

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE M 4 - 136

TITLE CLOSED EIGENVALUES - EIGENVECTORS

TYPE Closed

NUMBER OF WORDS 163

TEMPORARY STORAGE 0 through 19

DURATION $\sim 5n^3$ milliseconds per iteration. Number of iterations vary from 4 for $n = 3$ to 7 for $n = 23$

PRESET PARAMETERS S3: location of the symmetric matrix

PURPOSE To find the eigenvalues and eigenvectors of a symmetric matrix.

ACCURACY Depends upon conditioning of matrix. Usually 10 or 11 decimal places.

SCALING The sum of squares of the elements must be less than one-half.

METHOD OF USE The lower off-diagonal elements and diagonal elements should be stored consecutively beginning at location S3. This routine is called into use by

| | |
|-------|-------|
| p | x0 nF |
| | 50 pF |
| p + 1 | 26 -F |

where $x = J$ if only eigenvalues are desired
 $= 5$ if eigenvalues and eigenvectors are desired.
 $n =$ order of matrix.

RESULTS The eigenvalues replace the diagonal elements of the original matrix. The off diagonal elements of the original matrix are cleared to zero. The elements of the orthogonal matrix of eigenvectors are stored consecutively in locations beginning at $S3 + n(n + 1)/2$. Thus the matrix

$$\begin{matrix}
 a_{11} \\
 a_{21} \ a_{22} \\
 a_{31} \ a_{32} \ a_{33} \\
 \cdot \\
 \cdot \\
 \cdot \\
 a_{n1} \ a_{n2} \ a_{n3} \ \dots \ a_{nn}
 \end{matrix}$$

Becomes

$$\begin{matrix}
 e_1 \\
 0 \ e_2 \\
 0 \ 0 \ e_3 \\
 \cdot \\
 \cdot \\
 \cdot \\
 0 \ 0 \ 0 \ \dots \ e_n
 \end{matrix}$$

followed by

$$\begin{matrix}
 k_{11}, k_{12}, \dots, k_{1n} \\
 k_{21}, k_{22}, \dots, k_{2n} \\
 \cdot \\
 \cdot \\
 \cdot \\
 k_{n1}, k_{n2}, \dots, k_{nn}
 \end{matrix}$$

NOTES

- (1) If there is an arithmetical error ten F's will be punched out and then the computer will stop.
- (2) If the original matrix was scaled down by k then the eigenvalues will be scaled down by k.
- (3) The square root routine is at location 143L
- (4) This program is a modification of M3.

| | | | |
|---------------|----------------------|------|----------|
| DATE | 5/5/54 | Ret: | 10/13/55 |
| Programmed BY | <i>E. N. Roberts</i> | | |
| APPROVED BY | <i>J. Prash</i> | | |

| LOCATION | ORDER | | NOTES | PAGE 1 |
|----------|--------------------|-----------|-----------------------------------------|--------|
| 0 | K5 F 42 133L | | | |
| 1 | 46 153L 10 20F | | | |
| 2 | 42 153L 42 159L | | | |
| 3 | 40 F 50 153L | | | |
| 4 | S5 F 74 153L | | | |
| 5 | S5 F 10 1F | | $(n^2(n+1))/2$ | |
| 6 | L4 154L 42 158L | | | |
| 7 | 00 20F 46 158L | | | |
| 8 | L5 F 32 9L | | Test: Should only eigenvalues be found? | |
| 9 | 27 21L 41 F | from 8 | | |
| 10 | L5 158L 42 12L | from 20 | | |
| 11 | F5 12L 42 15L | | | |
| 12 | F5 160L 40 ()F | by 10 | Generate identity matrix | |
| 13 | F4 F 42 F | | | |
| 14 | L0 159L 32 20L | | | |
| 15 | 41 1F 41 ()F | by 11, 16 | From 18 | |
| 16 | F5 15L 42 15L | | | |

| LOCATION | ORDER | NOTES | PAGE 2 |
|----------|--------------------|----------|--------|
| 17 | F5 1F 40 1F | | |
| 18 | L0 159L 32 15L | | |
| 19 | L5 12L F4 159L | | |
| 20 | 22 10L L5-153L | From 14 | |
| 21 | L4 153L F4 160L | from 9 | |
| 22 | 40 155L 00 1F | | Waste |
| 23 | 41 18F 41 19F | | |
| 24 | 41 14F 41 15F | from 132 | |
| 25 | 41 4F 41 5F | from 112 | |
| 26 | L5 4F L4 152L | | |
| 27 | 40 4F L0 153L | | |
| 28 | 36 114L 50 5F | | |
| 29 | L5 5F 74 5F | | |
| 30 | 00 38F 42 157L | | |
| 31 | 00 20F 46 157L | | |
| 32 | L5 157L L4 154L | | |
| 33 | 46 95L L4 5F | | |

