \frac{m\alpha_0}{1-m\alpha_0}$$

Therefore, breakdown occurs at the point where $m\alpha_0 = 1$ which will be at a much lower voltage.

6.0 Punch-Through Voltage.

In some cases, particularly in narrow base, high frequency transistors, the emitter-collector impedance breaks down for another reason before avalanche occurs. This type of breakdown is known as punch through. We have already discussed how the space charge region at the collector causes the base region to become narrower. As V_c

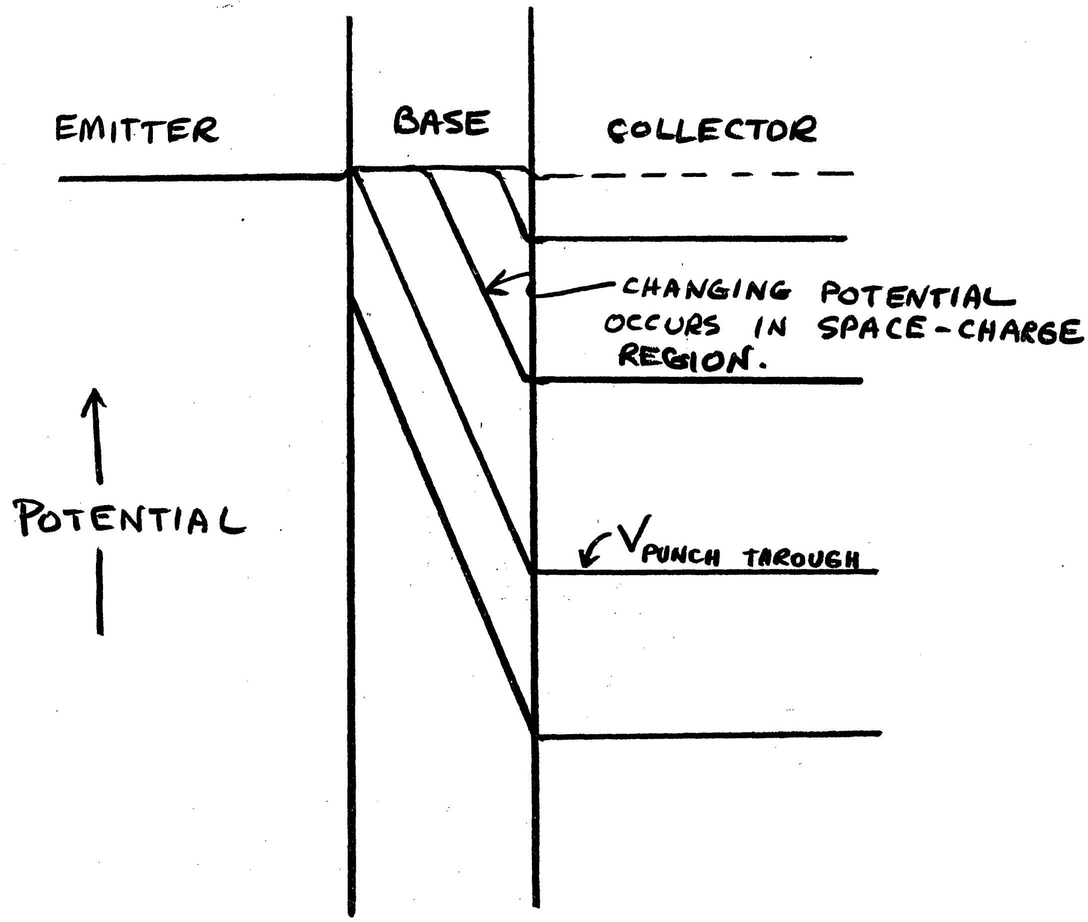

FIG. 8 - PUNCH-THROUGH VOLTAGE

increases the space charge region increases and base width decreases. Eventually a collector voltage is reached where the space charge region extends to the emitter. This is the punch-through voltage. Beyond this point the collector-emitter impedance is low and emitter voltage follows collector voltage.

7.0 Grounded-emitter r_{12} Characteristics.

These characteristics are shown in Fig. 9.

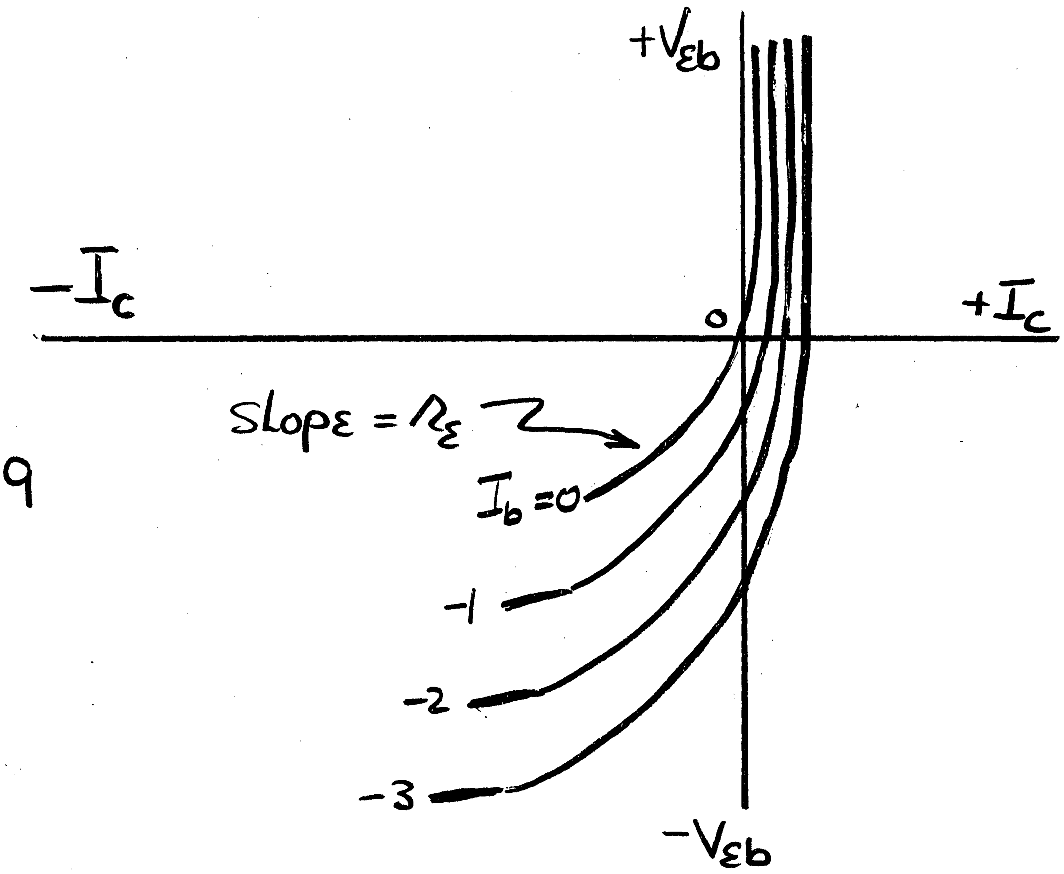

FIG. 9

The slopes of these curves give r_e.

$$\left|\frac{\partial V_b}{\partial I_c}\right|_{I_b} = r_e = \frac{kT}{qI_e} - (1-\alpha)h_{12}r_c.$$

In the saturation region where r_c is small the curves are approximately the theoretical $25/I_e$ ohms. In the active region the value is less.

The next part of this series will cover grounded-base, grounded-emitter, and grounded-collector amplifiers.

Signed: D. J. Eckl
D. J. Eckl

DJE/dg

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