UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE K 7 - 188

TITLE — Iterative Estimation of Communalities: Principal Axes Method of Factor Analysis

TYPE — Complete program

DURATION — Dependent on number of iterations and order of matrix. Each iteration takes one minute for order 11 and approximately five minutes for order 18 (the largest order matrix for which this program can be used).

DESCRIPTION — The code makes a principal axes factoring of the given matrix, re-estimates the communalities, substitutes the new estimates for the old, and prepares for another principal axes factoring. The latent roots and new communality estimates are punched during each iteration. The i^{th} communality estimates are

$$ {}^{(i)}h_j^2 = \sum_k^r {}^{(i)}f_{jk}^2 \qquad j = 1, \ldots, p $$

where p is the order of the matrix, r if the hypothesized rank, and the factors ${}^{(i)}\underset{\sim}{f}_j$ of the i^{th} iteration have been ordered in terms of their contribution to variance.

METHOD OF USE

1. The program tape is input until it stops.
2. The problem tape is input on the black switch.
3. The program tape is placed in the reader just after the word last read by the computer. Start with black switch to Stop Disable. Iteration will begin. When it is desired to end run, raise the black switch to Obey. The computer will stop on a "24 OF9" order at the end of the iteration it is going through. Now start with the white switch. The lower off-diagonal and diagonal elements of the correlation matrix will be punched by rows with the last communality estimates as diagonal entries. This latter output will simplify preparation of the problem tape that will be required for further iteration. Program ends on an "OF" order.

PREPARATION OF TAPES

The program tape can be copied from the master tape. The problem tape has the parameters followed by two copies of the lower off-diagonal and diagonal elements of the matrix punched by rows. The character "N" is punched after the last element of each copy. The elements of the matrix are punched in the form required for Library Routine N 3.

The parameters at the beginning of the problem tape are punched as follows:

d J p F r L

where d is the number of digits desired in the printed latent roots and communality estimates, p is the order of the matrix, and r is the rank of the matrix for which communality estimates are wanted.

COMMENTS

The squared multiple correlations may be suggested as initial communality estimates.[1]

The elements of the matrix should be multiplied (scaled) by one-tenth or by one-hundredth so that the sum of squares over the entire matrix is less than one-half.

The latent roots and communality estimates will be scaled by the same constant as the elements of the matrix (either one-tenth or one-hundredth).

EXAMPLE

We would like communality estimates for a set of four variables for which we have the following correlation matrix:

$$\begin{bmatrix} 1.00 & .73 & .44 & .25 \\ .73 & 1.00 & .49 & .36 \\ .44 & .49 & 1.00 & .37 \\ .25 & .36 & .37 & 1.00 \end{bmatrix}$$

We decide to retain two factors. We scale the elements of the matrix by one-tenth since the sum of squares for

[1] The formula is

$$h_i^2 = \frac{R^{ii} - 1}{R^{ii}},$$ where R^{ij} is the element in the i^{th} row and j^{th} column of the inverse of the correlation matrix; the inverse can be obtained with Library Routine M 2.

the matrix will then be less than one-half. Because we want communalities to four decimal places we must print five places since all answers will be scaled by the constant, one-tenth.

The problem tape is punched as follows:

5	results printed to five places
J	
4	matrix is of order four
F	
2	assumed rank is two
L	

```
+0543
+ 073      +0585
+ 044      + 049      +0298
+ 025      + 036      + 037     +0181N
spaces
+0543
+ 073      +0585
+ 044      + 049      +0298
+ 025      + 036      + 037     +0181N
```

The results for the first two iterations will be punched as:

```
-01952     +18103     -01352    +01271
+05882     +06775     +03555    +01892

-91459     +18721     +01615    -00774
+05993     +07231     +03650    +01847
```

DATE July 14, 1955
CODED BY R. A. Twery
APPROVED BY D. E. Muller

LOCATION	ORDER		NOTES	PAGE 1
	Routine X 1 - 18		Decimal Order Input	
	00 165K			
0	00 F			
	00 285F		First storage location	
	00 171K			
0	80 F			Constants
	00 F		(-1)	
1	00 1F			
	L5 ( )F	by 50F	Test word	
	00 100K			
	Routine N 3 - 23		Decimal Number Input	
	00 20K		Input routine	
0	41 2F			
	41 F	from 8L		
1	81 4F	from 5L		
	L0 28L			
2	32 5L			
	L4 28L			
3	50 F			
	74 28L			
4	S5 F		Input independent parameters	
	40 F			
5	26 1L			
	42 6L	from 26		
6	L5 F			
	40 (00)F	by 5L		
7	F5 2F			
	40 2F			
8	L0 29L			
	32 0L			
9	50 4F			
	75 4F			
10	S5 F			
	L4 4F			
11	10 1F			
	L4 165F			