| LOCATION | ORDER | NOTES | PAGE 3 |
|----------|---------|-------|--------|
| 34 | 46 51L | | |
| | 42 107L | | |
| 35 | 56 108L | | |
| | L5 4F | | |
| 36 | 50 4F | | |
| | 74 4F | | |
| 37 | 00 38F | | |
| | 42 157L | | |
| 38 | 00 20F | | |
| | 46 157L | | |
| 39 | L5 157L | | |
| | L4 154L | | |
| 40 | 42 95L | | |
| | L4 4F | | |
| 41 | 46 50L | | |
| | 42 109L | | |
| 42 | L0 4F | | |
| | L4 5F | | |
| 43 | 46 44L | | |
| | 46 110L | | |
| 44 | L5 ()F | by 43 | |
| | 40 6F | | |
| 45 | L5 15F | | |
| | 50 6F | | |
| 46 | 74 6F | | |
| | L4 14F | | |
| 47 | 40 14F | | |
| | S5 F | | |
| 48 | 40 15F | | |
| | L5 6F | | |
| 49 | 00 1F | | |
| | 40 6F | | |
| 50 | L5 ()F | By 41 | |
| | 40 8F | | |

| LOCATION | ORDER | NOTES | PAGE 4 |
|----------|-------------------|----------|--------|
| 51 | L5 ()F 40 7F | by 34 | |
| 52 | L0 8F 40 9F | | |
| 53 | L7 6F L2 9F | | |
| 54 | 36 136L L5 6F | | |
| 55 | 66 9F S5 F | | |
| 56 | 10 1F 40 3F | | |
| 57 | 19 1F 26 61L | | |
| 58 | L5 9F 10 1F | from 137 | |
| 59 | 66 6F S5 F | | |
| 60 | 40 3F 10 1F | from 137 | |
| 61 | 40 11F 50 3F | from 57 | |
| 62 | 7J 3F L4 156L | | |
| 63 | 22 63L 50 63L | | |
| 64 | 26 143L 40 12F | | |
| 65 | L3 11F 66 12F | | |
| 66 | S1 F 40 10F | | |
| 67 | 26 138L 50 67L | from 140 | |
| 68 | 26 143L 40 2F | | |

| LOCATION | ORDER | NOTES | PAGE 5 |
|----------|---------|----------|--------|
| 69 | L5 11F | from 142 | |
| | L0 156L | | |
| 70 | 32 74L | | |
| | L5 11F | | |
| 71 | 32 72L | | |
| | L1 156L | | |
| 72 | 26 73L | | |
| | 19 1F | From 71 | |
| 73 | 66 12F | from 72 | |
| | S5 F | | |
| 74 | 26 76L | | |
| | 50 3F | from 70 | |
| 75 | 75 10F | | |
| | 00 1F | | |
| 76 | 40 12F | from 74 | |
| | 66 2F | | |
| 77 | S5 F | | |
| | 40 9F | | |
| 78 | 41 3F | | |
| | 23 87L | | |
| 79 | L5 3F | from 100 | |
| | L0 153L | | |
| 80 | 32 113L | | |
| | L4 153L | from 113 | |
| 81 | 40 11F | | |
| | L5 5F | | |
| 82 | L0 3F | | |
| | 36 85L | | |
| 83 | L5 4F | | |
| | L0 3F | | |
| 84 | 36 86L | | |
| | 26 87L | | |
| 85 | L5 152L | from 82 | |
| | 46 11F | | |
| 86 | L5 152L | from 84 | |
| | 42 11F | | |