LOCATION	ORDER		NOTES PAGE 2
12	40 7F		
	26 30L		
13	L5 4F	from 31L	
	40 162F		
14	40 13F		
	L0 5F		
15	40 166F		
	L5 5F		
16	40 164F		
	L5 3F		
17	40 161F		Form and store dependent parameters
	00 20F		
18	46 218F		
	46 226F		
19	50 4F		
	75 4F		
20	S5 F		
	L4 163F		
21	40 167F		
	36 42F	Waste	
22	00 20F		
	46 25L		
23	L5 4F		
	00 1F		
24	40 5F		
	92 567F		Punch delay characters
25	40 ( )F	from 92F by 22L	
	50 25L		Input correlation matrix
26	26 100F		
	26 80F		
27	00 F		
	00 F		
28	00 F		
	00 10F		

LOCATION	ORDER		NOTES PAGE 3
29	80 F		
	00 3F		Constants for input routine
30	42 172F	from 12L	
	40 163F		
31	26 13L		
	00 F		
	00 258K		
	Routine P 2 - 52	from 227F from 219F from 276F	Print (A) with or without Sign to n Places
	00 80K		Check input of correlation matrix
0	L5 167F	from 46F	
	42 2L		
1	L0 165F		
	L4 163F		
2	22 12L		
	L5 (00)F	from 13L, 6L by 0L, 4L	
3	L4 59F		Form sum of matrix just input
	40 59F		
4	F5 2L		
	42 2L		
5	L0 14L		
	32 6L		
6	22 2L		
	L5 59F	from 5L	
7	L0 60F		
	40 F		Compare sums from the two correlation matrices
8	L3 F		
	34 999F		Return to input if sums do not agree
9	L5 59F		
	40 60F		
10	41 59F		
	92 451F		Print stroke character
11	L5 15L		Prepare to input correlation matrix once again
	40 11L		
12	26 45F		
	42 10F	from 2L	

LOCATION	ORDER		NOTES PAGE 4
13	42 14L		Addition to comparison sub-code
	22 2L		
14	22 12L		
	L5 (00)F	by 13L	Constants
15	0F F		
	26 45F		
	00 233K		Transfer correlations
0	L5 163F	from 276F	
	42 9L		
1	L5 165F		
	42 4L		Plant addresses
2	L5 167F		
	00 20F		
3	46 4L		
	36 4L	Waste	
4	L5 ( )F	from 8L, by 3L	
	40 ( )F	by 1L, 6L	
5	19 18F		
	F4 4L		
6	40 4L		Transfer correlation
	46 9L		
7	L0 9L		
	36 243F		
8	26 4L		
	36 9L	Waste	
9	L5 ( )F	by 6L	Test word
	40 ( )F	by 0L	
	00 243K		
0	L5 163F	from 230F	
	42 5L		
1	42 13L		
	41 169F		
2	41 168F	from 12L	
	L5 168F	from 9L	
3	L0 169F		
	40 170F		

LOCATION	ORDER	NOTES	PAGE 5
4	L3 170F		
	32 12L		
5	32 5L	Waste	
	41 ( )F	by 0L, 6L	
6	F5 5L	from 14L	
	42 5L		
7	42 13L		Generate identity matrix
	F5 168F		
8	40 168F		
	L0 162F		
9	36 10L		
	22 2L		
10	F5 169F	from 9L	
	40 169F		
11	L0 162F		
	36 21F		
12	26 2L		
	L5 171F	from 4L	
13	50 1F		
	40 ( )F	by 1L, 6L	
14	26 6L		
	00 F		
	00 222K		Put together and print latent roots
0	L5 165F		
	42 3L		
1	00 20F		Prepare addresses
	46 3L		
2	41 168F		
	92 135F		Punch two line-feed characters
3	L5 ( )F	from 10L by 6L, 9L	Transfer a latent root
	40 ( )F	by 0L, 10L	
4	50 (8)F	by 38F	
	50 4L		Print the latent root
5	26 258F		
	F5 168F		

LOCATION	ORDER		NOTES PAGE 6
6	40 168F		
	L0 162F		
7	36 175F		
	F5 168F		
8	00 20F		Increase counters and test for end
	L4 3L		
9	46 3L		
	F5 3L		
10	42 3L		
	26 3L		
	00 175K		Set (p-k) smallest roots equal to zero
0	41 168F	from 229F	
	41 173F		Clear counters
1	41 169F		
	L5 165F	from 13L	
2	L4 169F		
	42 6L		Plant address of next root
3	42 7L		
	42 9L		
4	42 16L		
	50 1023F		
5	L5 165F		
	L4 173F		Plant address of smallest root so far
6	42 8L		
	L3 ( )F	by 2L	
7	32 11L		
	L1 ( )F	by 3L	
8	32 15L		
	L4 ( )F	by 6L	Compare current root
9	36 14L		
	L4 ( )F	by 3L	
10	40 0F		
	L3 0F		
11	36 14L		
	F5 169F	from 7L, 15L 17L	

LOCATION	ORDER		NOTES PAGE 7
12	42 169F		Increase and test counter
	L0 162F		
13	32 17L		
	22 1L		Go back to finish a round of comparing
14	L5 169F	from 9L, 11L	Current root is smallest root, so far this round
	42 173F		
15	22 11L		
	L5 168F	from 8L	
16	42 168F		
	41 ( )F	by 4L	
17	22 11L		
	L5 165F	from 13L	
18	L4 173F		
	42 19L		One of the (p-k) smallest roots has been found
19	92 1F		
	41 ( )F	by 18L	
20	F5 168F		
	42 168F		
21	L0 166F		
	32 22L		
	00 197K		Estimate, transfer, and print communalities
0	22 175F		
	41 168F	from 196F	
1	92 131F		Punch one line-feed character
	19 38F		2^{-39} to 170F
2	40 170F		
	L5 167F		
3	L0 170F		Pre-set an address
	42 20L		
4	L5 20L	from 24L	
	F4 168F		
5	42 20L		
	41 170F		
6	41 169F		
	L5 165F		

LOCATION	ORDER		NOTES PAGE 8
7	42 12L		Set addresses
	50 162F		
8	75 168F		
	L5 163F		
9	S4 F		
	42 10L		
10	42 11L		
	50 (00)F	by 9L, 16L from 19L	
11	32 11L	Waste	
	7J (00)F	by 10, 16L	Form a squared factor loading and add to the sum across a row of the factor matrix
12	40 1023F		
	50 (00)F	by 7L, 15L	
13	7J 1023F		
	L4 170F		
14	40 170F		
	F5 12L		
15	42 12L		
	F5 10L		
16	42 10L		
	42 11L		Prepare to finish a communality estimate
17	F5 169F		
	40 169F		
18	L0 162F		
	32 19L		
19	22 10L		
	00 1F	from 18L	
20	L5 170F		
	40 (00)F	by 3L, 5L	
21	50 (8)F	by 38F	Print and store the communality estimate
	50 21L		
22	26 258F		
	F5 168F		
23	40 168F		Test for end
	L0 162F		

LOCATION	ORDER		NOTES PAGE 9
24	36 276F		
	26 4L		Return to estimate next communality
	00 800K		
	Routine X 2 - 108		Shifting Sum Check
	F68NL6S695		Sum check number
	24 20N		
	Problem tape is inserted here and read; the Program tape is then re-inserted at this point.		
	00 276K		
0	92 41F	from 221F	Punch delay characters
	24 233F		Stop before starting new iteration
1	L5 167F		
	42 2L		
2	92 191F		
	L5 (00)F	from 6L, by 1L, 5L	
3	50 7F		
	50 3L		
4	26 258F		
	F5 2L		Print correlation matrix with last
5	42 2L		communality estimates
	L0 8L		
6	36 7L		
	22 2L		
7	92 770F	from 6L	
	0F F		
8	92 191F		
	L5 SK		
	00 20K		
	Routine M 4 - 136		Closed Eigenvalues and Eigenvectors
	00 152K		
	Routine R 1 - 116		Square Root Routine
	26 233N		