| LOCATION | ORDER | NOTES | PAGE 6 |
|----------|---------|-----------|--------|
| 87 | L5 11F | from 84 | |
| | L4 95L | from 78 | |
| 88 | 40 95L | | |
| | 46 91L | | |
| 89 | 46 92L | | |
| | 42 93L | | |
| 90 | 42 97L | | |
| | 50 2F | | |
| 91 | 7J ()F | by 88 | |
| | 40 1F | | |
| 92 | 50 ()F | By 89 | |
| | 7J 9F | | |
| 93 | 40 F | | |
| | 50 ()F | By 89 | |
| 94 | 7J 9F | | |
| | L4 1F | | |
| 95 | 50 ()F | By 33, 88 | |
| | 50 ()F | By 40, 88 | |
| 96 | 7J 2F | | |
| | L0 F | | |
| 97 | 32 97L | | |
| | 50 ()F | by 90 | |
| 98 | L5 3F | | |
| | L4 152L | | |
| 99 | 40 3F | | |
| | L0 155L | | |
| 100 | 36 79L | | |
| | LJ 10F | | |
| 101 | 40 F | | |
| | L9 10F | | |
| 102 | 40 1F | | |
| | 50 6F | | |
| 103 | 7J 12F | | |
| | 40 11F | | |

M4

| LOCATION | ORDER | NOTES | PAGE 7 |
|----------|--------------------|---------------------------|--------|
| 104 | 50 7F 7J F | | |
| 105 | L4 11F 40 F | | |
| 106 | 50 8F 7J 1F | | |
| 107 | L4 F 40 ()F | by 34 | |
| 108 | L1 ()F L4 7F | by 35 | |
| 109 | L4 8F 40 ()F | by 41 | |
| 110 | 41 ()F L5 5F | by 43 | |
| 111 | L4 152L 40 5F | | |
| 112 | L0 4F 32 25L | | |
| 113 | 22 28L 23 80L | from 80 | |
| 114 | 41 16F 41 17F | from 28 | |
| 115 | L5 154L 42 117L | | |
| 116 | 46 117L L5 17F | from 122 | |
| 117 | 50 ()F 74 ()F | by 116,121 by 115, 121 | |
| 118 | L4 16F 40 16F | | |
| 119 | S5 F 40 17F | | |
| 120 | L5 117L L4 152L | | |

| LOCATION | ORDER | NOTES | PAGE 8 |
|----------|--------------------|--------------------------|--------|
| 121 | 40 117L L0 158L | | |
| 122 | 32 116L L5 19F | | |
| 123 | L0 17F 10 39F | | |
| 124 | L4 18F L0 16F | | |
| 125 | 32 132L 40 1F | | |
| 126 | L5 134L 26 135L | by 133,135 by 133,135 | |
| 127 | 19 34F L2 F | | |
| 128 | 36 130L 92 904F | | |
| 129 | OF F OF F | | |
| 130 | L5 16F 40 18F | from 135 | |
| 131 | L5 17F 40 19F | | |
| 132 | 26 24L L5 162L | from 125 | |
| 133 | 40 126L 22 ()F | by 0 | |
| 134 | L4 14F 40 F | | |
| 135 | 40 126L 26 130L | | |
| 136 | L3 6F 32 137L | from 54 | |
| 137 | 26 58L 27 60L | from 136 | |

| LOCATION | ORDER | NOTES | PAGE 9 |
|----------|--------------------|--------------------|--------|
| 138 | L5 10F L0 161L | from 67 | |
| 139 | 32 140L LJ 10F | | |
| 140 | 22 67L L5 161L | from 139 | |
| 141 | 40 10F L5 160L | | |
| 142 | 40 2F 26 69L | | |
| 143 | 40 1F K5 F | from 64 | |
| 144 | 42 151L 51 1F | | |
| 145 | 10 1F SJ F | | |
| 146 | 40 2F 50 F | from 150 | |
| 147 | L5 1F 66 2F | | |
| 148 | S5 F L0 2F | | |
| 149 | 10 1F 36 151L | | |
| 150 | L4 2F 26 146L | | |
| 151 | L5 2F 22 ()F | from 149 by 144 | |
| 152 | 00 1F 00 1F | | |
| 153 | 00 ()F 00 ()F | by 1 by 1 | |
| 154 | 00 S3 00 S3 | | |
| 155 | 80 ()F 00 ()F | by 22 by 22 | |

| LOCATION | ORDER | NOTES | PAGE 10 |
|----------|----------|----------|---------|
| 156 | 20 F | | |
| | 00 F | | |
| 157 | 00 F | | |
| | 00 ()F | by 30,37 | |
| 158 | J0 ()F | by 7 | |
| | 74 ()F | by 6 | |
| 159 | 80 F | | |
| | 00 ()F | by 2 | |
| 160 | 7L 4095F | | |
| | LL 4095F | | |
| 161 | 3L 4095F | | |
| | LL 4095F | | |
| 162 | L5 134L | | |
| | 26 135L | | |

M4

GHG/mge
10/13/55